Variational Methods for Computer Vision: Exercise Sheet 11

Exercise: January 23, 2017

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. The general formulation of the total variation applies also to discontinuous functions $u : \Omega \to \mathbb{R}$ and has the following form.

$$E(u) = \sup_{\varphi \in \mathcal{K}} \int_{\Omega} u \operatorname{div} \varphi \, \mathrm{d}x, \tag{1}$$

$$K = \{\varphi \in C_c^1(\Omega; \mathbb{R}^2) \mid |\varphi(x)| \leq 1 \; \forall x \in \Omega\}. \tag{2}$$

   Prove that the above formulation is a convex functional.

2. The unsmoothed structure tensor is a 2×2 matrix defined as $\nabla I \nabla I^\top$.

   (a) Show that the maximum rank of the structure tensor is 1.

   (b) Calculate the eigenvalues and eigenvectors of the structure tensor and interpret the result.

3. Let $I : \Omega \times \mathbb{R}_+ \to \mathbb{R}$ be an image sequence and $v = (v_1, v_2)^\top : \Omega \to \mathbb{R}^2$ be a vector field. Compute the Euler-Lagrange equation of the following functional.

$$E(v) = \int_{\Omega} (\langle \nabla I(x), v \rangle + I_t(x))^2 \, \mathrm{d}x + \alpha \int_{\Omega} |\nabla v_1(x)|^2 + |\nabla v_2(x)|^2 \, \mathrm{d}x. \tag{3}$$

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Implement the optical flow method of Lucas and Kanade and apply it to the images in the image sequence contained in the archive `vmcv_ex11.zip`.
2. Generate an image which color-codes all pixels moving to the left in white and pixels moving to the right in black.