Robotic 3D Vision

Lecture 10: Visual SLAM

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What We Will Cover Today

- Introduction to Visual SLAM
- Formulation of the SLAM Problem
- Full SLAM Posterior
- Bundle Adjustment (BA)
- Structure of the SLAM/BA Problem

What is Visual SLAM?

- Visual simultaneous localization and mapping (VSLAM)…
  - Tracks the pose of the camera in a map, and simultaneously
  - Estimates the parameters of the environment map (f.e. reconstruct the 3D positions of interest points in a common coordinate frame)
- Loop-closure: Revisiting a place allows for drift compensation
  - How to detect a loop closure?

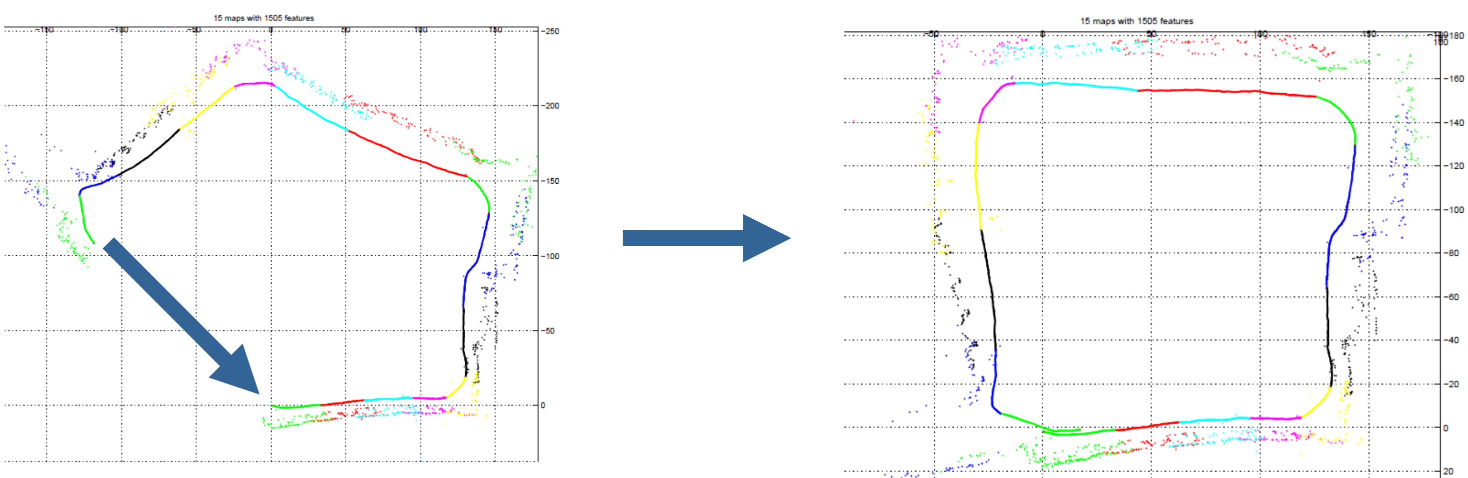

Image credit: Clemente et al., RSS 2007

What is Visual SLAM?

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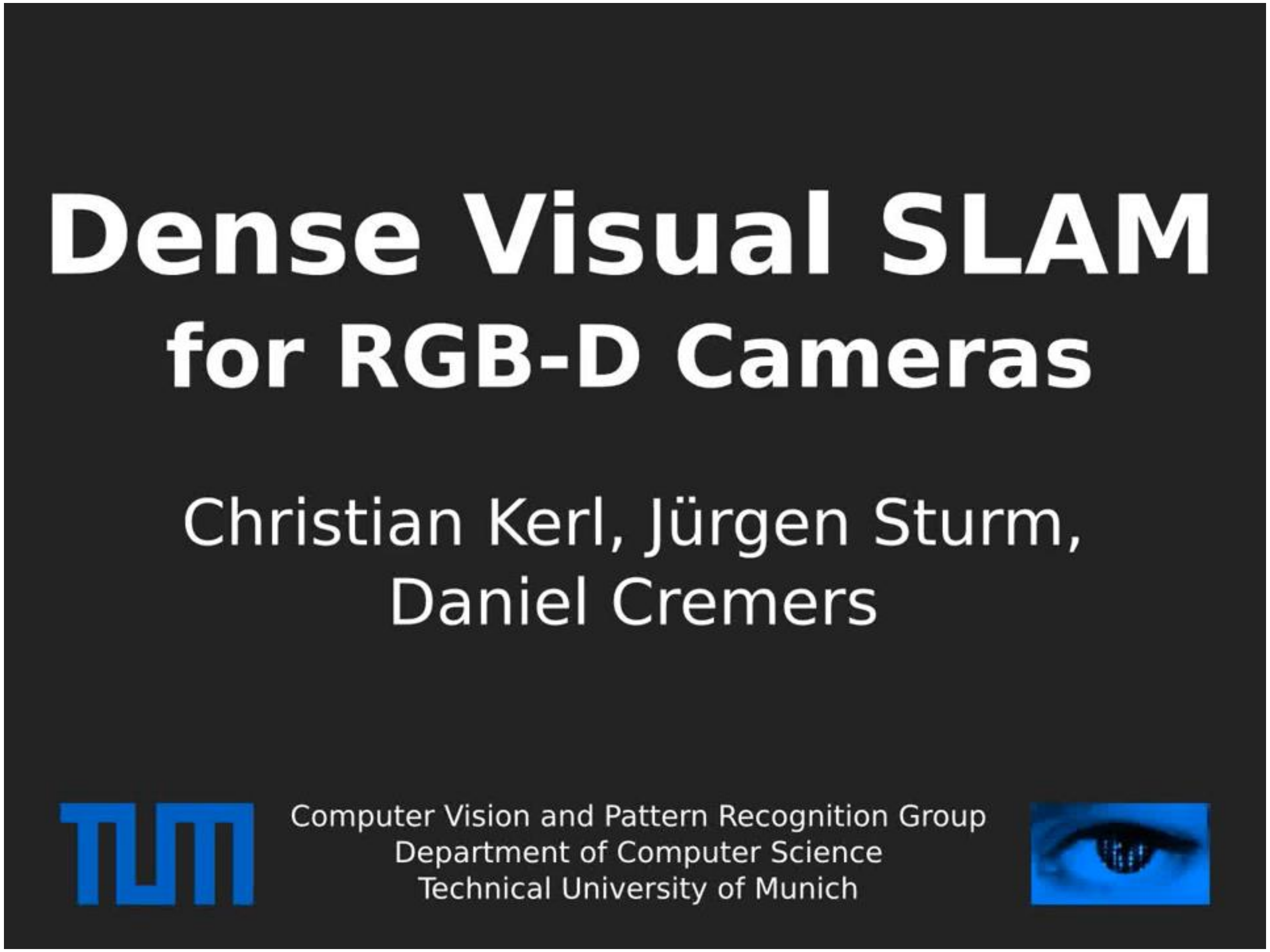
- Loop-closure: Revisiting a place allows for drift compensation
  - How to detect a loop closure?
- Global vs. local optimization methods
  - Global: bundle adjustment, pose-graph optimization, etc.
  - Local: incremental tracking-and-mapping approaches, visual odometry with local maps. Often designed for real-time.
  - Hybrids: Real-time local SLAM + global optimization in a slower parallel process (f.e. LSD-SLAM)

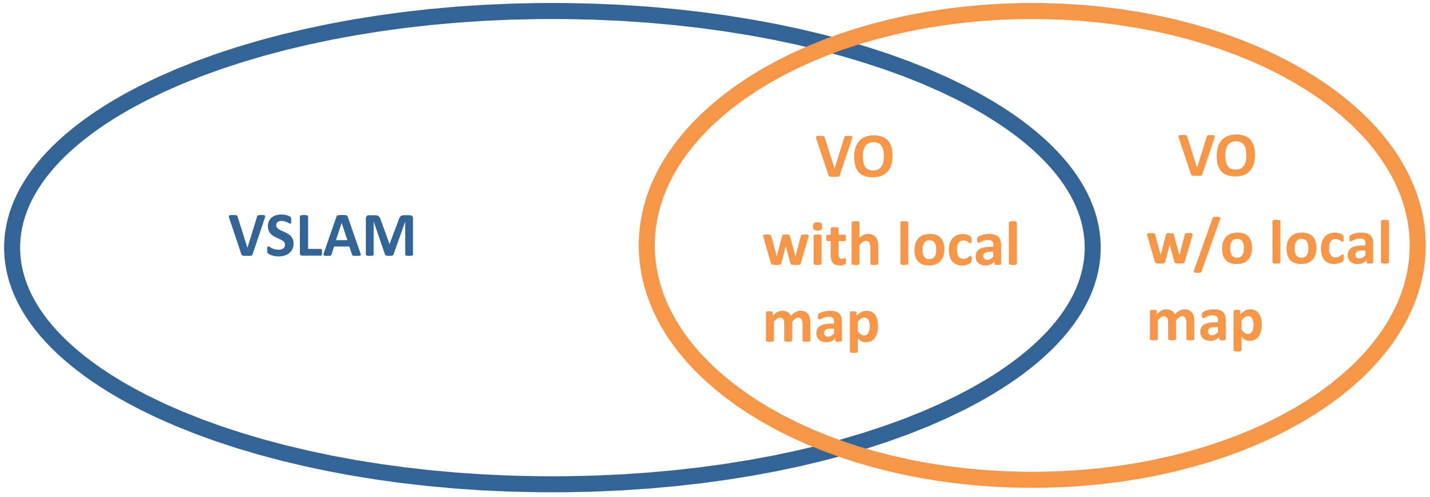
Visual SLAM with RGB-D Cameras



RGB-D SLAM by Map Deformation

Visual SLAM using Bundle Adjustment

VO vs. VSLAM



Structure from Motion

- Structure from Motion (SfM) denotes the joint estimation of
  - Structure, i.e. 3D reconstruction, and
  - Motion, i.e. 6-DoF camera poses,

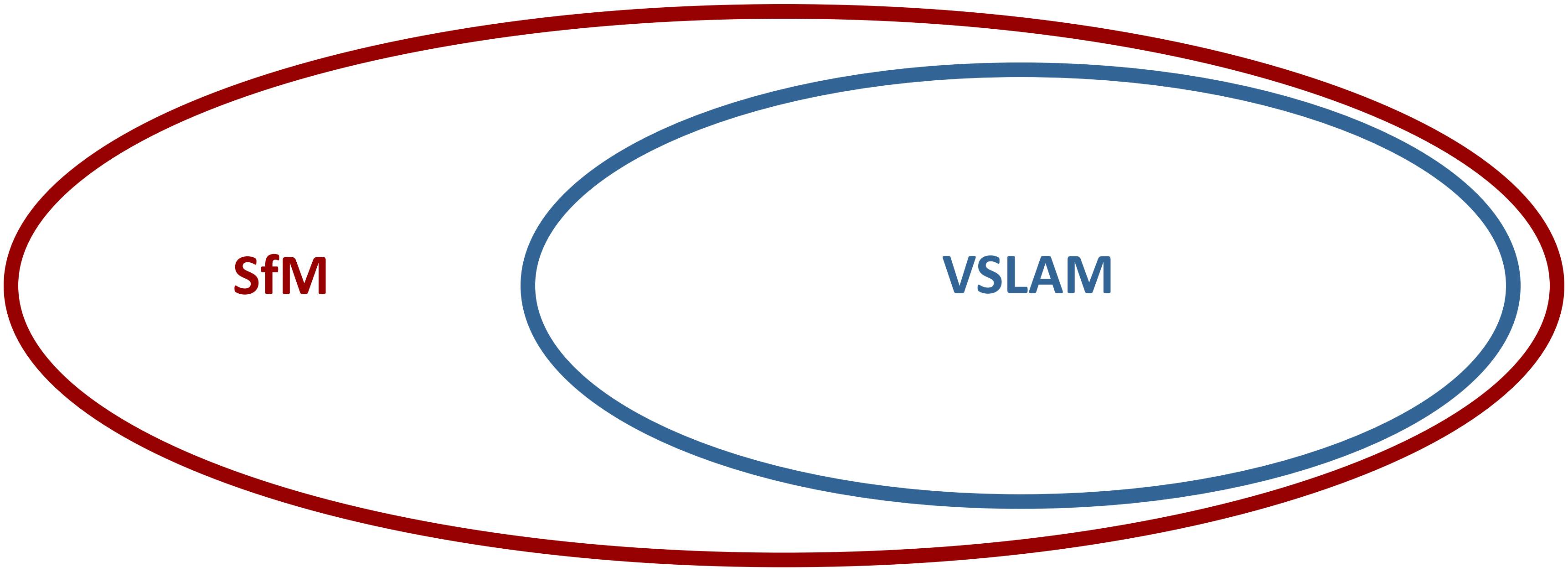
  from a collection (i.e. unordered set) of images

- Typical approach: keypoint matching and bundle adjustment

Structure from Motion

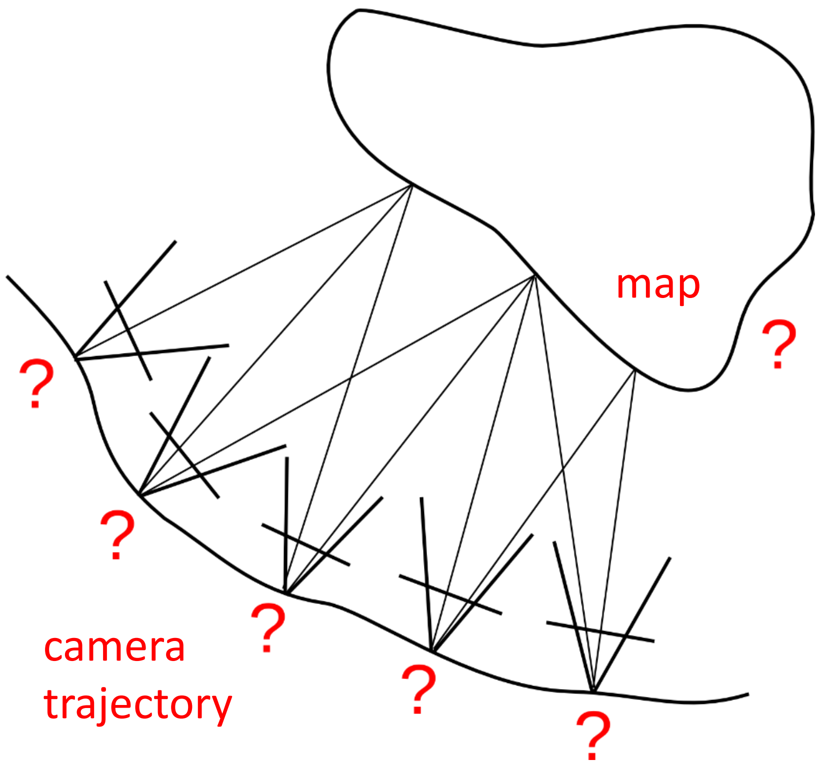
Agarwal et al., Building Rome in a Day, ICCV 2009, „Dubrovnik“ image set

VSLAM vs. SfM



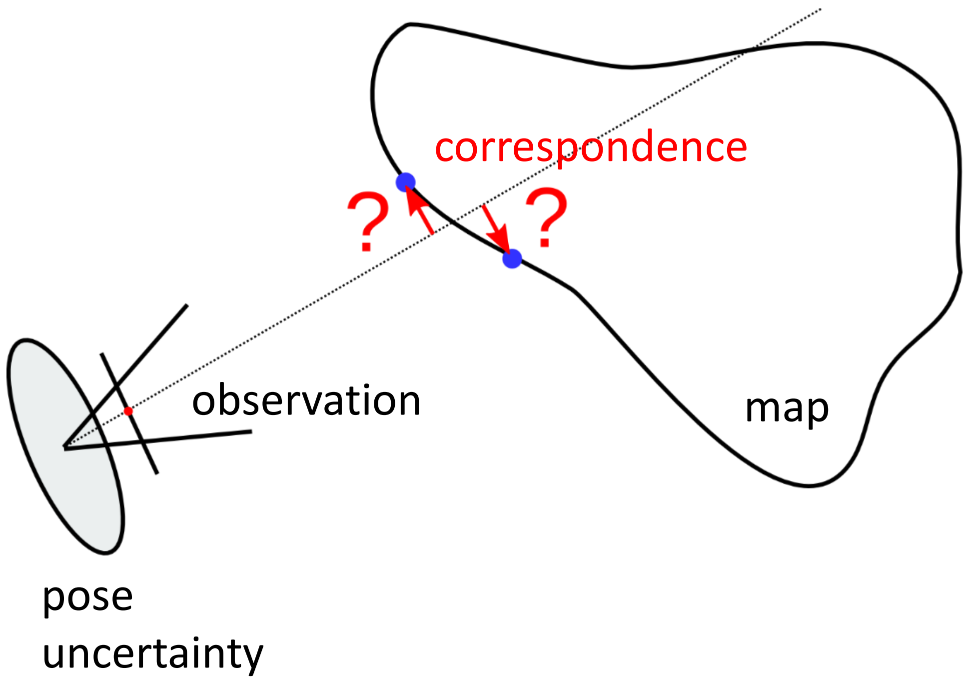
Why is SLAM difficult?

- Chicken-or-egg problem
  - Camera trajectory and map are unknown and need to be estimated from observations
  - Accurate localization requires an accurate map
  - Accurate mapping requires accurate localization



Why is SLAM difficult?

- Correspondences between observations and the map are unknown
- Wrong correspondences can lead to divergence of trajectory/map estimates
- Important to model uncertainties of observations and estimates in a probabilistic formulation of the SLAM problem



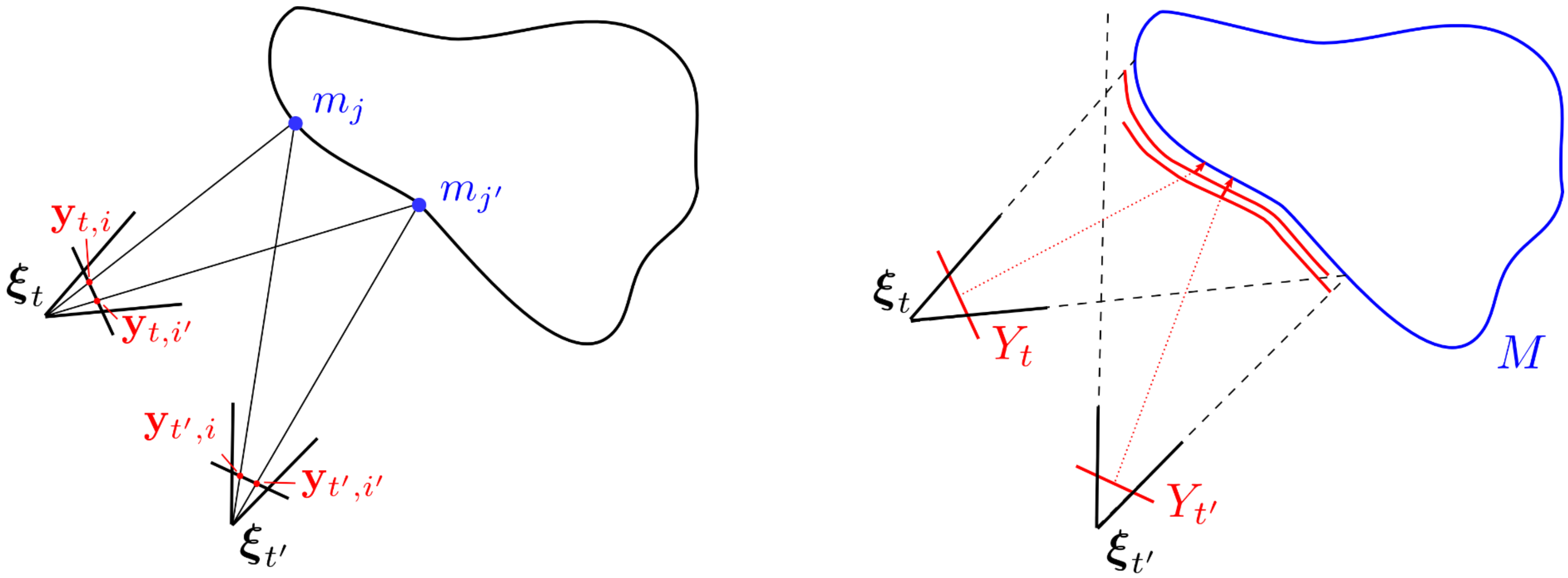
Definition of Visual SLAM

- Visual SLAM is the process of simultaneously estimating the egomotion of an object and the environment map using only inputs from visual sensors on the object and control inputs

- **Inputs:** images at discrete time steps t,
  - Monocular case: Set of images $I_{0:t} = \{I_0, \ldots, I_t\}$
  - Stereo case: Left/right images $I^l_{0:t} = \{I^l_0, \ldots, I^l_t\}$ $I^r_{0:t} = \{I^r_0, \ldots, I^r_t\}$
  - RGB-D case: Color/depth images $I_{0:t} = \{I_0, \ldots, I_t\}$ $Z_{0:t} = \{Z_0, \ldots, Z_t\}$

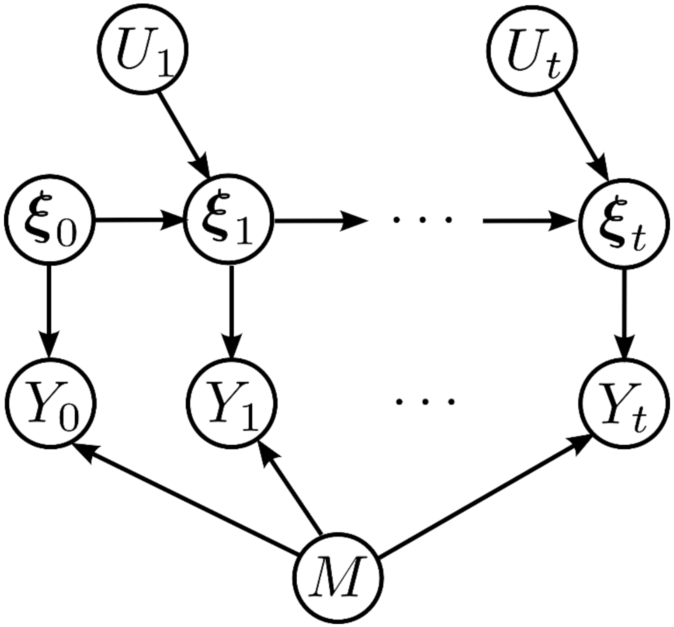
  - Robotics: **control inputs** $U_{1:t}$
- **Output**:
  - Camera pose estimates $\mathbf{T}_t \in \mathbf{SE(3)}$ in world reference frame. For convenience, we also write $\boldsymbol{\xi}_t = \boldsymbol{\xi}\left(\mathbf{T}_t\right)$
  - Environment map M

Map Observations in Visual SLAM



- With Y_t we denote observations of the environment map in image I_t , f.e.
  - Indirect point-based method: $Y_t = \{\mathbf{y}_{t,1}, \ldots, \mathbf{y}_{t,N}\}$ (2D or 3D image points)
  - Direct RGB-D method: $Y_t = \{I_t, Z_t\}$ (all image pixels)
  - …
- Involves data association to map elements $M = \{m_1, \ldots, m_S\}$
  - We denote correspondences by $c_{t,i} = j, 1 \leq i \leq N, 1 \leq j \leq S$

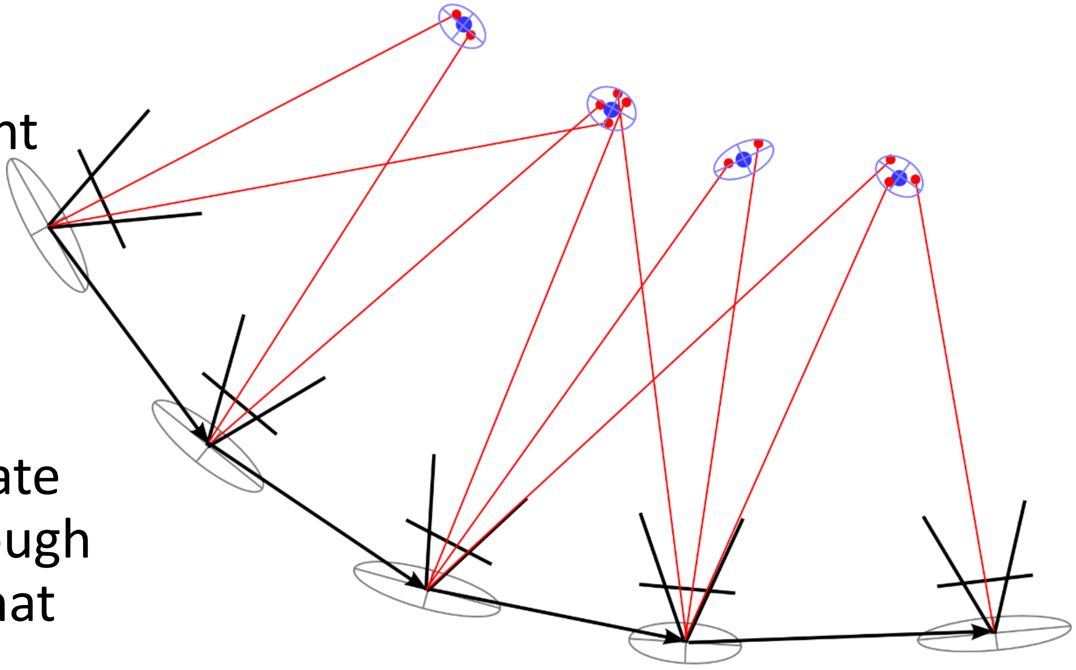
Probabilistic Formulation of Visual SLAM



- SLAM posterior probability: $p\left(\boldsymbol{\xi}_{0:t}, M \mid Y_{0:t}, U_{1:t}\right)$
- Observation likelihood: $p\left(Y_t \mid \boldsymbol{\xi}_t, M\right)$
- State-transition probability: $p\left(\boldsymbol{\xi}_t \mid \boldsymbol{\xi}_{t-1}, U_t\right)$

SLAM Graph Optimization

- Joint optimization for poses and map elements from image observations of map elements
  - Common map element observations induce constraints between the poses
  - Map elements correlate with each others through the common poses that observe them
  - No temporal sequence: Bundle Adjustment



Probabilistic Formulation

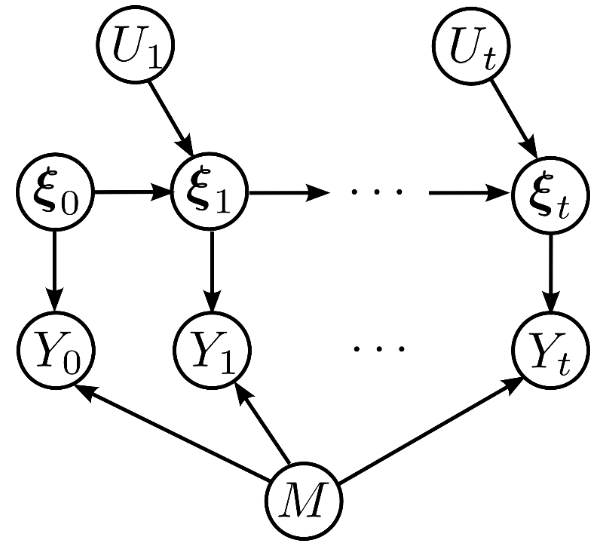
- SLAM posterior: $p\left(\boldsymbol{\xi}_{0:t}, M \mid Y_{0:t}, U_{1:t}, c_{0:t}\right)$
- Observation likelihood:

$$p\left(Y_t \mid \boldsymbol{\xi}_t, M, c_t\right) = p\left(Y_t \mid \boldsymbol{\xi}_t, m_{c_t}\right)$$

$$p(Y_t \mid \boldsymbol{\xi}_t, m_{c_t}) = \prod_i p(\mathbf{y}_{t,i} \mid \boldsymbol{\xi}_t, m_{c_{t,i}})$$

- State-transition probability:

$$p\left(\boldsymbol{\xi}_t \mid \boldsymbol{\xi}_{t-1}, U_t\right)$$



- SLAM posterior can be factorized:

$$p\left(\boldsymbol{\xi}_{0:t}, M \mid Y_{0:t}, U_{1:t}, c_{0:t}\right) = \eta\, p\left(Y_t \mid \boldsymbol{\xi}_t, m_{c_t}\right) p\left(\boldsymbol{\xi}_{0:t}, M \mid Y_{0:t-1}, U_{1:t}, c_{0:t-1}\right)$$

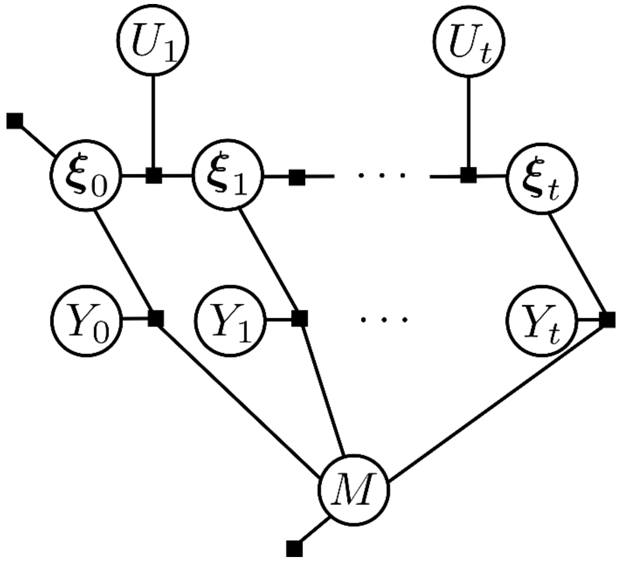
$$= \eta\, p\left(Y_t \mid \boldsymbol{\xi}_t, m_{c_t}\right) p\left(\boldsymbol{\xi}_t \mid \boldsymbol{\xi}_{t-1}, U_t\right) p\left(\boldsymbol{\xi}_{0:t-1}, M \mid Y_{0:t-1}, U_{1:t-1}\right)$$

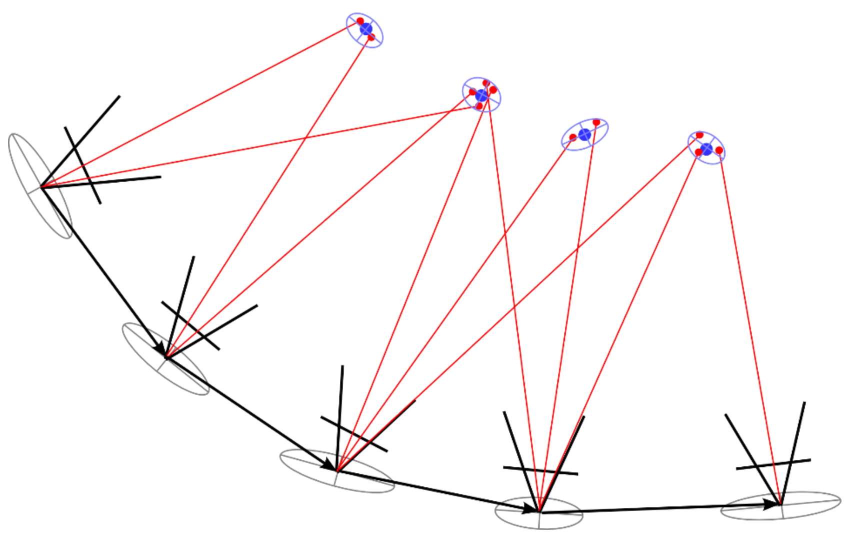
$$= \eta'\, p(\boldsymbol{\xi}_0)\, p(M) \prod_t p\left(Y_t \mid \boldsymbol{\xi}_t, m_{c_t}\right) p\left(\boldsymbol{\xi}_t \mid \boldsymbol{\xi}_{t-1}, U_t\right)$$

Factor Graph

- Factor graph representation of the full SLAM posterior

$$p\left(\boldsymbol{\xi}_{0:t}, M \mid Y_{0:t}, U_{1:t}, c_{0:t}\right) = \eta \; p(\boldsymbol{\xi}_0)\, p(M) \prod_t p\left(Y_t \mid \boldsymbol{\xi}_t, m_{c_t}\right) \, p\left(\boldsymbol{\xi}_t \mid \boldsymbol{\xi}_{t-1}, U_t\right)$$



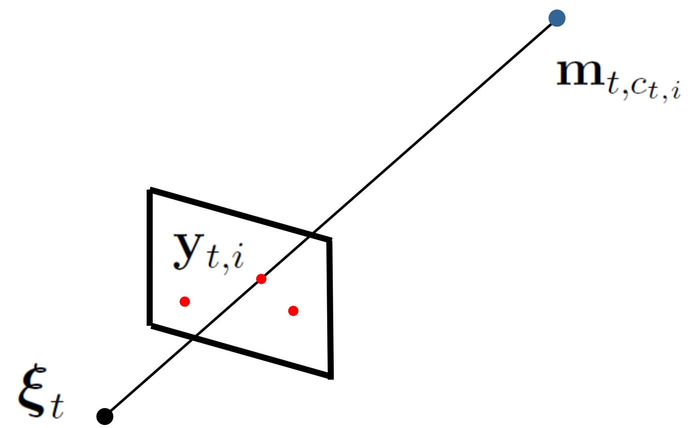


Explicit Model

- N_t noisy 2D point observation of 3D landmarks in each image, known data association

$$\mathbf{y}_{t,i} = h(\boldsymbol{\xi}_t, \mathbf{m}_{t,c_{t,i}}) + \boldsymbol{\delta}_t = \pi\left(\mathbf{T}(\boldsymbol{\xi}_t)^{-1}\overline{\mathbf{m}}_{t,c_{t,i}}\right) + \boldsymbol{\delta}_{t,i}$$

$$\boldsymbol{\delta}_{t,i} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{y}_{t,i}}\right)$$



- No control inputs

- Gaussian prior on pose $\xi_0 \sim \mathcal{N}\left(\boldsymbol{\xi}^0, \boldsymbol{\Sigma}_{0,\xi}\right)$

- Uniform prior on landmarks

Full SLAM Optimization as Energy Minimization

- Optimize negative log posterior probability (MAP estimation)

$$E(\boldsymbol{\xi}_{0:t}, M) = \frac{1}{2} \left(\boldsymbol{\xi}_0 \ominus \boldsymbol{\xi}^0 \right)^\top \Sigma_{0,\xi}^{-1} \left(\boldsymbol{\xi}_0 \ominus \boldsymbol{\xi}^0 \right)$$

$$+ \frac{1}{2} \sum_{\tau=0}^{t} \sum_{i=1}^{N_\tau} \left(\mathbf{y}_{\tau,i} - h(\boldsymbol{\xi}_\tau, \mathbf{m}_{c_{\tau,i}}) \right)^\top \Sigma_{\mathbf{y}_{\tau,i}}^{-1} \left(\mathbf{y}_{\tau,i} - h(\boldsymbol{\xi}_\tau, \mathbf{m}_{c_{\tau,i}}) \right)$$

- Non-linear least squares!! We know how to optimize this..
- Remark: noisy state transitions based on control inputs add further residuals between subsequent poses

Full SLAM Optimization as Energy Minimization

- Let‘s define the residuals on the full state vector

$$\mathbf{r}^0(\mathbf{x}) := \boldsymbol{\xi}_0 \ominus \boldsymbol{\xi}^0$$

$$\mathbf{r}^y_{t,i}(\mathbf{x}) := \mathbf{y}_{t,i} - h(\boldsymbol{\xi}_t, \mathbf{m}_{c_{t,i}})$$

$$\mathbf{x} := \begin{pmatrix} \boldsymbol{\xi}_0 \\ \vdots \\ \boldsymbol{\xi}_t \\ \mathbf{m}_1 \\ \vdots \\ \mathbf{m}_S \end{pmatrix}$$

- Stack the residuals in a vector-valued function and collect the residual covariances on the diagonal blocks of a square matrix

$$\mathbf{r}(\mathbf{x}) := \begin{pmatrix} \mathbf{r}^0(\mathbf{x}) \\ \mathbf{r}^y_{0,1}(\mathbf{x}) \\ \vdots \\ \mathbf{r}^y_{t,N_t}(\mathbf{x}) \end{pmatrix} \qquad \mathbf{W} := \begin{pmatrix} \boldsymbol{\Sigma}^{-1}_{0,\xi} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}^{-1}_{\mathbf{y}_{0,1}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{\Sigma}^{-1}_{\mathbf{y}_{t,N_t}} \end{pmatrix}$$

- Rewrite error function as $E(\mathbf{x}) = \frac{1}{2}\mathbf{r}(\mathbf{x})^\top \mathbf{W}\mathbf{r}(\mathbf{x})$

Recap: Gauss-Newton Method

- Idea: Approximate Newton's method to minimize E(x)
  - Approximate E(x) through linearization of residuals

$$
\begin{aligned}
\widetilde{E}(\mathbf{x}) &= \frac{1}{2}\widetilde{\mathbf{r}}(\mathbf{x})^\top \mathbf{W}\widetilde{\mathbf{r}}(\mathbf{x}) \\
&= \frac{1}{2}\left(\mathbf{r}(\mathbf{x}_k) + \mathbf{J}_k\left(\mathbf{x}-\mathbf{x}_k\right)\right)^\top \mathbf{W}\left(\mathbf{r}(\mathbf{x}_k) + \mathbf{J}_k\left(\mathbf{x}-\mathbf{x}_k\right)\right) \qquad \mathbf{J}_k := \nabla_{\mathbf{x}}\mathbf{r}(\mathbf{x})\big|_{\mathbf{x}=\mathbf{x}_k} \\
&= \frac{1}{2}\mathbf{r}(\mathbf{x}_k)^\top \mathbf{W}\mathbf{r}(\mathbf{x}_k) + \underbrace{\mathbf{r}(\mathbf{x}_k)^\top \mathbf{W}\mathbf{J}_k}_{=:\mathbf{b}_k^\top}\left(\mathbf{x}-\mathbf{x}_k\right) + \frac{1}{2}\left(\mathbf{x}-\mathbf{x}_k\right)^\top \underbrace{\mathbf{J}_k^\top \mathbf{W}\mathbf{J}_k}_{=:\mathbf{H}_k}\left(\mathbf{x}-\mathbf{x}_k\right)
\end{aligned}
$$

  - Find root of $\nabla_{\mathbf{x}}\widetilde{E}(\mathbf{x}) = \mathbf{b}_k^\top + \left(\mathbf{x}-\mathbf{x}_k\right)^\top \mathbf{H}_k$ using Newton's method, i.e.

$$
\nabla_{\mathbf{x}}\widetilde{E}(\mathbf{x}) = \mathbf{0} \text{ iff } \mathbf{x} = \mathbf{x}_k - \mathbf{H}_k^{-1}\mathbf{b}_k
$$

- Pros:
  - Faster convergence (approx. quadratic convergence rate)
- Cons:
  - Divergence if too far from local optimum (H not positive definite)
  - Solution quality depends on initial guess

Structure of the Bundle Adjustment Problem

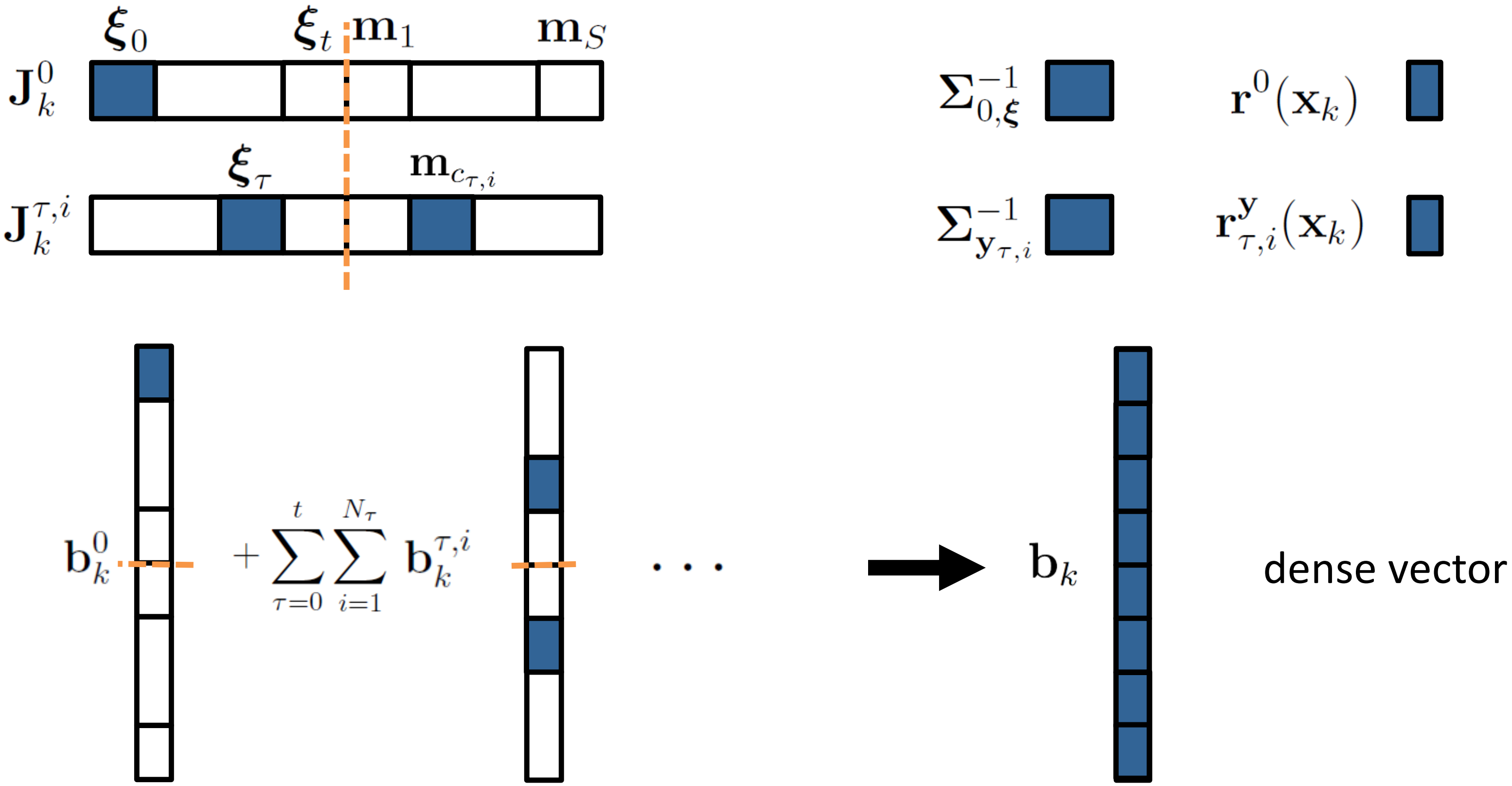
- $\mathbf{b}_k$ and $\mathbf{H}_k$ sum terms from individual residuals:

$$\mathbf{b}_k = \mathbf{b}_k^0 + \sum_{\tau=0}^{t} \sum_{i=1}^{N_\tau} \mathbf{b}_k^{\tau,i} = \left(\mathbf{J}_k^0\right)^\top \Sigma_{0,\boldsymbol{\xi}}^{-1} \mathbf{r}^0(\mathbf{x}_k) + \sum_{\tau=0}^{t} \sum_{i=1}^{N_\tau} \left(\mathbf{J}_k^{\tau,i}\right)^\top \Sigma_{\mathbf{y}_{\tau,i}}^{-1} \mathbf{r}_{\tau,i}^{\mathbf{y}}(\mathbf{x}_k)$$

$$\mathbf{H}_k = \mathbf{H}_k^0 + \sum_{\tau=0}^{t} \sum_{i=1}^{N_\tau} \mathbf{H}_k^{\tau,i} = \left(\mathbf{J}_k^0\right)^\top \Sigma_{0,\boldsymbol{\xi}}^{-1} \left(\mathbf{J}_k^0\right) + \sum_{\tau=0}^{t} \sum_{i=1}^{N_\tau} \left(\mathbf{J}_k^{\tau,i}\right)^\top \Sigma_{\mathbf{y}_{\tau,i}}^{-1} \left(\mathbf{J}_k^{\tau,i}\right)$$

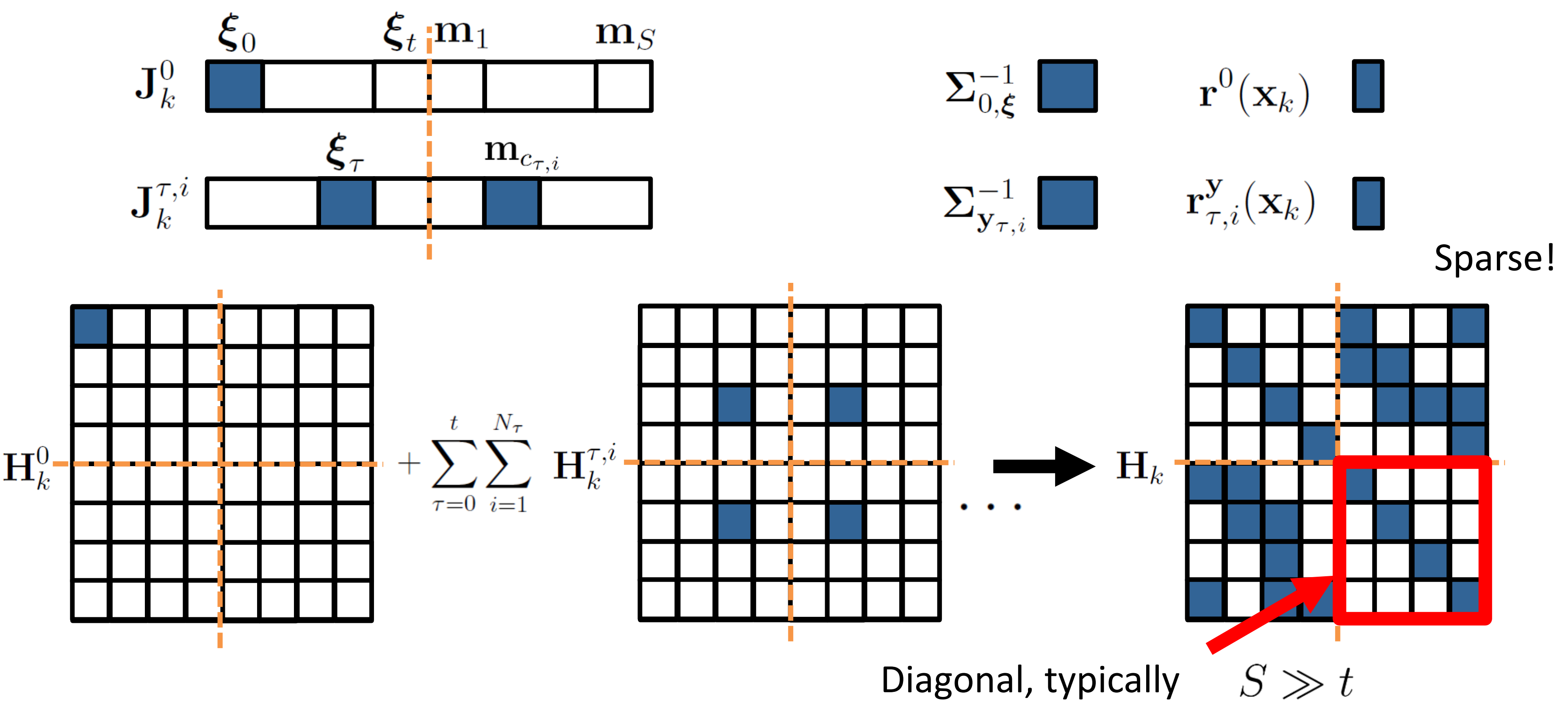
- What is the structure of these terms?

Structure of the Bundle Adjustment Problem



$$\mathbf{b}_k = \mathbf{b}_k^0 + \sum_{\tau=0}^{t}\sum_{i=1}^{N_\tau}\mathbf{b}_k^{\tau,i} = \left(\mathbf{J}_k^0\right)^\top \boldsymbol{\Sigma}_{0,\boldsymbol{\xi}}^{-1}\mathbf{r}^0(\mathbf{x}_k) + \sum_{\tau=0}^{t}\sum_{i=1}^{N_\tau}\left(\mathbf{J}_k^{\tau,i}\right)^\top \boldsymbol{\Sigma}_{\mathbf{y}_{\tau,i}}^{-1}\mathbf{r}_{\tau,i}^{\mathbf{y}}(\mathbf{x}_k)$$

Structure of the Bundle Adjustment Problem



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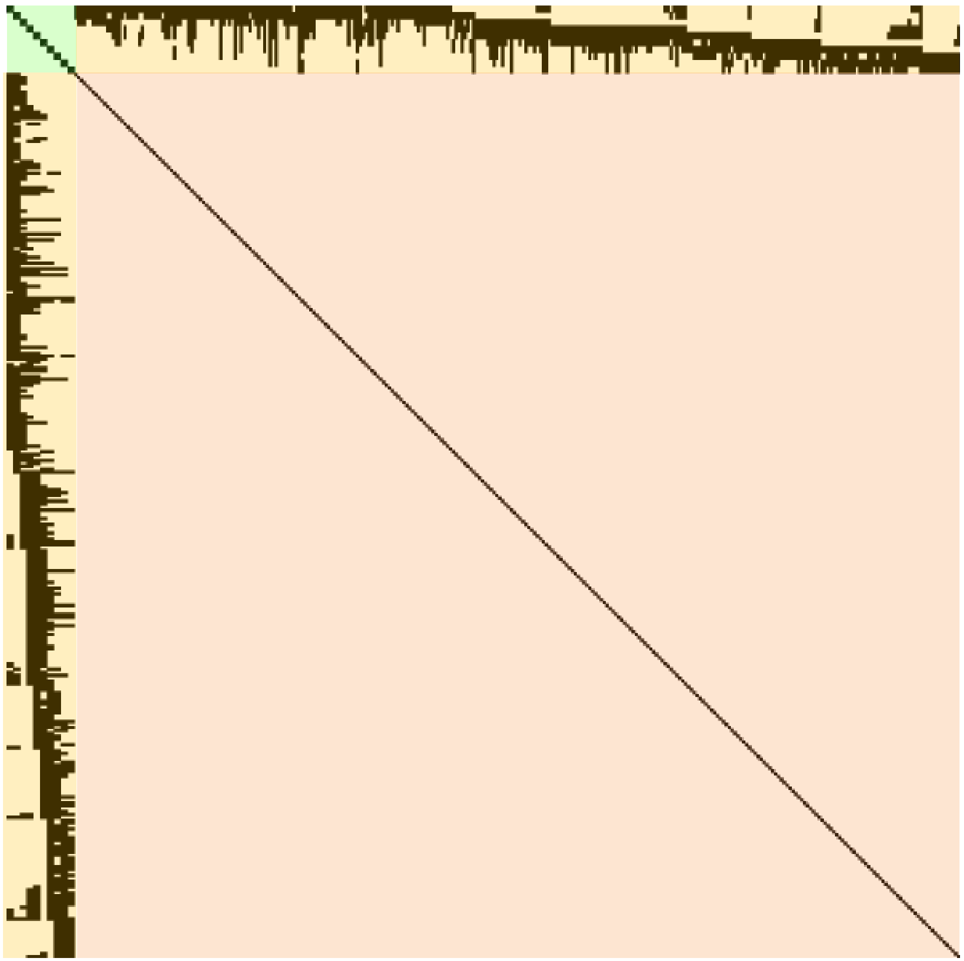
Example Hessian of a BA Problem

Pose dimensions

(10 poses)



Landmark dimensions

(982 landmarks)

Image source: Manolis Lourakis (CC BY 3.0)

Exploiting the Sparse Structure

- Idea:
  Apply the Schur complement to solve the system in a partitioned way

$$\mathbf{H}_k \Delta\mathbf{x} = -\mathbf{b}_k \quad \longrightarrow \quad \begin{pmatrix} \mathbf{H}_{\xi\xi} & \mathbf{H}_{\xi\mathrm{m}} \\ \mathbf{H}_{\mathrm{m}\xi} & \mathbf{H}_{\mathrm{mm}} \end{pmatrix} \begin{pmatrix} \Delta\mathbf{x}_\xi \\ \Delta\mathbf{x}_\mathrm{m} \end{pmatrix} = -\begin{pmatrix} \mathbf{b}_\xi \\ \mathbf{b}_\mathrm{m} \end{pmatrix}$$

$$\longrightarrow \quad \Delta\mathbf{x}_\xi = -\left(\mathbf{H}_{\xi\xi} - \mathbf{H}_{\xi\mathrm{m}}\mathbf{H}_{\mathrm{mm}}^{-1}\mathbf{H}_{\mathrm{m}\xi}\right)^{-1}\left(\mathbf{b}_\xi - \mathbf{H}_{\xi\mathrm{m}}\mathbf{H}_{\mathrm{mm}}^{-1}\mathbf{b}_\mathrm{m}\right)$$

$$\longrightarrow \quad \Delta\mathbf{x}_\mathrm{m} = -\mathbf{H}_{\mathrm{mm}}^{-1}\left(\mathbf{b}_\mathrm{m} + \mathbf{H}_{\mathrm{m}\xi}\Delta\mathbf{x}_\xi\right)$$

- Is this any better?

Exploiting the Sparse Structure

- What is the structure of the two sub-problems ?

$$\Delta \mathbf{x}_\xi = -\left(\mathbf{H}_{\xi\xi} - \mathbf{H}_{\xi \mathbf{m}} \mathbf{H}_{\mathbf{mm}}^{-1} \mathbf{H}_{\mathbf{m}\xi}\right)^{-1} \left(\mathbf{b}_\xi - \mathbf{H}_{\xi \mathbf{m}} \mathbf{H}_{\mathbf{mm}}^{-1} \mathbf{b}_{\mathbf{m}}\right)$$

- Poses:

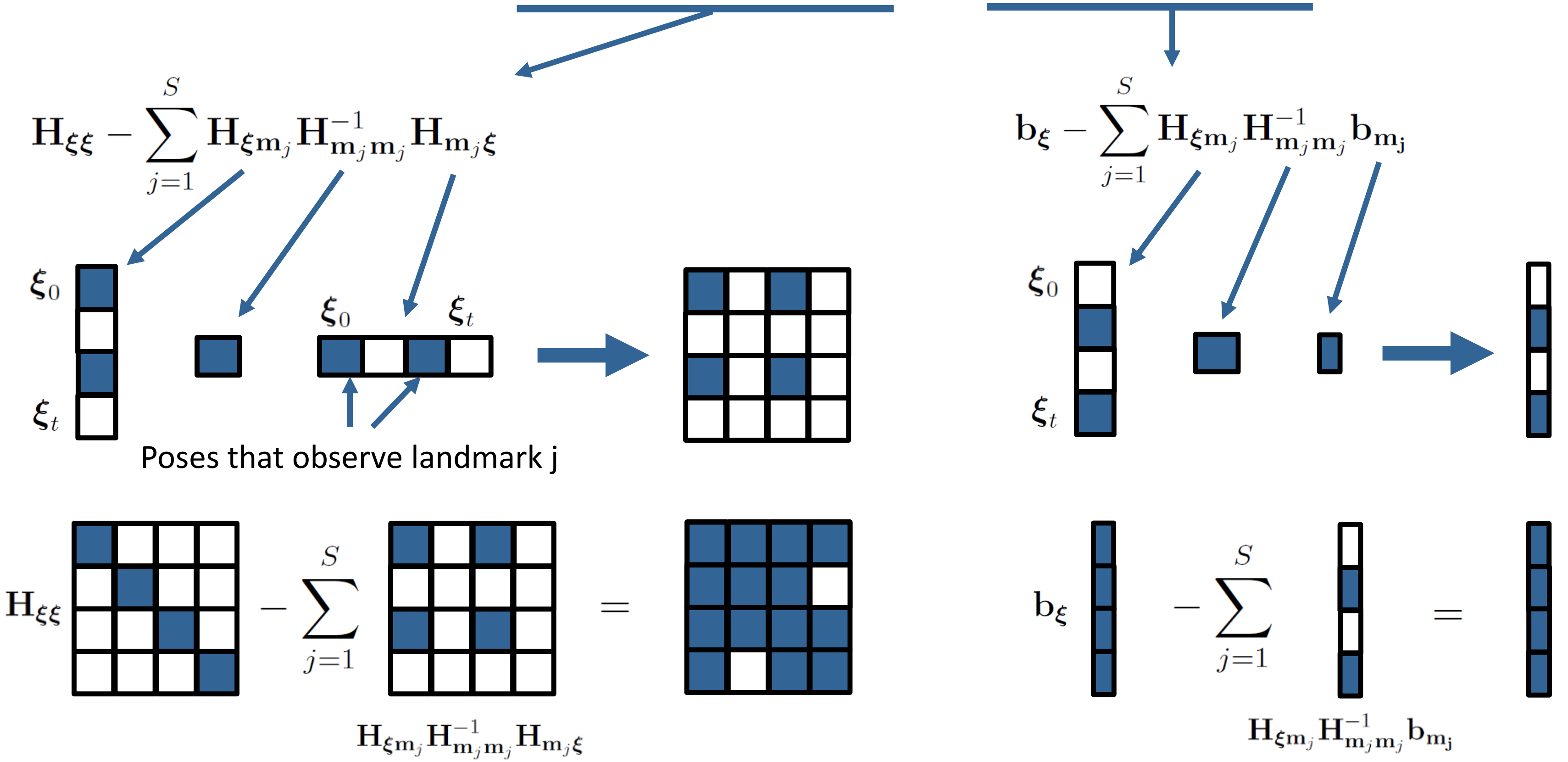
$$\mathbf{H}_{\xi\xi} - \mathbf{H}_{\xi \mathbf{m}} \mathbf{H}_{\mathbf{mm}}^{-1} \mathbf{H}_{\mathbf{m}\xi} = \mathbf{H}_{\xi\xi} - \sum_{j=1}^{S} \mathbf{H}_{\xi \mathbf{m}_j} \mathbf{H}_{\mathbf{m}_j \mathbf{m}_j}^{-1} \mathbf{H}_{\mathbf{m}_j \xi}$$

Reduced pose Hessian

$$\mathbf{b}_\xi - \mathbf{H}_{\xi \mathbf{m}} \mathbf{H}_{\mathbf{mm}}^{-1} \mathbf{b}_{\mathbf{m}} = \mathbf{b}_\xi - \sum_{j=1}^{S} \mathbf{H}_{\xi \mathbf{m}_j} \mathbf{H}_{\mathbf{m}_j \mathbf{m}_j}^{-1} \mathbf{b}_{\mathbf{m}_j}$$

Exploiting the Sparse Structure

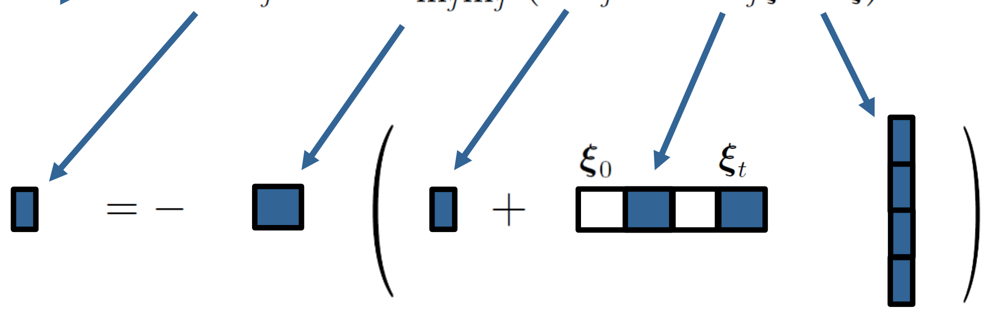
- What is the structure of the two sub-problems ?
- Poses: $\Delta \mathbf{x}_\xi = -\left(\mathbf{H}_{\xi\xi} - \mathbf{H}_{\xi\mathbf{m}}\mathbf{H}_{\mathbf{mm}}^{-1}\mathbf{H}_{\mathbf{m}\xi}\right)^{-1}\left(\mathbf{b}_\xi - \mathbf{H}_{\xi\mathbf{m}}\mathbf{H}_{\mathbf{mm}}^{-1}\mathbf{b}_\mathbf{m}\right)$



Exploiting the Sparse Structure

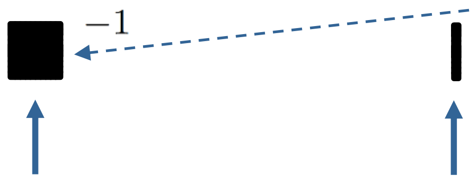
- What is the structure of the two sub-problems ?
- Landmarks: $\Delta\mathbf{x}_{\mathbf{m}} = -\mathbf{H}_{\mathbf{mm}}^{-1}\left(\mathbf{b}_{\mathbf{m}} + \mathbf{H}_{\mathbf{m}\xi}\Delta\mathbf{x}_{\xi}\right)$

$$\Rightarrow \quad \Delta\mathbf{x}_{\mathbf{m}_j} = -\mathbf{H}_{\mathbf{m}_j\mathbf{m}_j}^{-1}\left(\mathbf{b}_{\mathbf{m}_j} + \mathbf{H}_{\mathbf{m}_j\xi}\Delta\mathbf{x}_{\xi}\right)$$



  - Landmark-wise solution
  - Comparably small matrix operations
  - Only involves poses that observe the landmark

Exploiting the Sparse Structure



$$\Delta \mathbf{x}_\xi = -\left(\mathbf{H}_{\xi\xi} - \mathbf{H}_{\xi \mathrm{m}} \mathbf{H}_{\mathrm{mm}}^{-1} \mathbf{H}_{\mathrm{m}\xi}\right)^{-1} \left(\mathbf{b}_\xi - \mathbf{H}_{\xi \mathrm{m}} \mathbf{H}_{\mathrm{mm}}^{-1} \mathbf{b}_{\mathrm{m}}\right)$$

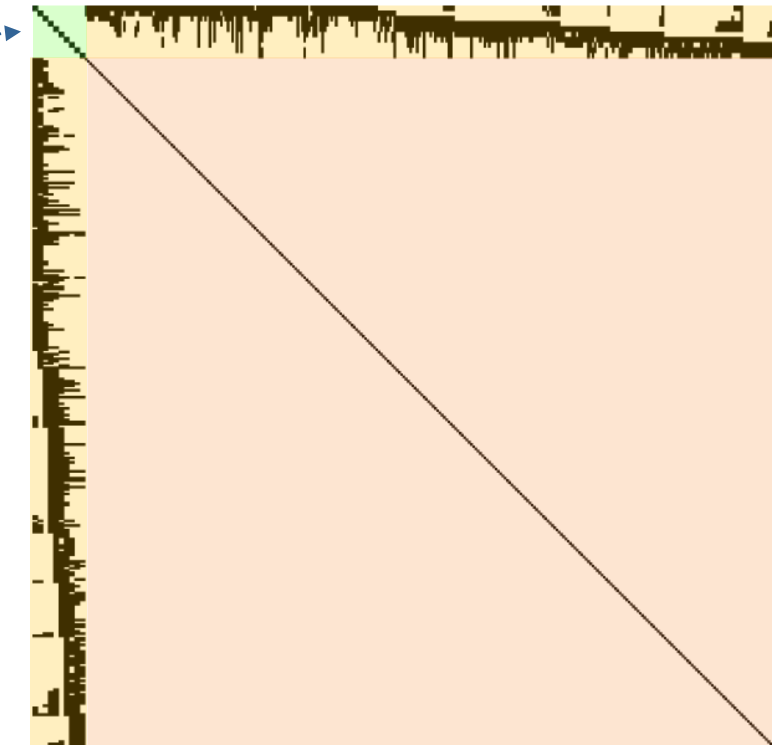
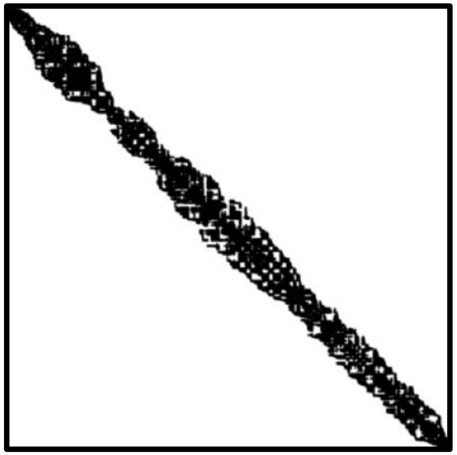
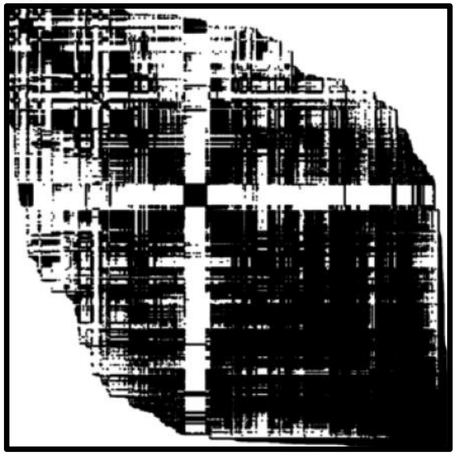


Image source: Manolis Lourakis (CC BY 3.0)

Exploiting the Sparse Structure



Camera on a moving vehicle
(6375 images)

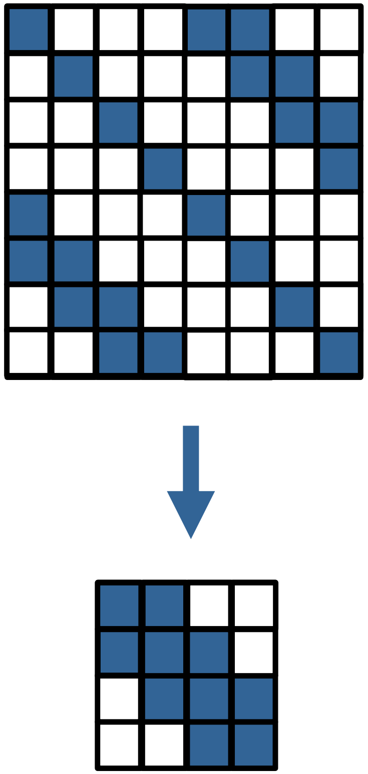


Flickr image search „Dubrovnik“
(4585 images)

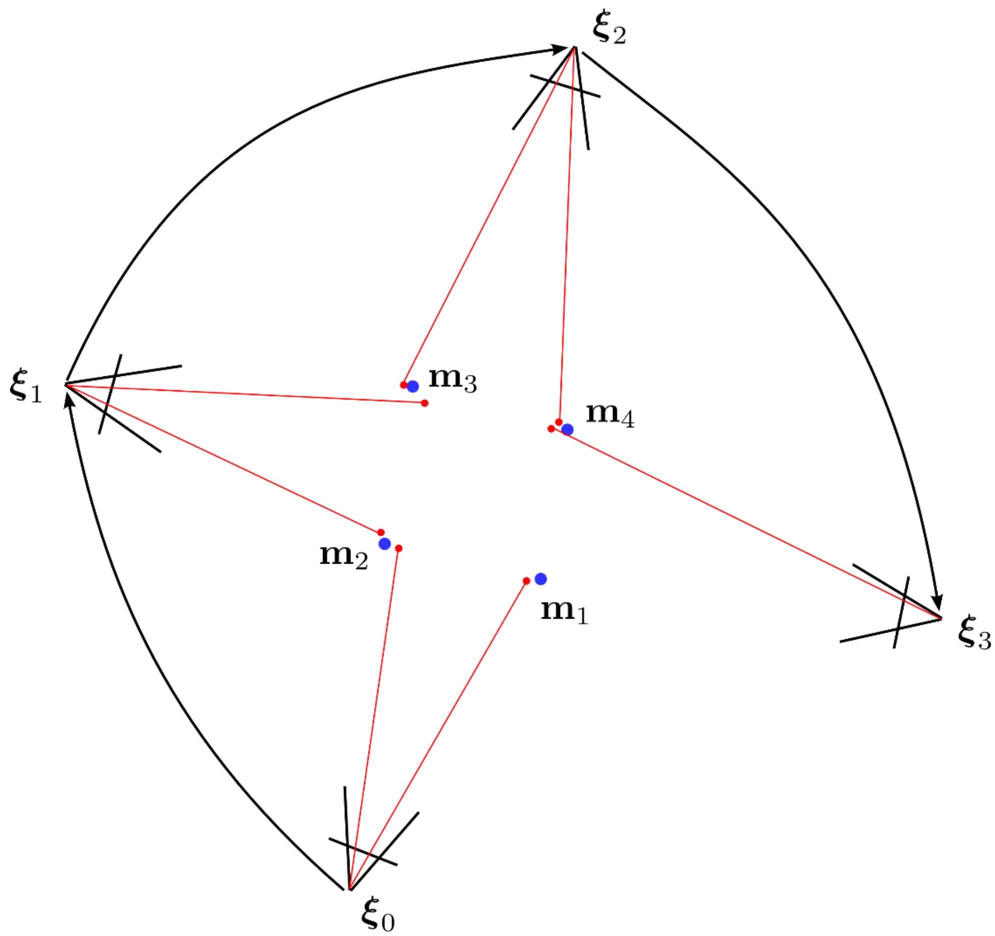
- Reduced pose Hessian can still have sparse structure
- However: For many camera poses with many shared observations, the inversion of the reduced pose Hessian is still computationally expensive!
- Exploit further structure, e.g., using variable reordering or hierarchical decomposition

Image from Agarwal et al., ICCV 2009

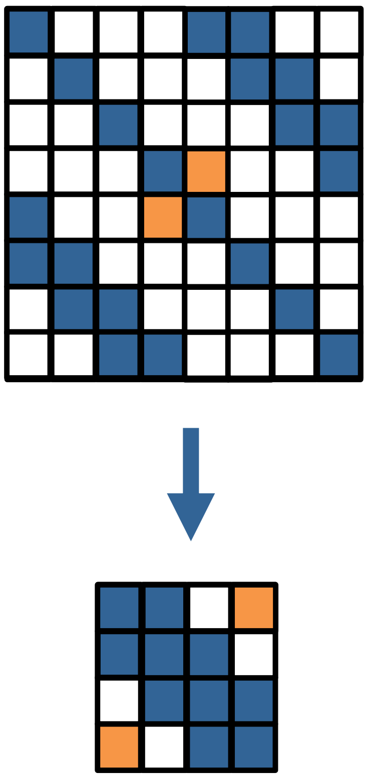
Effect of Loop-Closures on the Hessian



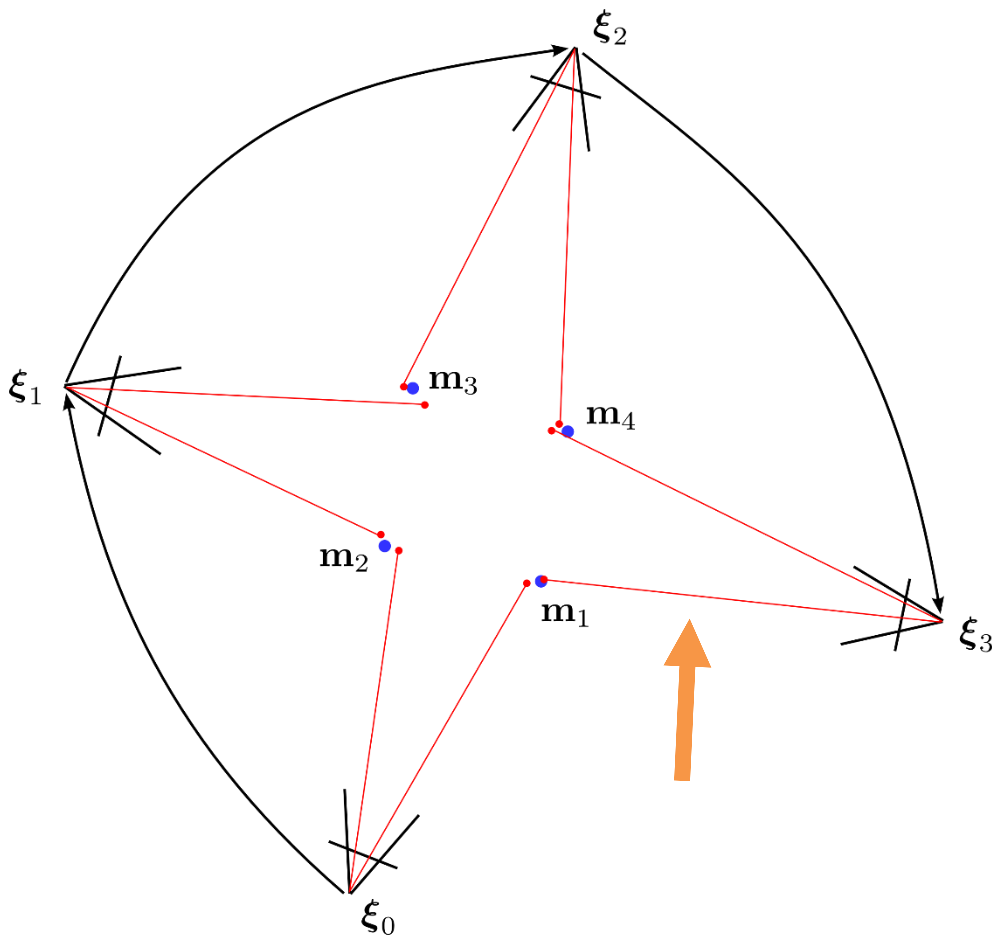
Band matrix



Effect of Loop-Closures on the Hessian



Not band matrix: costlier to solve



Further Considerations

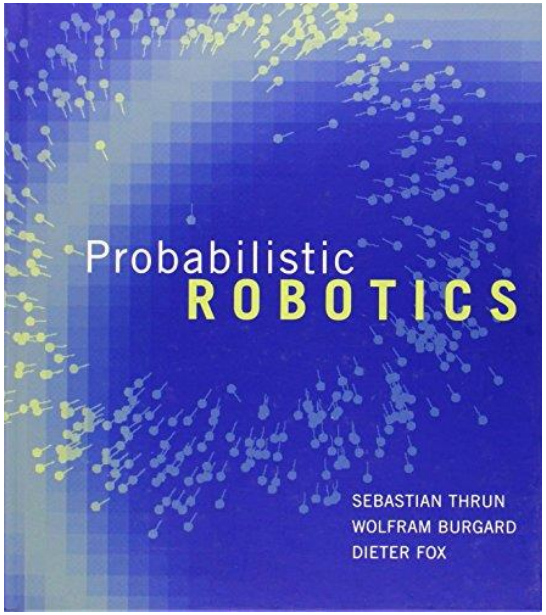
- Use matrix decompositions (f.e. Cholesky) to perform inversions
- Levenberg-Marquardt optimization improves basin of convergence
- Heavier-tail distributions / robust norms on the residuals can be implemented using Iteratively Reweighted Least Squares
- Twists are also a suitable pose parametrization for bundle adjustment: optimize increments on the twists
- Many further tricks to improve convergence/robustness/run-time efficiency, f.e.:
  - Preconditioning
  - Hierarchical optimization
  - Variable reordering
  - Delayed relinearization

Lessons Learned Today

- SLAM is a chicken-or-egg problem:
  - Localization requires map
  - Mapping requires localization
  - Unknown association of measurements to map elements
- Bundle Adjustment has a sparse structure that can be exploited for efficient optimization
- Reduction of BA to pose optimization problem through marginalization of landmarks (using the Schur complement)
- Loop closure constraints make SLAM optimization problem less efficient to solve (but reduce drift!)

Further Reading

- Probabilistic Robotics textbook



Probabilistic Robotics,
S. Thrun, W. Burgard, D. Fox,
MIT Press, 2005

- Triggs et al., Bundle Adjustment – A Modern Synthesis, 2002

Thanks for your attention!