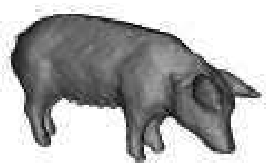
Combinatorial Optimization in Computer Vision (IN2245)

Frank R. Schmidt
Csaba Domokos

Winter Semester 2015/2016

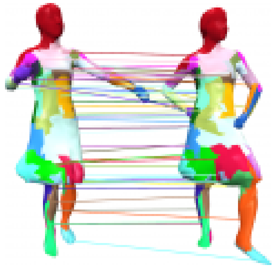
Computer Vision

We are always looking for master and bachelor students!



3D Reconstruction

Optical Flow



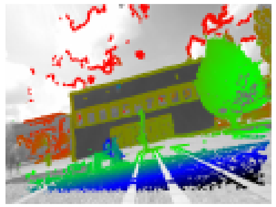
Shape Analysis

Robot Vision

RGB-D Vision

Image Segmentation

Convex Relaxation



Visual SLAM

Please fill out the application form: `https://vision.in.tum.de/application`

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Schedule 4 / 37

IN2245

The lecture **Combinatorial Optimization in Computer Vision** will be organized as following:

- **Tuesday Lecture:** 10-11 and 11-12 in Room 02.09.023
- **Wednesday Lecture:** 14-15 and 15-16 in Room 02.09.023
- **Tuesday Tutorial:** 14-16 in Room 02.05.014

The tutorial combines theoretical and programming assignments:

- **Assignment Distribution:** Wednesday 15:00-15:15 in Room 02.09.023
- **Theoretical Assignment Due:** Tuesday 11:00-11:15 in Room 02.09.023
- **Assignment Presentation:** Tuesday 14-16 in Room 02.05.014

October

October 2015						
Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20 Lecture 1	21 Lecture 2	22	23	24	25
26	27 Lecture 3 Tutorial 1	28 Lecture 4	29	30	31	

November

November 2015						
Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
						1
2	3 Lecture 5 Tutorial 2	4 Lecture 6	5	6	7	8
9	10 Lecture 7 Tutorial 3	11 Lecture 8	12	13	14	15
16	17 Lecture 9 Tutorial 4	18 Lecture 10	19	20	21	22
23	24 Lecture 11 Tutorial 5	25 Lecture 12	26	27	28	29
30						

December

December 2015						
Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
	1 Lecture 13 Tutorial 6	2 Lecture 14	3	4	5	6
7	8 Lecture 15 Tutorial 7	9 Lecture 16	10	11	12	13
14	15 Lecture 17 Tutorial 8	16 Lecture 18	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

January

January 2016						
Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
				1	2	3
4	5	6	7	8	9	10
11	12 Lecture 19 Tutorial 9	13 Lecture 20	14	15	16	17
18	19 Lecture 21 Tutorial 10	20 Lecture 22	21	22	23	24
25	26 Lecture 23 Tutorial 11	27 Lecture 24	28	29	30	31

February

February 2016						
Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1	2 Lecture 25 Tutorial 12	3 Lecture 26	4	5	6	7
8	9	10	11	12	13	14
15	16	17 Week of the Exam	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Exams

Requirements for being admitted to the exam:

- **Registration:** Students need to be registered prior to the exam:
  November, 10^{th} – January, 15^{th} via TUM online.
- **Exam:** In the week of February, 15^{th} – 19^{th}.

Participation at the tutorial:

- **Not mandatory, but highly recommended:**
  Theoretical assignments will help to understand the topics of the lecture.
  Programming assignments will help to apply the theory to practical computer vision problems.
- **Bonus:** Active students who solve 60% of the assignments earn a bonus.
- **Exam:** If one receives a mark between 1.3 and 4.0 in the exam, the mark will be improved by 0.3 resp. 0.4. Marks of 1.0 or 5.0 cannot be improved.

Bonus

To achieve the bonus, the following requirements have to be fulfilled:

Theory

- 60% of all theoretical assignments have to be solved.
  (Submissions happen only Tuesdays from 11:00-11:15)
- At least one theoretical exercise has to be presented in front of the class.

Programming

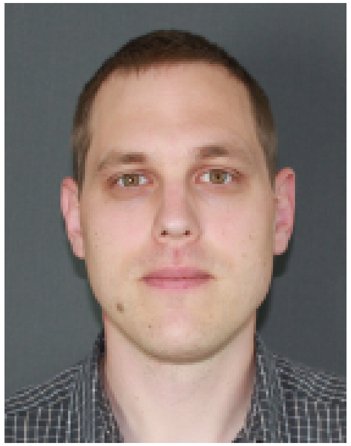
- 60% of all programming assignments have to be presented during the tutorial.
- At least one programming exercise has to be explained to one of the TAs.

To promote team work, form groups of **two** or **three** students in order to solve and submit the assignments.

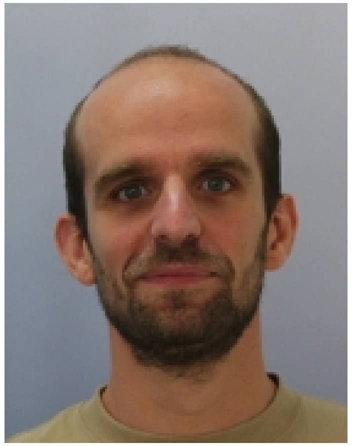
Team

Lecturers | **Teaching Assistants**

Dr. Frank R. Schmidt



Dr. Csaba Domokos



Thomas Windheuser

Thomas Möllenhoff

Please do not hesitate to contact us in order to set up an appointment:

- f.schmidt@in.tum.de
- csaba.domokos@in.tum.de
- thomas.windheuser@in.tum.de
- thomas.moellenhoff@in.tum.de

Course Material

Course Page

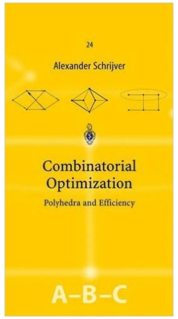
On the internal site of the course page you have access to extra course material. `https://vision.in.tum.de/teaching/ws2015/cocv_2245`

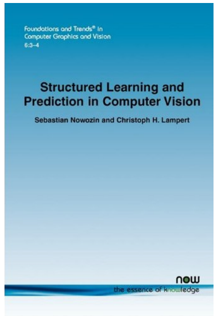
Password `PBO+MRF`

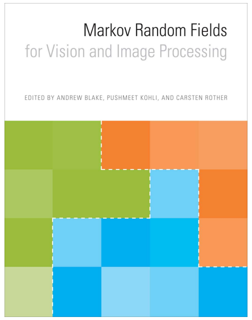
- Printer friendly slides for each lecture (Available prior to the lecture)
- Assignment Sheets (Available after the Wednesday lecture)
- Solution Sheets (Available after the Tuesday tutorial)

The course page will also be used for extra announcements.

Literature







- Alexander Schrijver, *Combinatorial Optimization*, ISBN 978-3-540-44389-6.
  3 Volumes, 83 Chapters, 1882 pages, over 4000 references.
- Sebastian Nowozin, Christoph Lampert, *Structured Learning and Prediction in Computer Vision*, ISBN 978-1-601-98456-2.
- Andrew Blake, Pushmeet Kohli, Carsten Rother, *Markov Random Fields for Vision and Image Processing*, ISBN 978-0-262-01577-6.

In addition, we will mention current conference and journal papers.

Topics of the Lecture

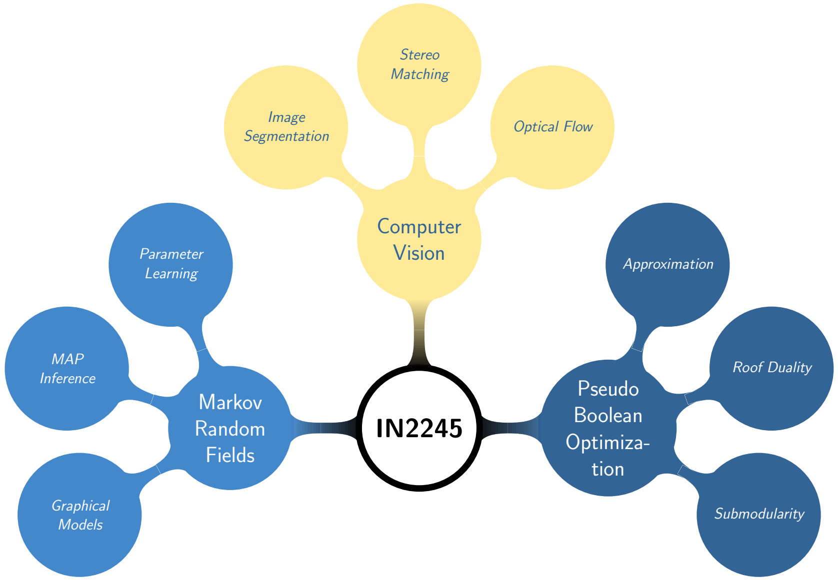
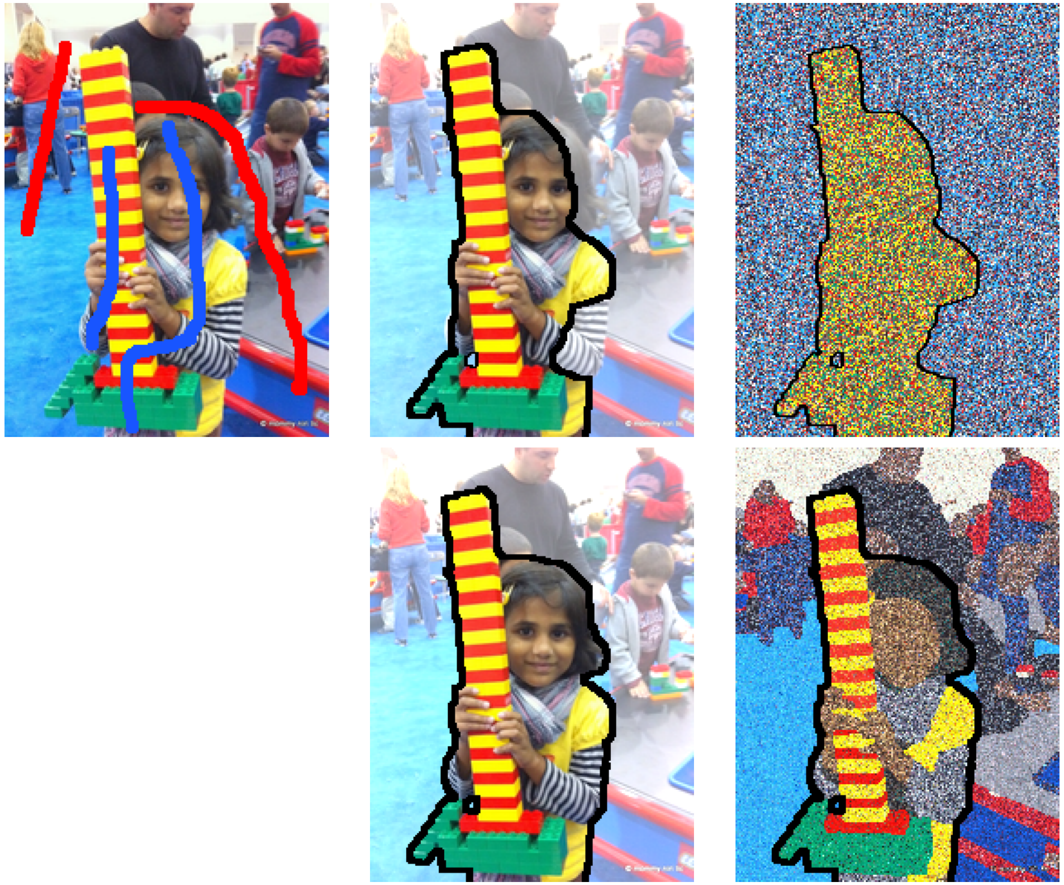
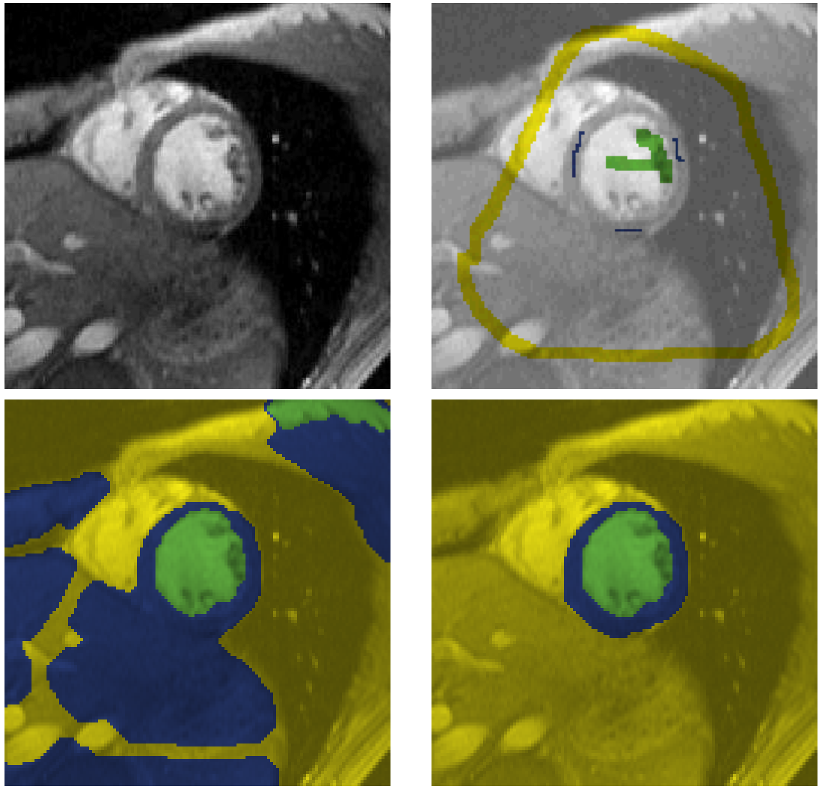


Image Segmentation

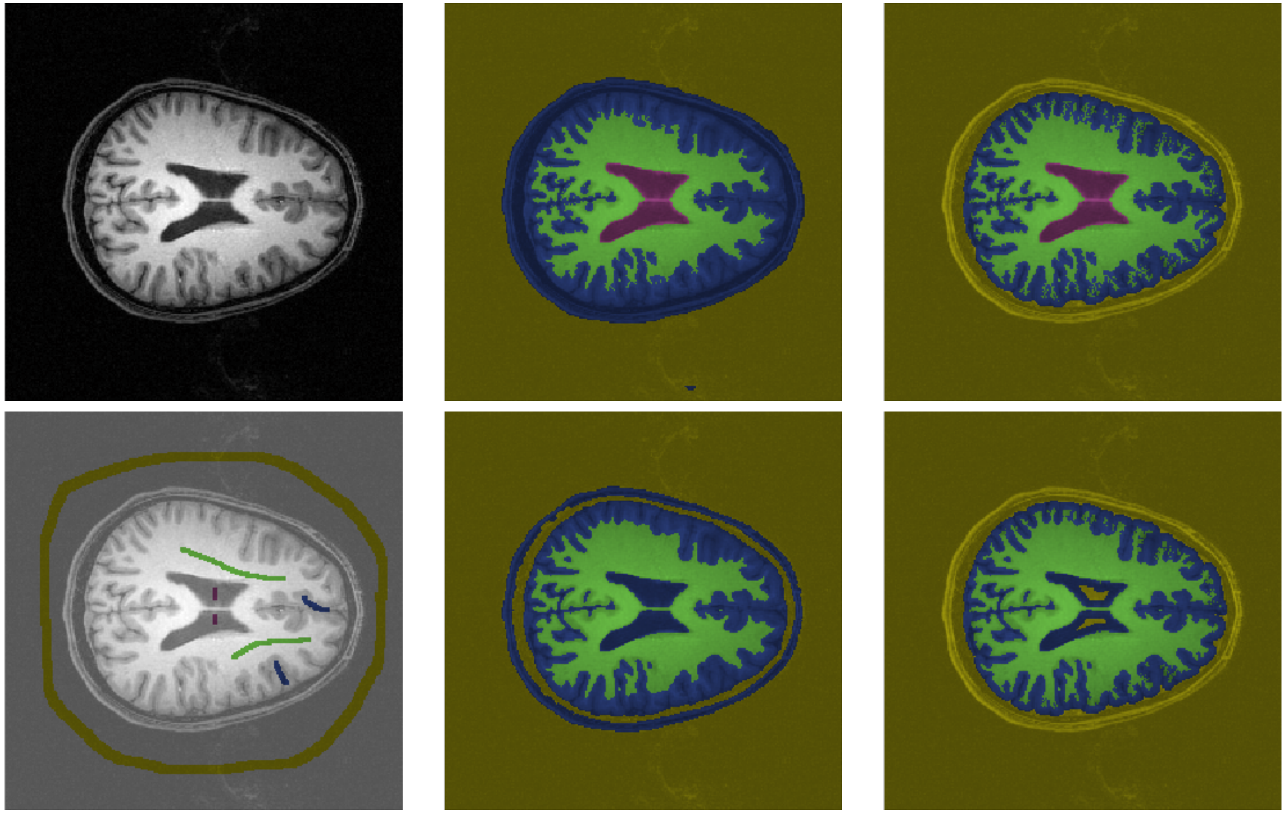


Medical Image Segmentation (2D)



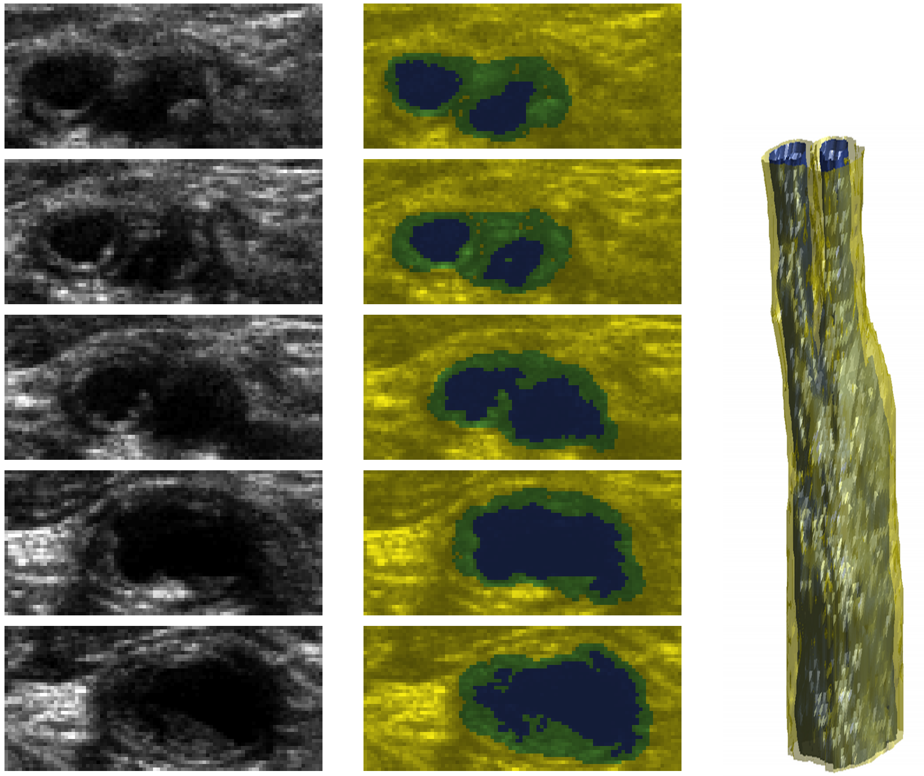
Minimal Distance Constraint   Maximal Distance Constraint

Medical Image Segmentation (2D)



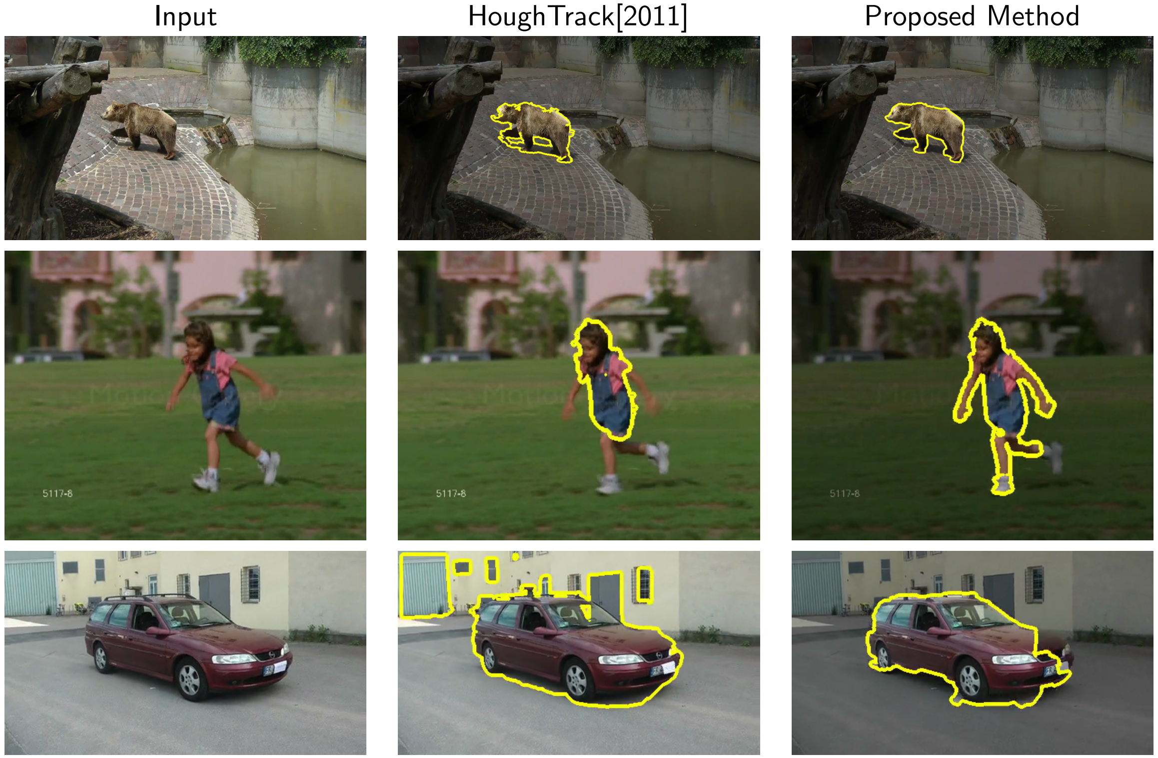
Input Minimal Distance Maximal Distance

Medical Image Segmentation (3D)



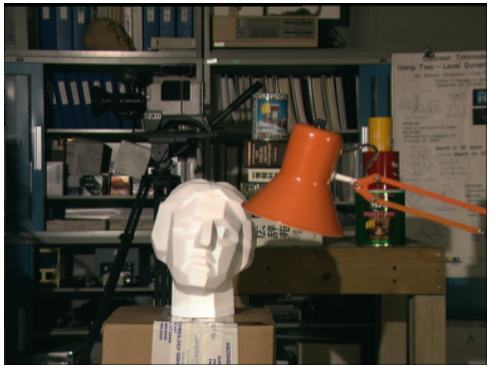
Images 2D-Segmentation 3D-Segmentation

Video Segmentation



Stereo Matching

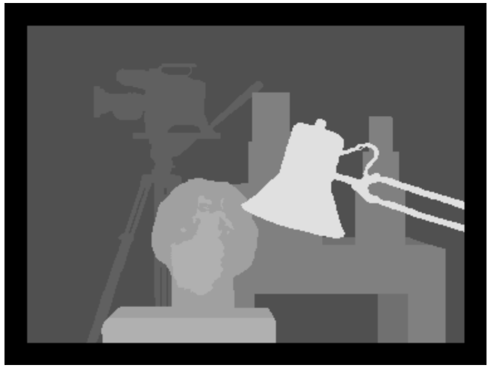
Left Image



Right Image

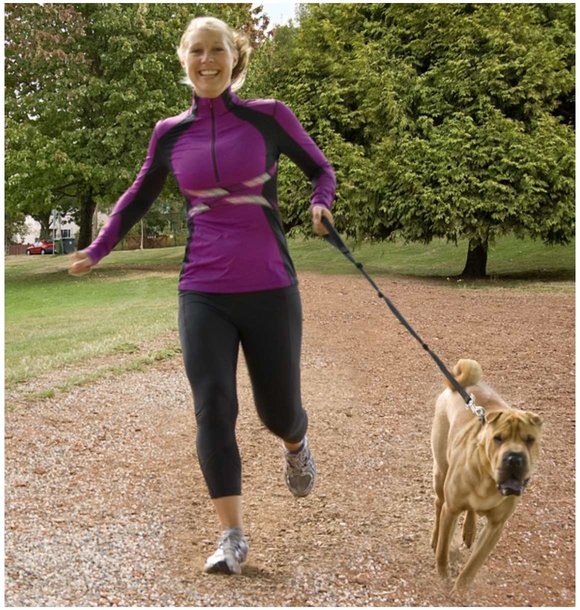
Combinatorial Optimization

Ground Truth

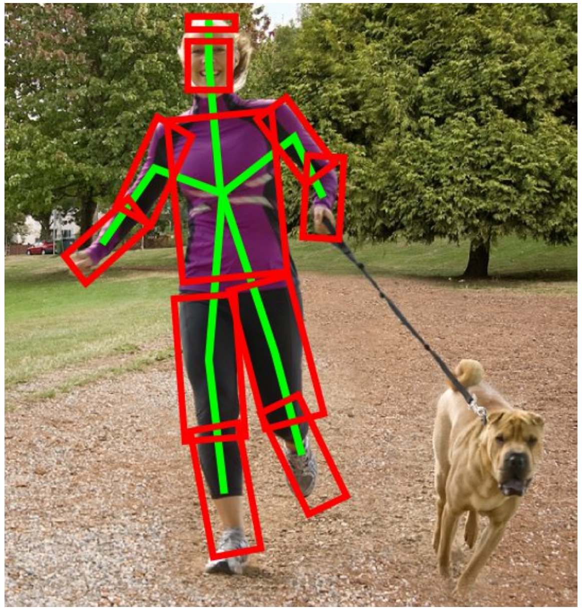


Human Pose Estimation

Input Image



Human Pose



Naïve Set Theory

The concepts of sets was first introduced by Georg Cantor, starting with
Cantor, Ueber eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen, J. reine und angewandte Mathematik, 1874

Some of the powerful tools of Cantor's set theory was the concept of the *cardinality* as well as the *powerset* $\mathcal{P}(\Omega)$, which is the set of all subsets of a given set Ω.

Russell presented in 1901 the paradox that the existence of the *set of all sets* leads to a contradiction. In 1899 Cantor obtained a similar paradox when he studied the *cardinality of the set of all sets*.

This led to different axiomatic set theories. The most popular is the set theory according to Zermelo (1908) and Fraenkel (1922), referred to as ZFC.

ZFC

Essentially the axioms of ZFC can be summarized in the following sense:

1. Two sets A, B are equal iff $x \in A \Leftrightarrow x \in B$.
2. There is *an empty set* (denoted as $\emptyset$) for which $x \in \emptyset$ is always false.
3. For x, y, there is a set (denoted as $\{x, y\}$) that only contains x and y.
4. For set $\mathcal{A}$, there exists a set (denoted as $\cup\mathcal{A}$) that contains the elements of $\mathcal{A}$'s elements. $[A \cup B := \cup\{A, B\}]$
5. There exists a set N that contains $\emptyset$ and for each set $A \in N$ it also contains its succesor $A' := A \cup \{A\}$. $[0 := \emptyset$, $1 := 0'$, etc.]
6. For set A, there exists the *powerset* $\mathcal{P}(A)$ that contains all subsets of A.
7. For set $A \neq \emptyset$, there exists $a \in A$ with $a \cap A = \emptyset$.
8. For set A and predicate $F(\cdot)$, there exists the set $\{x \in A | F(x)\}$.
9. For set A and predicate $F(\cdot, \cdot)$, there exists the set $\{y | \exists x \in A : F(x, y)\}$.
10. **If $\mathcal{A}$ is a set of non-empty sets, then there is a function $F\colon \mathcal{A} \to \cup\mathcal{A}$ such that for all $A \in \mathcal{A} : F(A) \in A$.**

Ordered Sets

Given a set Ω, a relationship $\preceq$ is called **partial order** if

$x \preceq x$	for all $x \in \Omega$	(Identity)
$x \preceq y, y \preceq x \Rightarrow x = y$	for all $x, y \in \Omega$	(Antisymmetry)
$x \preceq y, y \preceq z \Rightarrow x \preceq z$	for all $x, y, z \in \Omega$	(Transitivity)

$(\Omega, \preceq)$ is called a **partially ordered set** or a **poset**.

A partial order $\preceq$ is called a **total order** if also the following holds:

$$x \preceq y \text{ or } y \preceq x \qquad \text{for all } x, y \in \Omega$$

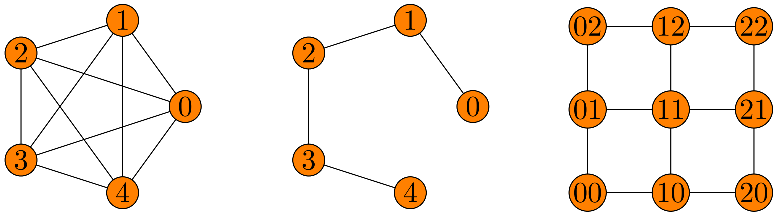
$(\Omega, \preceq)$ is called a **totally ordered set**.

A subset $C \subset \Omega$ of a poset $(\Omega, \preceq)$ that is totally ordered is called a **chain**.

Hasse Diagram

Given a poset $(\Omega, \preceq)$, the **Hasse diagram** is a directed graph (Ω, E) with

$$(x, y) \in E \quad :\Leftrightarrow x \prec y \text{ and } \neg (\exists z \in \Omega : x \prec z \prec y)$$



Totally Ordered Set $\{0, \dots, 4\}$ — Hasse Diagram $\{0, \dots, 4\}$ (Chain) — Hasse Diagram $\{0, \dots, 2\}^2$

Distributive Lattice

Let $(\Omega, \preceq)$ be a poset. For two elements $x, y \in \Omega$, we call $z \in \Omega$

the **meet** of x and y iff	$a \preceq z$	$\Leftrightarrow$	$a \preceq x$ and $a \preceq y$
the **join** of x and y iff	$a \succeq z$	$\Leftrightarrow$	$a \succeq x$ and $a \succeq y$

Note that if *meet* and *join* exist, they are unique. They are denoted as $x \wedge y$ and $x \vee y$ respectively. Meet and join are both associative and commutative.

Ω is called a **lattice** if *meet* and *join* exist for all $x, y \in \Omega$. If they are distributive, i.e.,

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z),$$

the lattice Ω is called a **distributive lattice**.

Partial Order and Distributive Lattice

Given a set Ω, the poset $(\mathcal{P}(\Omega), \subset)$ is a distributive lattice with
$A \wedge B = A \cap B$ and $A \vee B = A \cup B$.

If $(\Omega, \preceq)$ is a poset, we call $I \subset \Omega$ a **lower ideal** if

$$y \in I \Rightarrow [x \in I \text{ for all } x \preceq y] \,.$$

Let $\mathcal{I}_\Omega$ be the set of all lower ideals. $(\mathcal{I}_\Omega, \subset)$ is also a distributive lattice.

If $(L, \preceq)$ is a distributive lattice, we call $u \in L$ **join-irreducible** if

$$x, y \in L \backslash \{u\} \Rightarrow u \neq x \vee y.$$

Let $\mathcal{J}_L$ be the set of all join-irreducible elements of L.
$(\mathcal{J}_L, \preceq)$ is a poset.

Partial Order and Distributive Lattice

Theorem 1. *Let* $(\Omega, \preceq)$ *be a finite poset. Then* $(\Omega, \preceq) \approx (\mathcal{J}_{\mathcal{I}_\Omega} \backslash \{\varnothing\}, \subset)$.

Proof. For each $y \in \Omega$ we denote $y_\preceq := \{x \in \Omega | x \preceq y\}$. Let us now assume that we have a non-empty $A \in \mathcal{I}_\Omega$. Then there exists a maximal $y \in A$, i.e.,

$$y \preceq x \Rightarrow x = y \qquad \text{for all } x \in A$$

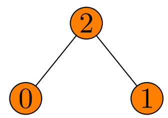
If $A \neq y_\preceq$, A is join-reducible via $A = (A \backslash \{y\}) \cup y_\preceq$. Otherwise it is join-irreducible. Therefore

$$\mathcal{J}_{\mathcal{I}_\Omega} \backslash \{\varnothing\} = \{y_\preceq | y \in \Omega\}$$

with $x \preceq y$ iff $x_\preceq \subset y_\preceq$.

Example

$\preceq$	0	1	2
0	X		X
1		X	X
2			X



Let us assume that $\Omega = \{0, 1, 2\}$ is poset with the partial ordering $\preceq$ (s.a.).
Then we have

$$\begin{aligned} \mathcal{P}(\Omega) &= \{\varnothing, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\} \\ \mathcal{I}_\Omega &= \{\varnothing, \{0\}, \{1\}, \{0,1\}, \{0,1,2\}\} \\ \mathcal{J}_{\mathcal{I}_\Omega} &= \{\varnothing, 0_\preceq, 1_\preceq, 2_\preceq\} \end{aligned}$$

Partial Order and Distributive Lattice

Theorem 2. *Let* $(L, \preceq)$ *be a distr. lattice. Then* $(L, \preceq) \approx (\mathcal{I}_{\mathcal{J}_L} \backslash \{\emptyset\}, \subset)$.

Proof. Let $y_{\preceq} := \{x \in \mathcal{J}_L | x \preceq y\}$ for any $y \in L$. We have to show that each non-empty lower ideal $I \subset \mathcal{J}_L$ can be represented as $y_{\preceq}$. To this end, let

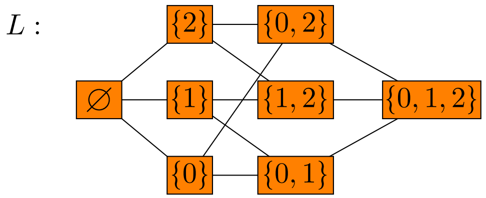
$$y := \bigvee_{x \in I} x.$$

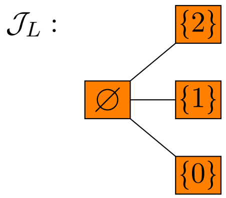
$I \subset y_{\preceq}$ follows from the definition of y. Let therefore be $z \in y_{\preceq}$. Since L is distributive, we have

$$z = z \wedge y = z \wedge \left(\bigvee_{x \in I} x \right) = \bigvee_{x \in I} (z \wedge x).$$

Since z is join-irreducible, there is one $x \in I$ with $z = z \wedge x$.
Hence $z \preceq x$ and thus, $z \in I$.

Example





Consider the distributive lattice

$$L = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$$

Then we have

$$\mathcal{J}_L = \{\emptyset, \{0\}, \{1\}, \{2\}\}$$

$$\mathcal{I}_{\mathcal{J}_L} = \{\emptyset, \emptyset_\subset, \{0\}_\subset, \{1\}_\subset, \{2\}_\subset, \{0\}_\subset \cup \{1\}_\subset, \{0\}_\subset \cup \{2\}_\subset, \{1\}_\subset \cup \{2\}_\subset, \{0\}_\subset \cup \{1\}_\subset \cup \{2\}_\subset\}$$

Literature

Set Theory

- Cantor, *Ueber eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen*, 1874, J. Reine Angew. Math (77), 258–262.
- Zermelo, *Untersuchungen über die Grundlagen der Mengenlehre*, 1908, Math. Annalen (65), 261–281.
- Fraenkel, *Zu den Grundlagen der Cantor-Zermeloschen Mengenlehre*, 1922, Math. Annalen (86), 230–237.
- Fraenkel et al., *Foundations of Set Theory*, 1973 (1958), North-Holland.

Distributive Lattice

- Birkhoff, *Lattice Theory*, AMS Colloquium Publiations, 25, 1967.
- Schrijver, *Combinatorial Optimization*, Chapter 14.