Machine Learning for Computer Vision
Winter term 2016

December 13, 2016
Graphical Models, Hidden Markov Models

Probabilistic Graphical Models

Exercise 1: Reading a graphical model

We have the following graphical model:
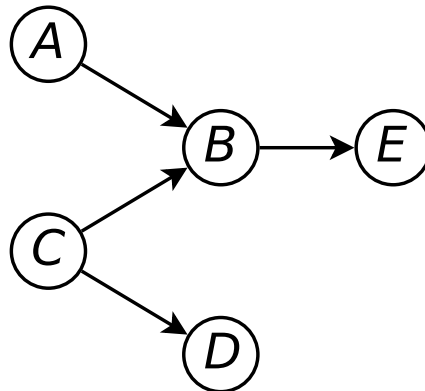

Abbildung 1: Graphical model.

a) Write the joint probability distribution corresponding to the graphical model depicted in Fig. 1.

b) What are the conditional independence assumptions of this model?

c) Which of the following assertions are true, and why?

- B is d-separated from D by C,
- A is d-separated from C by E,
- A is d-separated from C by D,
- E is d-separated from D by B,
- E is d-separated from D by A.

Exercise 2: Markov Chain

We have the following Markov Chain:

$$A \text{---} B \text{---} C \text{---} D$$
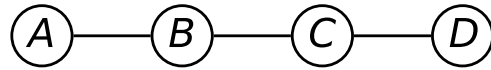
a) Write the joint probability distribution associated to this Markov Chain.

b) Each variable can take value 0 or 1, and we want to express that it is 9 times more probable that neighboring variables have equal values than they have different value. Give the potential functions of this Markov Chain.

c) Compute the probability distributions $p(A)$ and $p(C)$.

d) Now, we observe that D is 1, recompute the distributions over A and C: $p(A \mid [D = 1])$ and $p(C \mid [D = 1])$.
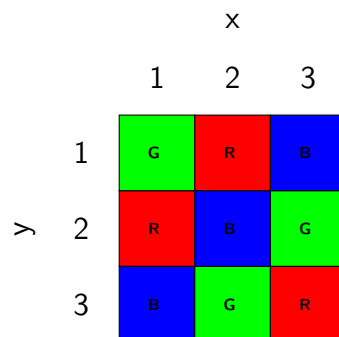
e) Compute $p(C \mid [A = 0], [D = 1])$.

Hidden Markov Models

Exercise 3: Viterbi Algorithm

We play again with our robot from the first homework assignment. As we mentioned back then the robot has a camera with an observation model that looks as follows:

Sensed color \ Actual color	R	G	B
R	0.8	0.1	0.1
G	0.1	0.6	0.2
B	0.1	0.3	0.7

This time we put the robot in a room where the floor looks like this:

a) What is the state space? What is the observation space? Draw the trellis diagram.

b) Assume the robot can only move vertically and horizontally. We let the robot move randomly. If the attempted move leads outside of the bounds of the room the robot stays at its current position. Compute the state transition matrix.

c) After 3 time steps, what is most likely the path that the robot followed if the camera reads $\{z_1 = R,\ z_2 = G,\ z_3 = G\ \}$? Assume the robot's initial position is unknown.