

**Machine Learning for Computer Vision**  
**Winter term 2017**

November 21, 2017  
Topic: Metric Learning

**Exercise 1: Metric Learning**

- a) Given a valid metric  $D_M$ , is  $D_M^2$  also a metric? Why?
- b) Given a matrix  $X$  of  $n$  data points  $x_i \in \mathbb{R}^d$ , show how computing the eigen-decomposition of the covariance of  $X$  is equivalent to computing the singular value decomposition of  $X$ .
- c) What is the difference between metric learning and kernel learning? When would you prefer to use a kernel method over a metric learning method?
- d) In Neighborhood Component Analysis, we define a stochastic neighbor selection rule. The probability that a data point  $j$  is selected as neighbor of point  $i$  is given by:

$$p_{ij} = \frac{\exp\{-\|Lx_i - Lx_j\|^2\}}{\sum_{k \neq i} \exp\{-\|Lx_i - Lx_k\|^2\}} \quad (1)$$

namely a softmax over the squared distances to all points in the transformed space. The goal is to maximize

$$f(L) = \sum_i \sum_{j \in C_i} p_{ij} \quad (2)$$

namely the probability that the neighbors that will be selected for each point  $i$  will belong to the same class  $C_i$ . Can you derive the gradient of  $f(L)$ ?

- e) What is the difference between LDA and NCA?
- f) The KL-divergence measures the similarity of two probability distributions. It is defined as:

$$D_{KL}(p||q) = \int p(x) \log \left( \frac{p(x)}{q(x)} \right) dx$$

Is the KL-divergence a metric? Why?