Variational Methods for Computer Vision: Exercise Sheet 5

Exercise: November 21, 2016

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $u \in C^2(\Omega; \mathbb{R})$ be a twice differentiable real-valued function and $\Omega \subset \mathbb{R}^2$. And let

$$E(u) = \int_\Omega \mathcal{L}(u(x), \nabla u(x), \nabla^2 u(x)) \, \mathrm{dx}$$

   be a real valued Gâteaux differentiable functional which depends on u, ∇u and the Hessian $\nabla^2 u$. Calculate the Gâteaux derivative of E.

2. Calculate the Euler-Lagrange equation of the following energy functional:

$$E(u) = \frac{1}{2\lambda} \int_\Omega (k * u - f)^2 + \int_\Omega |\nabla u| \, \mathrm{dx}$$

   Where $\Omega \subset \mathbb{R}^2$ represents the image domain, $u : \Omega \to \mathbb{R}$ denotes the optimization variable, $f : \Omega \to \mathbb{R}$ stands for the input image and k denotes a convolution kernel (not necessarily symmetrical).

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Finish the exercises from last week.