Computer Vision Group
Institut für Informatik
Technische Universität München

Weekly Exercises 11

Room: 02.09.023

Tuesday, 19.01.2016, 14:15-15:45

Submission deadline: Tuesday, 26.01.2016, 11:15 , Room 02.09.023

Fast Trust-Region (6 Points)

Exercise 1 (Bhattacharyya Distance, 3 Points)**.** Let $K = \{1, \dots, k\}, k \in \mathbb{N}$ and let $\mathcal{D} = \{p : K \to \mathbb{R}_{\geq 0} | \sum_i p(i) = 1\}$ be the set of all probability distributions on K. Define the *Bhattacharyya distance* $D : \mathcal{D} \times \mathcal{D} \to \mathbb{R}$ by

$$D(p, q) = -\log \left(\sum_i \sqrt{p(i)q(i)} \right).$$

1. Show that the Bhattacharyya distance D is symmetric and positive.
2. Show that $D(p, q) = 0 \Leftrightarrow p = q$.
3. Give an example where the triangle inequality does not hold for D.

For the next exercise we need the *isoperimetric inequality*:

Theorem (Isoperimetric Inequality)**.** Consider a closed curve $\alpha \subset \mathbb{R}^2$. Let $\mathcal{L}$ be the length of α and let $\mathcal{A}$ be the area of the region enclosed by α. It holds:

$$4\pi\mathcal{A} \leq \mathcal{L}^2.$$

Equality holds if and only if α is a circle.

Exercise 2 (3 Points)**.** Consider a minimizer $S^* \subset \mathbb{R}^2$ of the minimization problem

$$\min_{S \subset \mathbb{R}^2} \left(\int_S 1 dx - V_0 \right)^2 + \mathcal{L}(S),$$

where $V_0 \in \mathbb{R}_{>0}$ and $\mathcal{L}(S)$ is the length of the boundary of S.

1. Show that S^* is a disk.
2. Show that the radius r^* of the disk S^* satisfies

$$r^* = \arg\min_{r \geq 0} \left(\pi r^2 - V_0\right)^2 + 2\pi r.$$

3. Find the optimal radius for $V_0 = 1000$. *Hint:* Compute the derivative and use your favourite software (e.g. Matlab or Wolfram α) to find the roots of the resulting third-degree polynomial.

Programming (10 Points)

Exercise 3 (Fast Trust Region Method, 10 Points)**.** In this exercise we apply the *fast trust region method* to find the minimizer of energy function

$$E(x) = (\langle \mathbf{1}, x \rangle - V_0)^2 + \mathcal{L}(x),$$

where $x \in \{0, 1\}^n$ are binary variables describing a segmentation of a pixel grid. $V_0 = 1000$ and $\mathcal{L}(x)$ is the boundary length of the segmentation.