

# Combinatorial Optimization in Computer Vision (IN2245)

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Medical Imaging

Anatomy and Multi-Labeling

Surface Segmentation

Minimal Distance

## 16. Minimal Distance Constraint

Medical Imaging

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## Medical Imaging

Medical Imaging

## Classical Image Segmentation

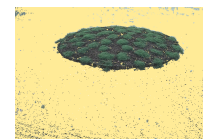
Anatomy and Multi-Labeling

Surface Segmentation

Minimal Distance



Given Image



Data Term



Data + Regularizer

The overall energy we like to minimize is

$$E(x) = \sum_{i=1}^n f_i(x_i) + \underbrace{\sum_{i=1}^n \sum_{j \in \mathcal{N}(i)} f_{ij} \cdot \delta(x_i, x_j)}_{\text{weighted length}}, \quad x \in \mathbb{B}^n.$$

This **submodular energy** can be globally minimized via graph-cut  
if  $\delta_{\ell_1, \ell_2} = \llbracket \ell_1 \neq \ell_2 \rrbracket$ .

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## Medical and RGB Images

Medical Imaging

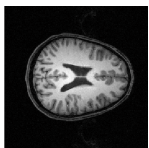
Anatomy and Multi-Labeling

Surface Segmentation

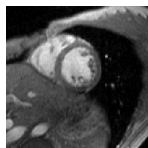
Minimal Distance

Differences to color-images are

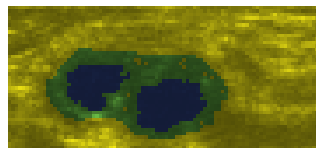
- In general, grayscale images (intensities of magnetic field etc.)
- Boundaries are often difficult to detect
- Color models for *foreground* and *background* tend to look *very similar*
- Interested in multi-label segmentation



Brain



Heart



Carotid Artery

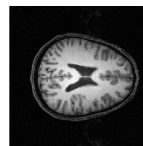
Medical Imaging

## Medical Imaging and Multilabeling

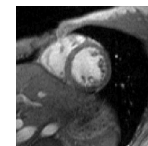
Anatomy and Multi-Labeling

Surface Segmentation

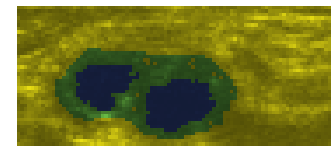
Minimal Distance



Brain



Heart



Carotid Artery

$$E(x) = \sum_{i=1}^n f_i(x_i) + \sum_{i=1}^n \sum_{j \in \mathcal{N}(i)} f_{ij} \cdot \delta(x_i, x_j), \quad x \in \{\mathbf{0}, \dots, \mathbf{k}\}^n$$

This problem is NP-hard for the Potts model  $\delta(\ell_1, \ell_2) = \llbracket \ell_1 \neq \ell_2 \rrbracket$  and  $k > 1$ .

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## Anatomy and Multi-Labeling

Medical Imaging

## Anatomy vs. Geometry \*

Anatomy and Multi-Labeling

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The term **anatomy** is derived from Greek:

ἀνά "upwards"

τέμνω "to cut"

and refers to the study about the internal structure of organisms.

Anatomy refers to the true nature of the internal structure.

Very often, we cannot detect this true nature.

Instead, we like to enforce certain geometrical properties.

The term **geometry** is also derived from Greek:

γῆ "earth"

μέτρον "measurement"

and refers to the study about **shape** and **relative positions** of objects.

While we are interested in an anatomical model, we usually enforce geometrical constraints.

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If we want to segment a medical observation into its components, we can cast this as a multilabeling problem. Enforcing geometrical constraints is equivalent to restricting the set  $\mathcal{L}^n$  of feasible labelings.

Given  $x \in \mathcal{L}^n$  and  $\ell \in \mathcal{L}$ , we refer to  $S_\ell := \{i | x_i = \ell\}$  as the **region of  $\ell$** .

Given two different regions  $S_\alpha$  and  $S_\beta$ , one might be interested in the following geometrical constraints:

$$\begin{aligned} S_\alpha &\subset S_\beta && \text{(inclusion constraint)} \\ S_\alpha &\supset S_\beta && \text{(exclusion constraint)} \\ \text{dist}(S_\alpha, S_\beta) &\geq d && \text{(minimal distance constraint)} \\ \text{dist}(S_\alpha, S_\beta) &\leq d && \text{(maximal distance constraint)} \end{aligned}$$

There may be different distance functions  $\text{dist}(\cdot, \cdot)$  that we can use.

Using Bayes' rule, we can write any multilabeling problem as a probability maximization problem

$$\begin{aligned} \arg\max_{x \in \mathcal{L}^n} P(x|I) &= \arg\max_{x \in \mathcal{L}^n} \frac{P(I|x) \cdot P(x)}{P(I)} \\ &= \arg\min_{x \in \mathcal{L}^n} -\log(P(I|x)) - \log(P(x)) \end{aligned}$$

The *likelihood*  $P(I|x)$  tells us how well a segmentation  $x \in \mathcal{L}^n$  fits to the observed image  $I$ . The *prior*  $P(x)$  tells us how likely a certain segmentation  $x \in \mathcal{L}^n$  is. It does not depend on the observation.

If we want to exclude a certain labeling  $x \in \mathcal{L}$ , its probability is "0" and we have  $-\log(P(x)) = +\infty$ .

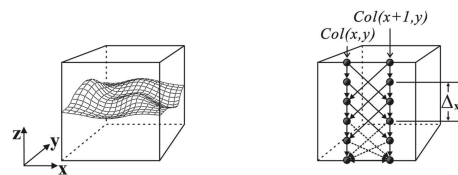
## Surface Segmentation

## Multisurface Segmentation

The vision group of Sonka proposed in 2006 a method for medical surface segmentation that bears a certain resemblance to the Ishikawa construction.

They considered **parametrized** surfaces, i.e., mappings  $f: \Omega_{xy} \rightarrow \mathbb{N}_0$ .

Besides the vertical infinity costs, they also added additional infinity costs that assures that the surface does not change too much in the  $x$ - and  $y$ -direction.



## Data & Regularity Term

Since they only wanted to employ a data term  $c_{(x,y,z)}$  for the surface that is related to the gradient of the image, they used the following data term

$$f_{(x,y,z)}(1) = \begin{cases} c_{(x,y,z)} & \text{if } z=0 \\ c_{(x,y,z)} - c_{(x,y,z-1)} & \text{if } z>0. \end{cases}$$

These data terms proved to result in faster algorithms than the original idea of Ishikawa to use vertical edges between neighboring layers.

By adding infinity edges

$$\begin{aligned} [(x,y,z), (x \pm 1, y, z - \Delta_x)] & \quad \text{and} \quad [(x,y,z), (x \pm 1, y, z + \Delta_x)] \\ [(x,y,z), (x, y \pm 1, z - \Delta_y)] & \quad \text{and} \quad [(x,y,z), (x, y \pm 1, z + \Delta_y)] \end{aligned}$$

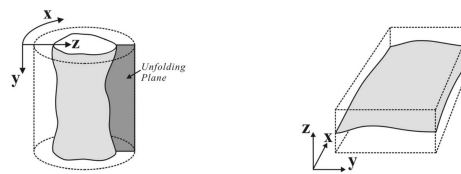
the surface can only vary by at least  $\Delta_x$  or  $\Delta_y$  in the  $x$ - resp.  $y$ -direction.

## Closed Surface

In order to also handle closed surfaces, they proposed the technique of **unfolding** in order to obtain a surface that can be parametrized.

By doing so, they introduced in fact **cylindrical coordinates**.

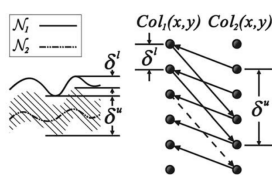
A disadvantage of this approach is the explicit knowledge of the central axis of the medical object. Also, the data term has to be transformed into the cylindrical coordinates which may result in small inaccuracies.



## Multiple Surfaces

If one wants to handle multiple surfaces with different data term, the surface construction has to be repeated for each surface. Sonka's group assumed that all surfaces can be parametrized with the same unfolding technique.

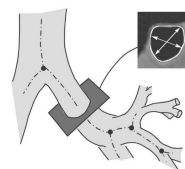
Afterwards, additional edges between corresponding columns enforce the minimal and maximal distance constraint. This constraint does not depend on the final segmentation but on the chosen unfolding technique.

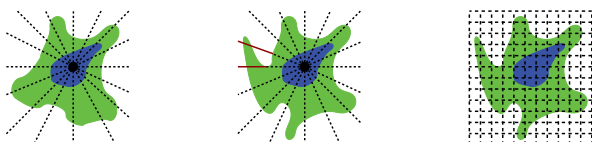


## Results

The results are competitive, but the implementation is in general difficult

- The central structure should be easy to detect (pre-processing)
- Gradient information have to be recomputed with respect to the unfolding technique (pre-processing).
- The results are given in cylindrical coordinates. The actual image segmentation has to be derived from it (post-processing).





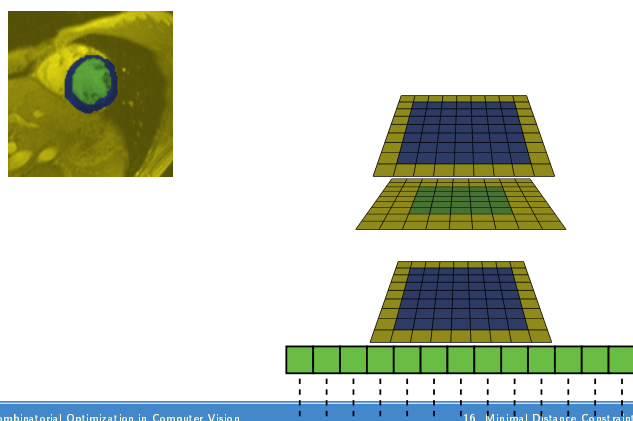
Every star-shaped region can be modelled, but the enforced distance constraints might be limited to the straight lines leaving the center.

Only star-shaped regions can be modelled.  
This limits the set of feasible labelings.

We plan to expand the space of possible labelings and enforcing a minimal distance as a well as a maximal distance constraint.

## Minimal Distance

## Ishikawa Construction



## Minimal Distance Constraint



We address the nested multilabeling problem



s.t. minimal tubular distance

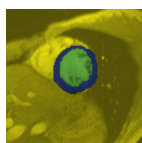
The **tubular distance**  $t$  between  $S_\alpha \subset S_\beta$  is the minimal value  $r$  such that

$$S_\alpha \oplus B_r \subset S_\beta$$

$$A \oplus B = \bigcup_{b \in B} \{a + b | a \in A\}$$

$$B_r = \{x \in \mathbb{R}^d | \|x\| \leq r\}$$

## Nested Labeling



We are interested in the following label penalty

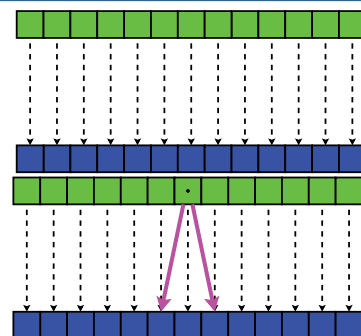
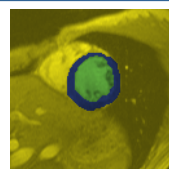
$$\delta(\ell_1, \ell_2) = \begin{cases} 0 & , |\ell_1 - \ell_2| = 0 \\ 1 & , |\ell_1 - \ell_2| = 1 \\ \infty & , \text{otherwise} \end{cases}$$

In practice,  $\infty$  is a large constant  $K$  and one can formulate  $\delta$  as

$$\delta(\ell_1, \ell_2) = \begin{cases} 0 & , |\ell_1 - \ell_2| = 0 \\ 1 + (\Delta - 1) \cdot K & , |\ell_1 - \ell_2| = \Delta \end{cases}$$

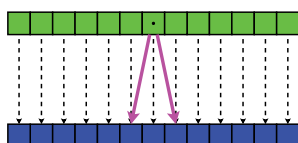
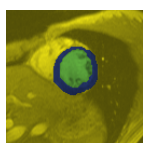
This penalty is convex in  $|\ell_1 - \ell_2|$ .

## Nested Labeling (Ishikawa)

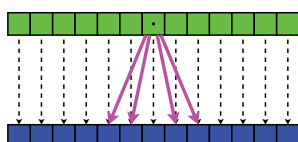
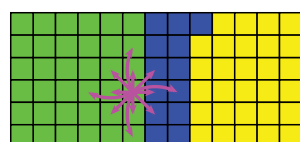


Given the labelspace  $\mathcal{L} = \{0, \dots, k\}$ , the involved variables are  $k$  copies of the  $n$  pixels in the image domain, resulting in  $n \cdot k$  vertices.

## Minimal Distance Constraint



Increasing the neighborhood enforces larger minimal distance constraints.



## Heart Segmentation

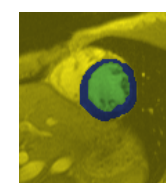
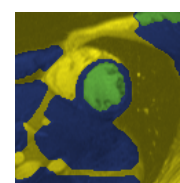
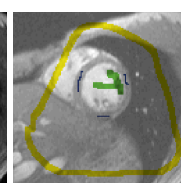
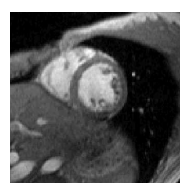


Image Seeds

Minimal Distance

Maximal Distance

A. Delong, Y. Boykov: ICCV, 2009  
F. R. Schmidt, Y. Boykov: ECCV, 2012

### Distance Constraints

- Li, Wu, Chen, Sonka, *Optimal Surface Segmentation in Volumetric Images – A Graph-Theoretical Approach*, 2006, IEEE TPAMI 28, 119–134.
- Delong, Boykov, *Globally Optimal Segmentation of Multi-Region Objects*, 2009, IEEE ICCV, 285–292.
- Schmidt, Boykov, *Hausdorff Distance Constraint for Multi-Surface Segmentation*, 2012, ECCV, 598–611.