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Machine Learning for Computer Vision
Winter term 2016

November 1, 2016
Regression, Kernels and Gaussian Processes

Exercise 1: Bayesian Update

Consider a linear regression model with basis functions $\phi(x)$ as presented in the lecture. We assume a Gaussian prior distribution for the weights:

$$p(\mathbf{w}) = \mathcal{N}(\mathbf{w}|m_0, S_0)$$

Suppose we have already observed N data points, so the posterior distribution is

$$p(\mathbf{w}|\mathbf{t}) = \mathcal{N}(\mathbf{w}|m_N, S_N)$$

with

$$m_N = S_N(S_0^{-1}m_0 + \sigma^{-2}\Phi^T\mathbf{t}) \quad \text{and} \quad S_N = S_0^{-1} + \sigma^{-2}\Phi^T\Phi.$$

Now, we observe a new data point (x_{N+1}, t_{N+1}). What is the new posterior?

Exercise 2: Constructing kernels

Let k_1 and k_2 be kernels, $f : \mathbb{R}^n \to \mathbb{R}$ an arbitrary function . Show that we can construct new kernels via

a) $k(x_1, x_2) = k_1(x_1, x_2) + k_2(x_1, x_2)$

b) $k(x_1, x_2) = k_1(x_1, x_2)k_2(x_1, x_2)$

c) $k(x_1, x_2) = f(x_1)k_1(x_1, x_2)f(x_2)$

d) $k(x_1, x_2) = \exp(k_1(x_1, x_2))$

e) $k(x_1, x_2) = x_1^T A x_2$, where A symmetric, positive semi-definite $n \times n$ matrix

Exercise 3: Polynomial kernel

Let $x_i, x_j \in \mathbb{R}^2$

a) Show (by induction) that $k_d(x_i, x_j) = (x_i^T x_j)^d$ is a kernel for every $d \geq 1$.

b) Find $\phi_d(x)$ such that $k_d(x_i, x_j) = \phi_d(x_i)^T \phi_d(x_j)$.

c) Find $\tilde{\phi}_2(x)$ for $\tilde{k}_2(x_i, x_j) = (x_i^T x_j + d)^2$ $(d > 0)$.

Exercise 4: Gaussian Processes Regression (Programming)

Download the file *train_ data.txt*. The file contains pairs of inputs and targets you can use to train a Gaussian Process model. Can you fit a function to these points? Start by using a squared-exponential kernel with length scale $l = 3$, $\sigma_f = 1$ and $\sigma_n = 0.5$. Try different values for the length scale hyperparameter. How does the model change?

The next exercise class will take place on **November 11th, 2016**.

For downloads of slides and of homework assignments and for further information on the course see

`https://vision.in.tum.de/teaching/ws2016/mlcv16`
