Variational Methods for Computer Vision: Exercise Sheet 8

Exercise: December 12, 2016

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $\Omega = [-5; 5] \times [-5; 5]$ be a rectangular area and let $I : \Omega \to [0, 1]$ be an image given by

$$I(x, y) = \begin{cases} 1, & \text{if} \quad x^2 + y^2 \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

   Furthermore let $C : [0, 1] \to \Omega$ be a curve represented by a circle centered at the origin having radius r.

   (a) Following the lecture, write down the Gâteaux derivatives of the Mumford-Shah functional for 2 regions

$$E(\{u_1, u_2\}, C) = \int_{\Omega_i} (I(x) - u_1)^2 + (I(x) - u_2)^2 \mathrm{d}x + \nu |C|,$$

   for the two cases $r > 1$ and $r \leq 1$.

   (b) Show that the Gâteaux derivative at $r = 1$ is not continuous. Why is $\nu \leq 1$ a good choice in order to obtain good segmentation results? What is the ideal choice for ν in our example?

2. Consider the the geodesic active contour functional,

$$E(C) = \int_0^1 g(C(s))|C'(s)|ds, \tag{1}$$

   where $C : [0, 1] \to \Omega \subset \mathbb{R}^2$ and $g : \Omega \to \mathbb{R}$ denotes some edge indicator function. Convince yourself that the gradient descent equation for C is given by:

$$\frac{dC}{dt} = (g\kappa - \langle \nabla g, n \rangle) n.$$

   by calculating the Euler-Lagrange equation of (1). Here κ denotes the curvature and n the normal vector of the parametrized curve $C : [0, 1] \to \Omega$.

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Finish the practical exercises from the previous exercise sheets.