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Robotic 3D Vision

Lecture 14: Visual SLAM 5 – DSO, SLAM Overview

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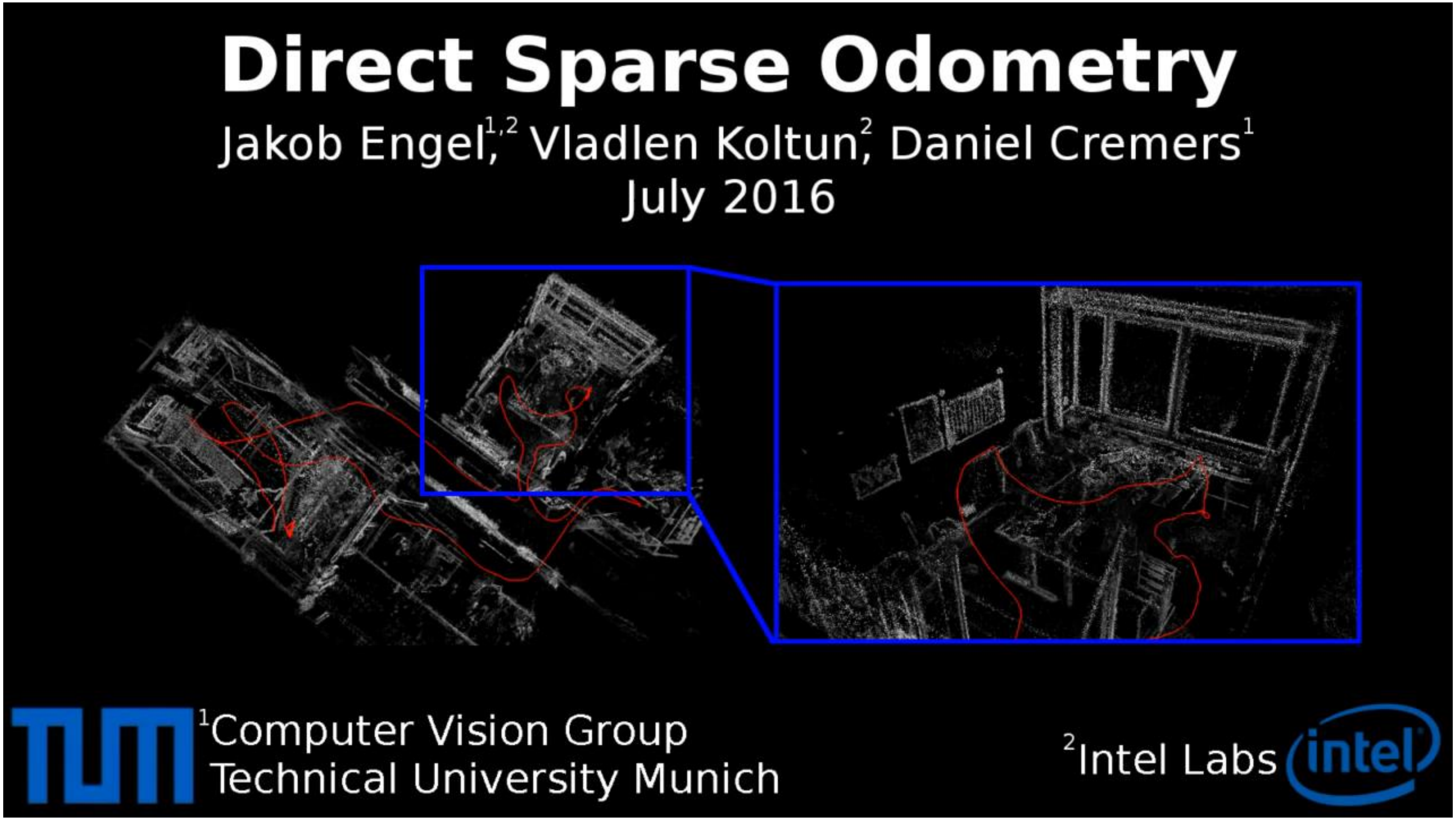
What We Will Cover Today

- Direct Sparse Odometry

- Overview on Visual Odometry and SLAM

Recap: Direct Sparse Odometry (DSO)



Direct Sparse Odometry

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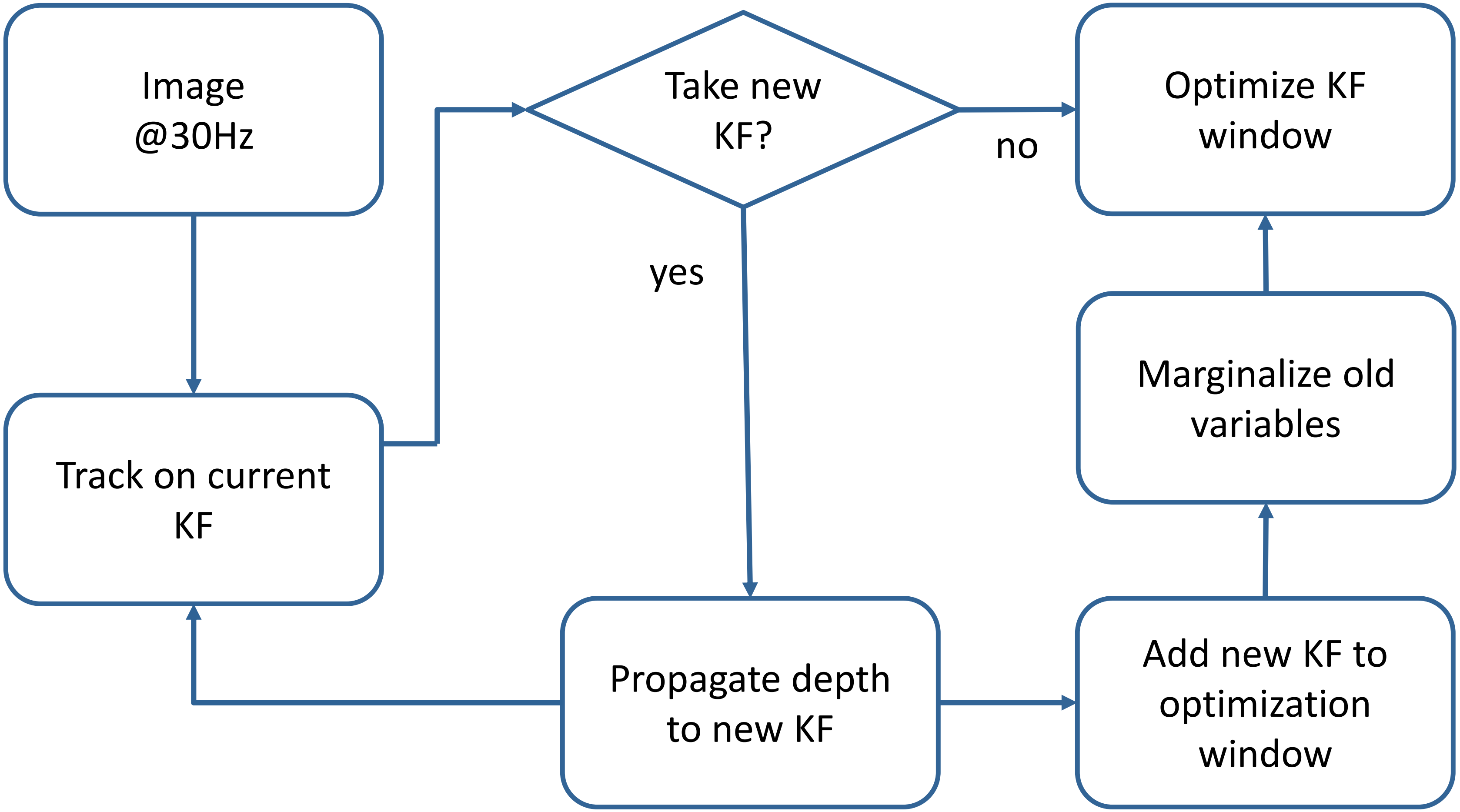
Recap: DSO Algorithm Overview

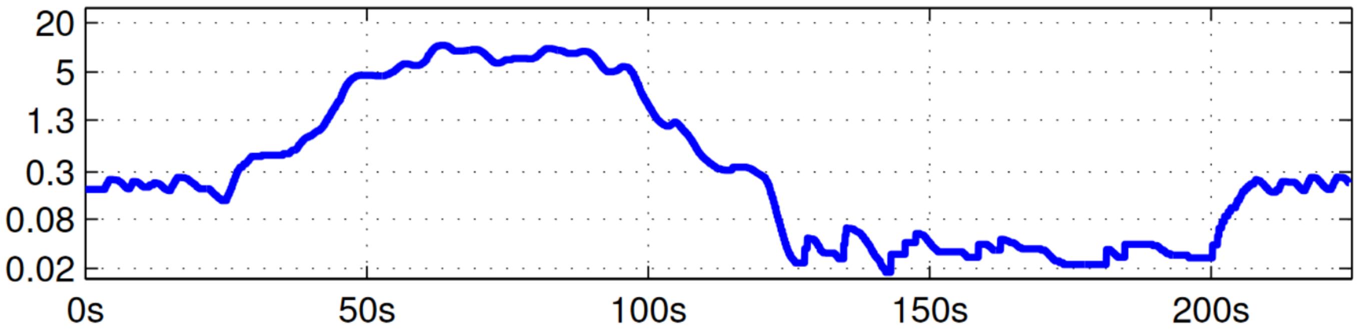


Brightness Constancy Assumption Revisited

- Camera images include vignetting effects and non-linear camera response function
- Idea: invert vignetting and camera response function using a known calibration
- Perform direct image alignment on irradiance images:

$$I'(\mathbf{y}) = tB(\mathbf{y}) = \frac{G^{-1}(I(\mathbf{y}))}{V(\mathbf{y})}$$

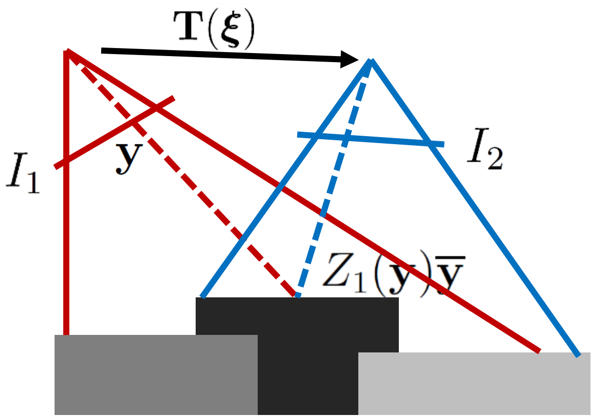
Brightness Constancy Assumption Revisited



- Automatic exposure and gain adjustment needed in realistic environments
- Add exposure and affine gain parameters explicitly to objective function:

$$r_I = \left(I_2(\omega(\mathbf{y}, \boldsymbol{\xi}, Z_1(\mathbf{y}))) - b_2\right) - \frac{t_2 \exp(a_2)}{t_1 \exp(a_1)} \left(I_1(\mathbf{y}) - b_1\right)$$

Tracking on Keyframe



$\mathbf{T}(\boldsymbol{\xi})$
I_1
$\mathbf{y}$
I_2
$Z_1(\mathbf{y})\bar{\mathbf{y}}$

- Direct image alignment of current frame to most recent keyframe

$$\zeta^* = \arg\min -\log(p(\zeta)) - \sum_{\mathbf{y}\in\Omega_Z} \log p(r(\mathbf{y},\zeta) \mid \zeta)$$

- Photometric residuals with photometric calibration

$$r_I = \left(I_2(\omega(\mathbf{y},\boldsymbol{\xi},Z_1(\mathbf{y}))) - b_2\right) - \frac{t_2\exp(a_2)}{t_1\exp(a_1)}\left(I_1(\mathbf{y}) - b_1\right)$$

- Optimized parameters ζ now include photometric calibration

Tracking on Keyframe

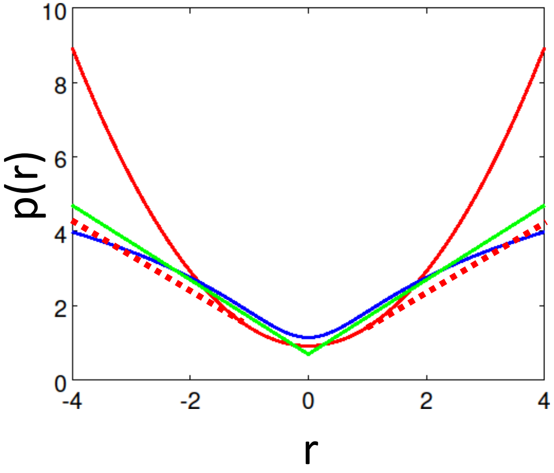
- Residual distribution

$$E(\zeta) = \sum_{\mathbf{y} \in \Omega_Z} w_{\mathbf{y}} \, \| r(\mathbf{y}, \zeta) \|_\delta$$

  - Huber loss on residuals
  - Additional gradient dependent weight

$$w_{\mathbf{y}} := \frac{c^2}{c^2 + \| \nabla I_1(\mathbf{y}) \|_2^2}$$

- Solved using iteratively reweighted least squares



- Normal distribution
- Laplace distribution
- Student-t distribution

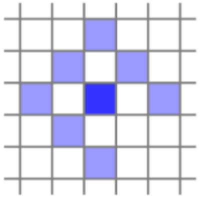
┈┈┈┈ Huber-loss for δ = 1

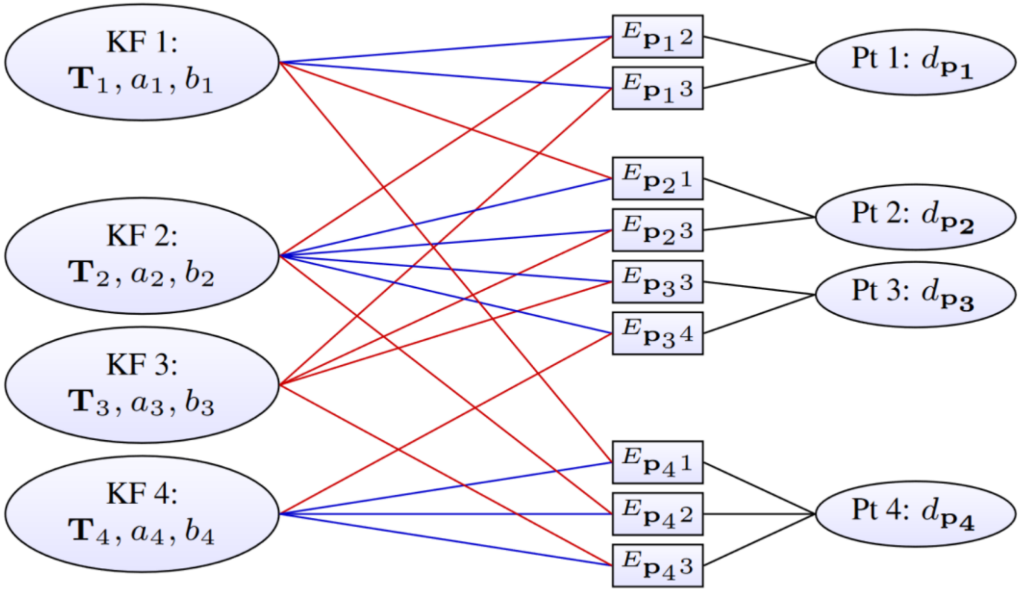
Fixed-Lag Smoothing

- Optimize direct image alignment error function
- Optimize in a recent window for
  - keyframe poses and photometric calibration
  - inverse depth of sparse set of active points
- Pose in SE(3)
- Marginalization of old variables

$$E_{\text{photo}} := \sum_{i\in\mathcal{F}} \sum_{\mathbf{p}\in\mathcal{P}_i} \sum_{j\in\text{obs}(\mathbf{p})} E_{\mathbf{p}j}$$

$$E_{\mathbf{p}j} := \sum_{\mathbf{p}\in\mathcal{N}_{\mathbf{p}}} w_{\mathbf{p}} \left\| r(\mathbf{p},\zeta_{ij}) \right\|_{\delta}$$

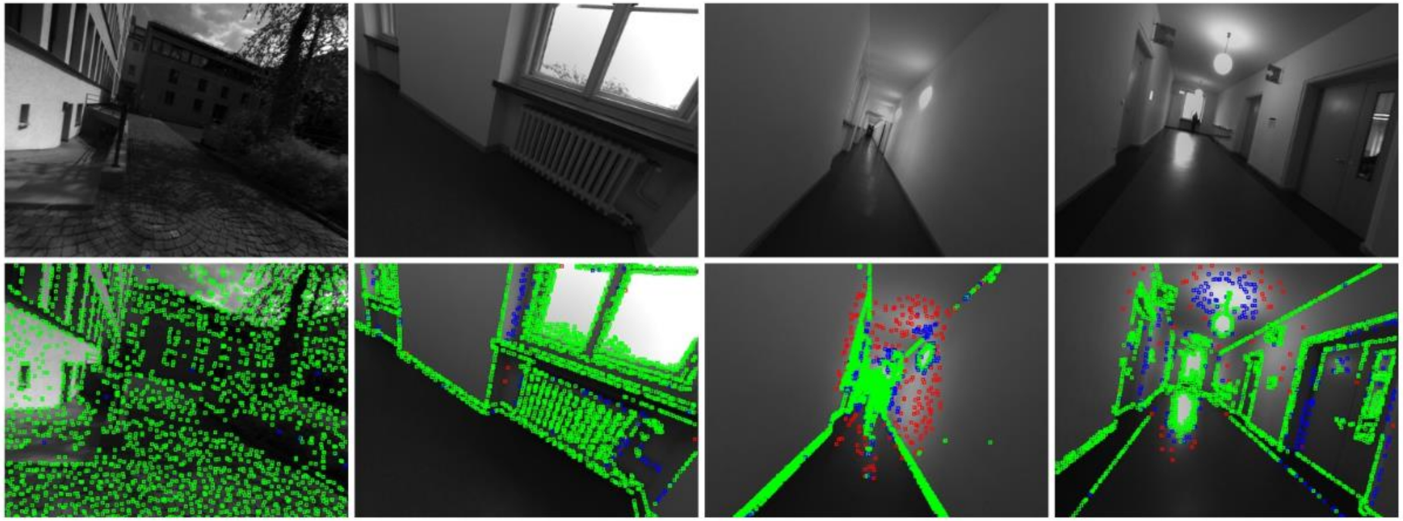




Depth Estimation

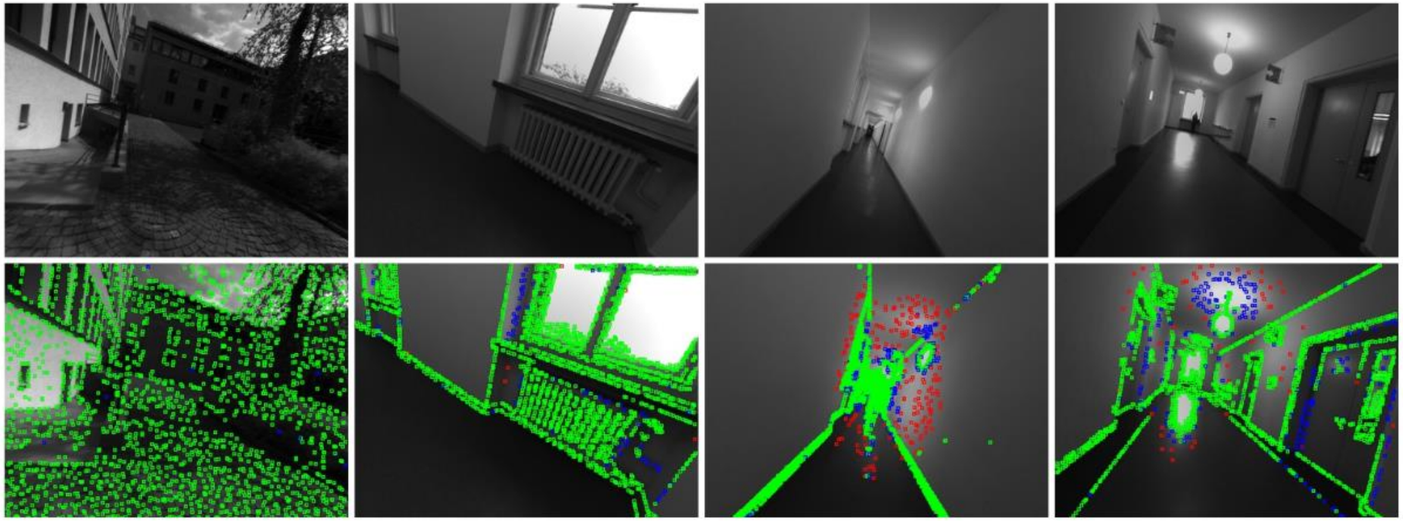
- Optimize inverse depth of a set of N_p points in all keyframes in bundle adjustment window

- Initialization of inverse depth of new points by fusion of short-baseline stereo comparisons from subsequent frames (similar to LSD-SLAM)



Depth Estimation

- Candidate point selection
  - Region-adaptive gradient magnitude threshold in 32x32 gridcells
  - Adaptive block size d to subdivide image into dxd blocks, select pixel with largest gradient magnitude above adaptive cell threshold
  - Adapt block size to obtain N_p pixels in each keyframe



Keyframe Selection

- Several criteria to decide when to create new keyframe
  - Mean square optical flow of points in latest keyframe towards current frame during tracking

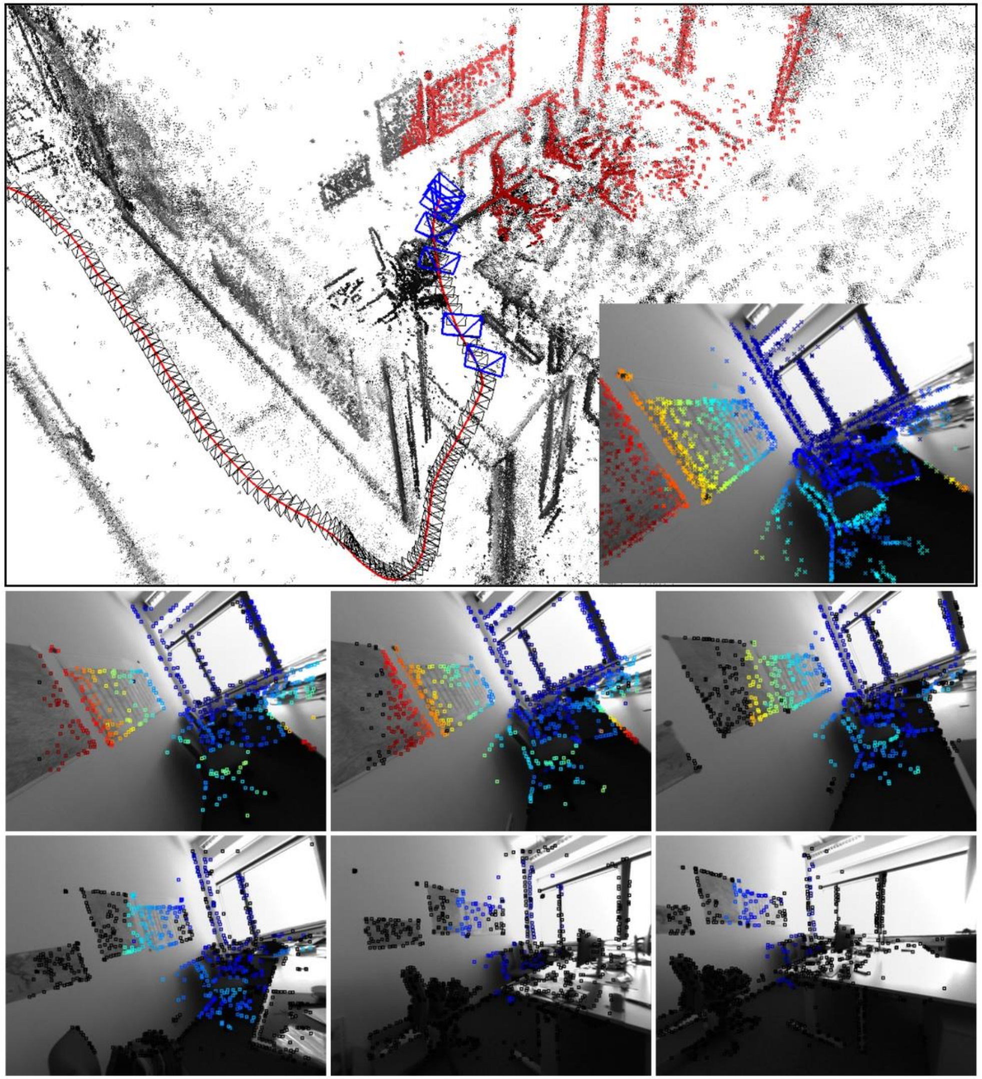
$$f := \left(\frac{1}{n} \sum_{i=1}^{n} \|\mathbf{p} - \mathbf{p}'\|^2\right)^{\frac{1}{2}}$$

  - Mean flow without rotation (translations cause occlusion effects, despite low f)
  - Relative brightness factor between keyframe and current frame

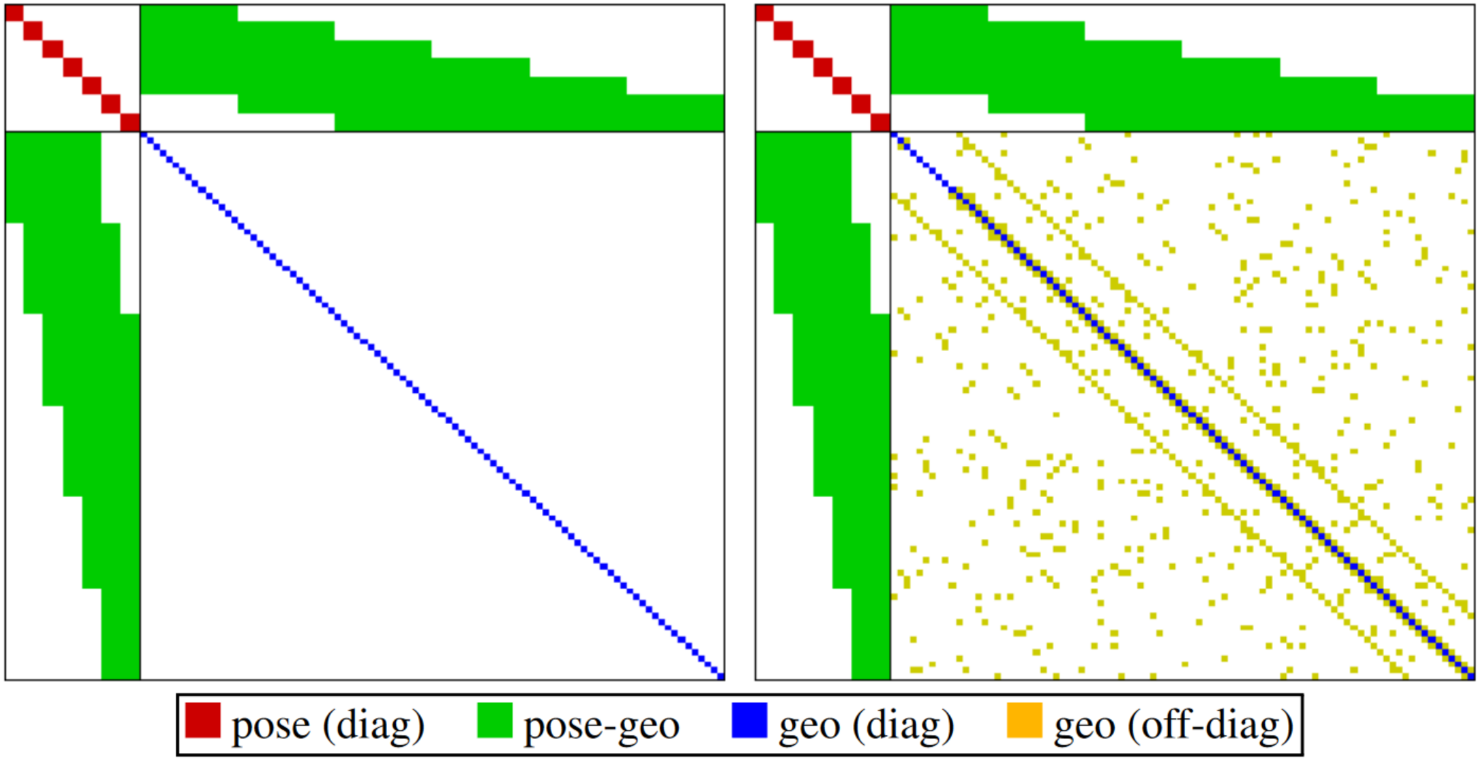
$$a := \left|\log(e^{a_j - a_i} t_j t_i^{-1})\right|$$

- Threshold linear combination of criteria

Keyframe Selection



Structure of the Hessian



- DSO neglects spatial correlations of depth estimates in image
- Hessian block on depths is diagonal

Marginalization

- Goal of marginalization is
  - to keep information of old poses and depths as prior without relinearizing and updating old variables
  - to maintain the structure of the Hessian (no fill-in for depths!)

- Marginalization of a keyframe proceeds by
  - First marginalize all points hosted in the keyframe before the keyframe pose
  - Marginalize points without observations in last two keyframes
  - Drop observations of points from other keyframes in the marginalized keyframe to keep sparsity of Hessian

Recap: Gauss-Newton Method

- Approximate Newton's method to minimize E(x)
  - Approximate E(x) through linearization of residuals

$$\begin{aligned}\widetilde{E}(\mathbf{x}) &= \frac{1}{2}\widetilde{\mathbf{r}}(\mathbf{x})^\top \mathbf{W}\widetilde{\mathbf{r}}(\mathbf{x}) \\ &= \frac{1}{2}\left(\mathbf{r}(\mathbf{x}_k) + \mathbf{J}_k\left(\mathbf{x}-\mathbf{x}_k\right)\right)^\top \mathbf{W}\left(\mathbf{r}(\mathbf{x}_k) + \mathbf{J}_k\left(\mathbf{x}-\mathbf{x}_k\right)\right) \qquad \mathbf{J}_k := \nabla_{\mathbf{x}}\mathbf{r}(\mathbf{x})\big|_{\mathbf{x}=\mathbf{x}_k} \\ &= \frac{1}{2}\mathbf{r}(\mathbf{x}_k)^\top \mathbf{W}\mathbf{r}(\mathbf{x}_k) + \underbrace{\mathbf{r}(\mathbf{x}_k)^\top \mathbf{W}\mathbf{J}_k}_{=:\mathbf{b}_k^\top}\left(\mathbf{x}-\mathbf{x}_k\right) + \frac{1}{2}\left(\mathbf{x}-\mathbf{x}_k\right)^\top \underbrace{\mathbf{J}_k^\top \mathbf{W}\mathbf{J}_k}_{=:\mathbf{H}_k}\left(\mathbf{x}-\mathbf{x}_k\right)\end{aligned}$$

  - Find root of $\nabla_{\mathbf{x}}\widetilde{E}(\mathbf{x}) = \mathbf{b}_k^\top + \left(\mathbf{x}-\mathbf{x}_k\right)^\top \mathbf{H}_k$ using Newton's method, i.e.

$$\nabla_{\mathbf{x}}\widetilde{E}(\mathbf{x}) = \mathbf{0} \text{ iff } \mathbf{x} = \mathbf{x}_k - \mathbf{H}_k^{-1}\mathbf{b}_k$$

- Pros:
  - Faster convergence (approx. quadratic convergence rate)
- Cons:
  - Divergence if too far from local optimum (H not positive definite)
  - Solution quality depends on initial guess

Marginalization

- More formally, consider GN method for error function E(x)

$$\nabla_{\mathbf{x}} \widetilde{E}(\mathbf{x}) = \mathbf{0} \text{ iff } \mathbf{x} = \mathbf{x}_k - \mathbf{H}_k^{-1} \mathbf{b}_k$$

- Split into variables $\mathbf{x}_\alpha$ to keep and $\mathbf{x}_\beta$ to marginalize

$$\begin{pmatrix} \mathbf{H}_{\alpha\alpha} & \mathbf{H}_{\alpha\beta} \\ \mathbf{H}_{\beta\alpha} & \mathbf{H}_{\beta\beta} \end{pmatrix} \begin{pmatrix} \boldsymbol{\Delta}\mathbf{x}_\alpha \\ \boldsymbol{\Delta}\mathbf{x}_\beta \end{pmatrix} = - \begin{pmatrix} \mathbf{b}_\alpha \\ \mathbf{b}_\beta \end{pmatrix}$$

- Applying the Schur complement yields

$$\widehat{\mathbf{H}}_{\alpha\alpha} = \mathbf{H}_{\alpha\alpha} - \mathbf{H}_{\alpha\beta} \mathbf{H}_{\beta\beta}^{-1} \mathbf{H}_{\beta\alpha}$$

$$\widehat{\mathbf{b}}_\alpha = \mathbf{b}_\alpha - \mathbf{H}_{\alpha\beta} \mathbf{H}_{\beta\beta}^{-1} \mathbf{b}_\beta$$

- Add as additional prior to GN optimization

Marginalization

- Several criteria to decide when to marginalize a keyframe
  - Always keep the latest two keyframes
  - Keyframes with less than 5% visible points are marginalized
  - If more than N_f keyframes, marginalize keyframe which maximizes

$$s(I_i) = \sqrt{d(i,1)} \sum_{j \in [3,n] \setminus \{i\}} (d(i,j) + \epsilon)^{-1}$$

translation distance between frame i and j

small constant

Comparison of Direct SLAM Methods

DVO-SLAM	LSD-SLAM	DSO
+ RGB-D cameras	+ monocular cameras + stereo cameras	+ monocular cameras + stereo cameras
+ global consistency	+ global consistency	- no global consistency
camera pose tracking towards keyframe	camera pose tracking towards keyframe	camera pose tracking towards keyframe
+ depth from sensor	+ depth from stereo comparisons & filtering	++ depth optimization using photometric residuals in local keyframe window
tracking-only & pose graph optimization	tracking-and-mapping & pose graph optimization	tracking-and-mapping & direct sparse bundle adjustment in local keyframe window with marginalization
+ local accuracy	+ local accuracy	++ local accuracy

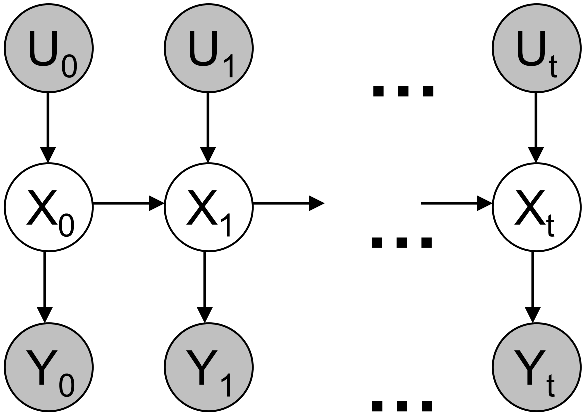
VO / VSLAM Overview

- Lecture blocks so far
  - Image formation and multiple view geometry
  - Probabilistic state estimation
  - Visual and visual-inertial odometry
  - Visual SLAM

- Outlook
  - 3D object detection and tracking
  - Dense reconstruction and map representations
  - Introduction to non-rigid reconstruction

Probabilistic State Estimation

- Probabilistic formulation of visual odometry and SLAM algorithms as inference in hidden Markov models



- Observation model $p\left(Y_t \middle| X_{0:t}, U_{0:t}, Y_{0:t-1}\right) = \boxed{p\left(Y_t \middle| X_t\right)}$

- State-transition model $p\left(X_t \middle| X_{0:t-1}, U_{0:t}\right) = \boxed{p\left(X_t \middle| X_{t-1}, U_t\right)}$

Probabilistic State Estimation

- Filtering: recursive estimation of most recent state (f.e. most recent camera pose)
  - Recursive Bayesian filter
  - (Extended) Kalman filter
  - Particle filter

Predict: $$p(X_t \mid y_{0:t-1}, u_{0:t}) = \int p(X_t \mid X_{t-1}, u_t)\, p(X_{t-1} \mid y_{0:t-1}, u_{0:t-1})\, dX_{t-1}$$

observation

y_t

action

u_t

Correct: $$p(X_t \mid y_0, \ldots, y_t) = \frac{p(y_t \mid X_t)\, p(X_t \mid y_{0:t-1}, u_{0:t})}{\int p(y_t \mid X_t)\, p(X_t \mid y_{0:t-1}, u_{0:t})\, dX_t}$$

Probabilistic State Estimation

- Full state posterior estimation
  - Gaussian noise models, non-linear models leads to non-linear least squares
  - Gauss-Newton method, typically offline
  - Other noise models: Iteratively reweighted least squares

$$p\left(X_{0:t} \middle| U_{1:t}, Y_{0:t}\right) = p\left(X_0\right)\left(\prod_{\tau=0}^{t} \eta_\tau\, p\left(Y_\tau \middle| X_\tau\right)\right)\left(\prod_{\tau=1}^{t} p\left(X_\tau \middle| X_{\tau-1}, U_\tau\right)\right)$$

$$\Downarrow$$

$$\arg\min_{\mathbf{x}} E(\mathbf{x}) = \tfrac{1}{2}\mathbf{r}(\mathbf{x})^\top \mathbf{W}\mathbf{r}(\mathbf{x})$$

Probabilistic State Estimation

- Fixed-lag smoothing:
  - Inference of a window of recent states
  - Marginalization of remaining states
  - Trade-off between recursive filtering (faster) and full state posterior estimation (more accurate)

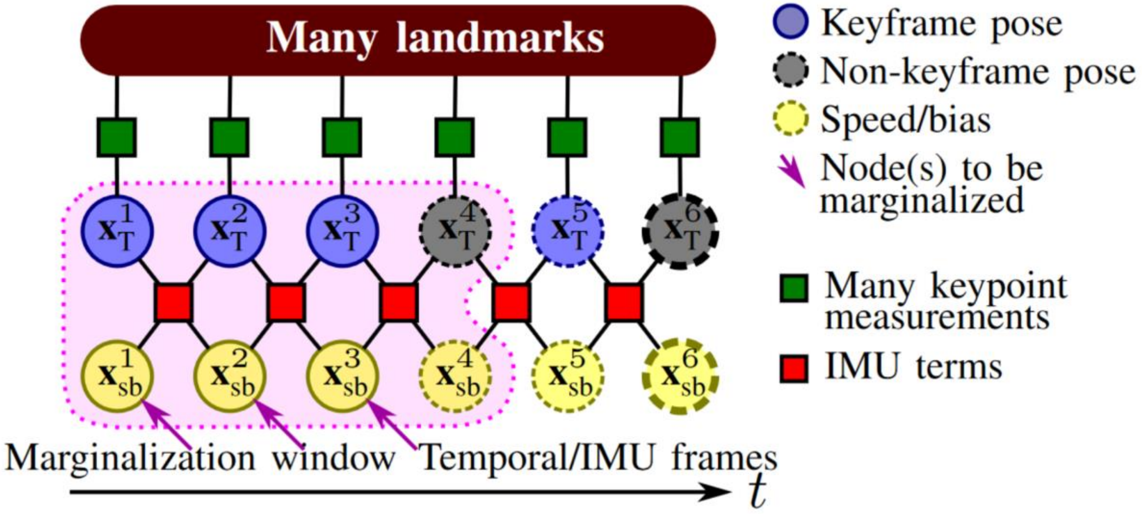


Image source: Leutenegger et al., IJRR 2015

State Estimation Approaches

Filtering	Fixed-Lag Smoothing	Full State Posterior Estimation
Recursive Bayesian filtering of the most recent state (e.g. Kalman Filter)	Optimize window of states through non-linear optimization and marginalization of old states	Full posterior optimization of all states through non-linear least squares
- Single linearization	+ Relinearize (in window)	+ Relinearize
- Accumulation of linearization errors	- Accumulation of linearization errors	+ Sparse Matrices
- Gaussian approximation of marginalized states	- Gaussian approximation of marginalized states	+ Highest Accuracy
+ Faster	+ Fast	+ Slow

Visual Odometry vs. SLAM

Visual Odometry	Visual SLAM
Estimate motion of object from measurements of visual sensor on the object	Estimation motion of object and map of environment from measurements of visual sensor on the object
Real-time requirements	Real-time tracking, lower frame-rate loop closing and global optimization
Local consistency, drift	Local and/or global consistency
Map/3D reconstruction as a side-product	Concurrent accurate map estimation/3D reconstruction

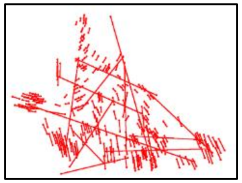
Indirect vs. Direct Methods

Indirect

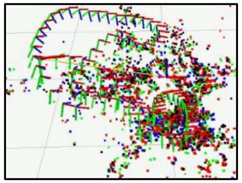
Input images

Extract and match keypoints (SIFT,BRIEF,…)

Track: min. reprojection error (point distances)



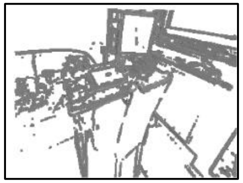
Map: estimate keypoint parameters (f.e. 3D coordinates)



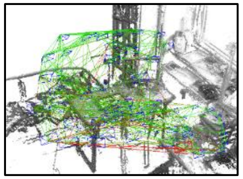
Direct

Input images

Track: min. photometric/ geometric error pixel-wise



Map: estimate per-pixel depth from photoconsistency

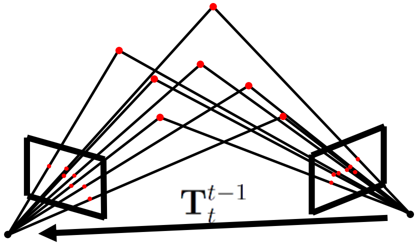


Motion Estimation from Point Correspondences

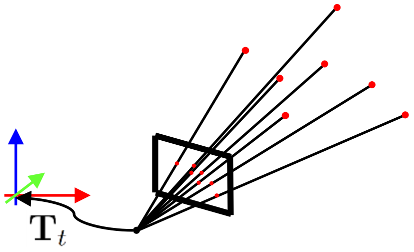
- **2D-to-2D**
  - Reproj. error:

$$E\left(\mathbf{T}_t^{t-1}, X\right) = \sum_{i=1}^{N} \left\|\bar{\mathbf{y}}_{t,i} - \pi\left(\bar{\mathbf{x}}_i\right)\right\|_2^2 + \left\|\bar{\mathbf{y}}_{t-1,i} - \pi\left(\mathbf{T}_t^{t-1}\bar{\mathbf{x}}_i\right)\right\|_2^2$$

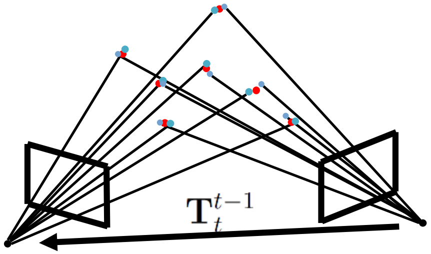
  - Linear algorithm: 8-point



- **2D-to-3D**
  - Reprojection error: $E(\mathbf{T}_t) = \sum_{i=1}^{N} \left\|\mathbf{y}_{t,i} - \pi(\mathbf{T}_t\bar{\mathbf{x}}_i)\right\|_2^2$
  - Linear algorithm: DLT PnP



- **3D-to-3D**
  - Reprojection error: $E\left(\mathbf{T}_t^{t-1}\right) = \sum_{i=1}^{N} \left\|\bar{\mathbf{x}}_{t-1,i} - \mathbf{T}_t^{t-1}\bar{\mathbf{x}}_{t,i}\right\|_2^2$
  - Linear algorithm: Arun's method

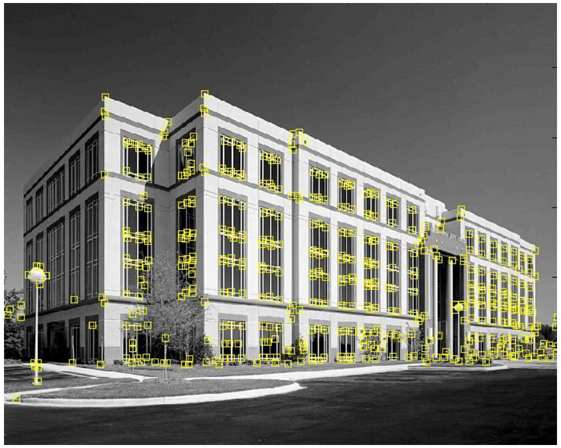


Motion Estimation for Camera Type

Correspondences	Monocular	Stereo	RGB-D
2D-to-2D	X	X	X
2D-to-3D	X	X	X
3D-to-3D		X	X

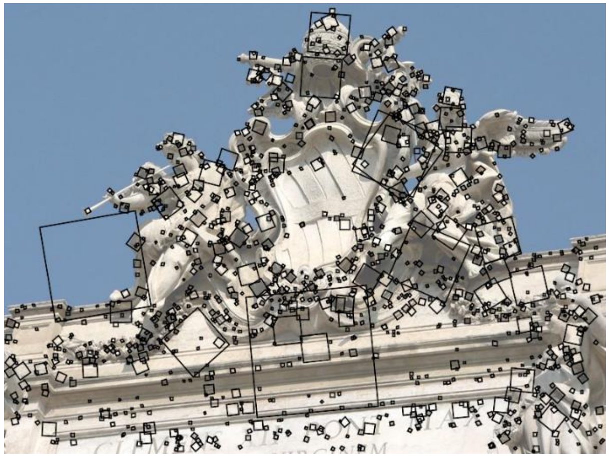
Keypoint Detection

- Desirable properties of keypoint detectors for visual odometry:
  - high repeatability,
  - localization accuracy,
  - robustness,
  - invariance,
  - computational efficiency



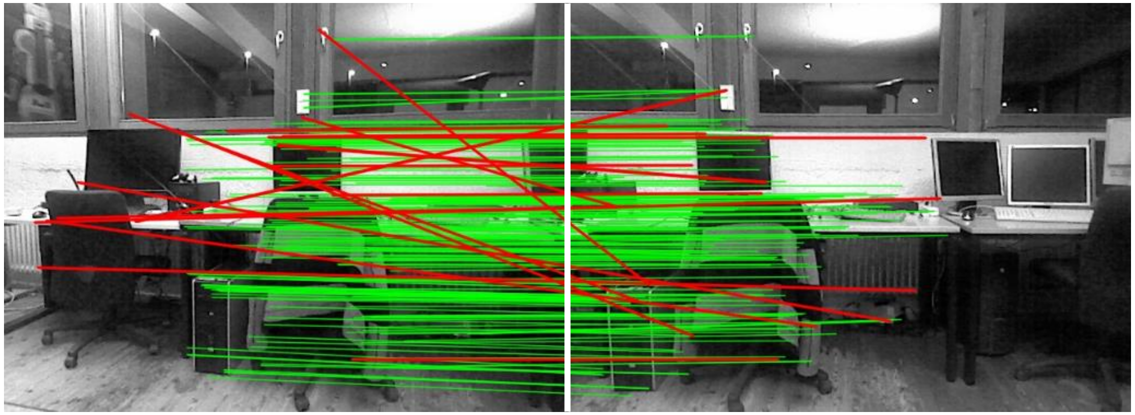
Harris Corners

Image source: Svetlana Lazebnik



DoG (SIFT) Blobs

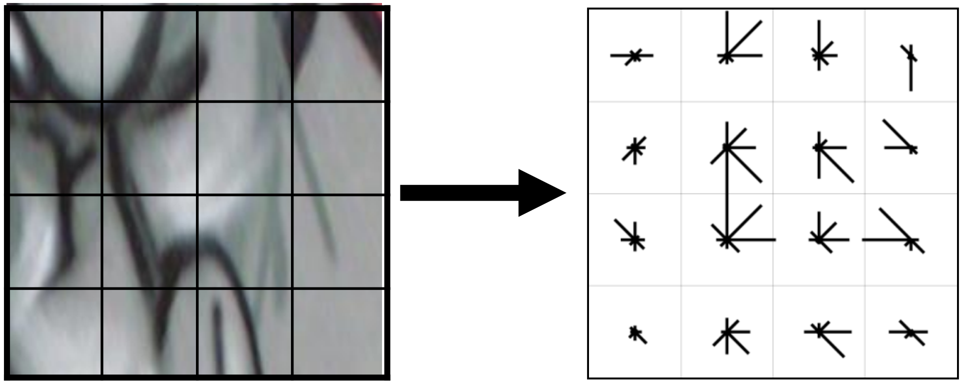
Keypoint Matching



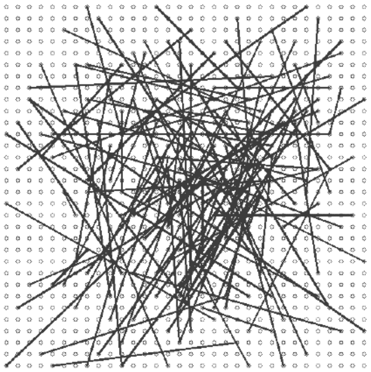
- Desirable properties for VO:
  - High recall
  - Precision
  - Robustness
  - Computational efficiency
- One possible approach to keypoint matching: by descriptor
- Robustness: RANSAC

Keypoint Descriptors

- Desirable properties for VO: distinctiveness, robustness, invariance
- Extract signatures that describe local image regions, examples:
  - Histograms over image gradients (SIFT)
  - Histograms over Haar-wavelet responses (SURF)
  - Binary patterns (BRIEF, BRISK, FREAK, etc.)
  - Learning-based descriptors (f.e. Calonder et al., ECCV 2008)
- Rotation-invariance: Align with dominant orientation in local region
- Scale-invariance: Adapt described region extent to keypoint scale



SIFT gradient pooling



BRIEF test locations

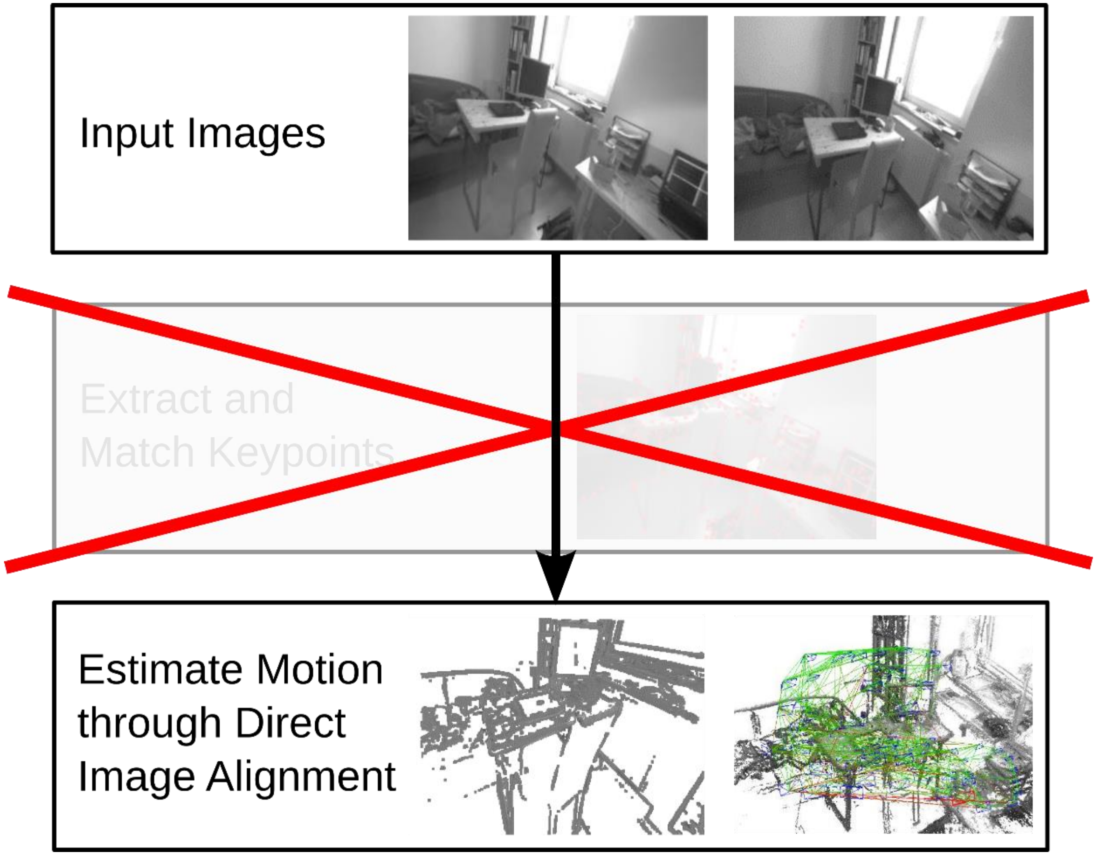
Image source: Svetlana Lazebnik / Calonder et al., ECCV 2010

Direct Visual Odometry Pipeline

- Avoid manually designed keypoint detection and matching
- Instead: direct image alignment

$$E(\boldsymbol{\xi}) = \int_{\mathbf{u}\in\Omega} |\mathbf{I}_1(\mathbf{u}) - \mathbf{I}_2(\omega(\mathbf{u},\boldsymbol{\xi}))| \, d\mathbf{u}$$

- Warping requires depth
  - RGB-D
  - Fixed-baseline stereo
  - Temporal stereo, tracking and (local) mapping

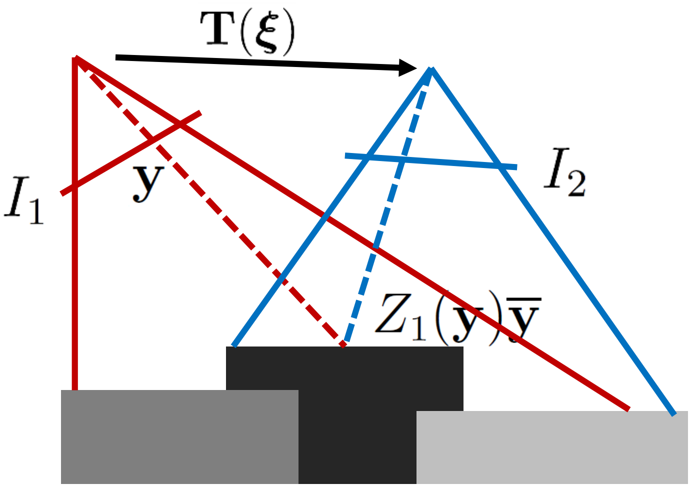


Input Images

~~Extract and Match Keypoints~~

Estimate Motion through Direct Image Alignment

Probabilistic Direct Image Alignment


$\mathbf{T}(\boldsymbol{\xi})$
I_1
$\mathbf{y}$
I_2
$Z_1(\mathbf{y})\overline{\mathbf{y}}$

- Measurements are affected by noise

$$I_1(\mathbf{y}) = I_2\left(\pi\left(\mathbf{T}(\boldsymbol{\xi})Z_1(\mathbf{y})\overline{\mathbf{y}}\right)\right) + \epsilon$$

- A convenient assumption is Gaussian noise

$$\epsilon \sim \mathcal{N}(0, \sigma_I^2)$$

- If we further assume that pixel measurements are stochastically independent, we can formulate the a-posteriori probability

$$p(\boldsymbol{\xi} \mid I_1, I_2) \propto p(I_1 \mid \boldsymbol{\xi}, I_2)p(\boldsymbol{\xi})$$

$$\propto p(\boldsymbol{\xi}) \prod_{\mathbf{y}\in\Omega} \mathcal{N}\left(I_1(\mathbf{y}) - I_2\left(\pi\left(\mathbf{T}(\boldsymbol{\xi})Z_1(\mathbf{y})\overline{\mathbf{y}}\right)\right); 0, \sigma_I^2\right)$$

Optimization Approach

- Optimize negative log-likelihood
  - Product of exponentials becomes a summation over quadratic terms
  - Normalizers are independent of the pose

$$E(\boldsymbol{\xi}) = \sum_{\mathbf{y} \in \Omega} \frac{r(\mathbf{y}, \boldsymbol{\xi})^2}{\sigma_I^2}$$

, stacked residuals: $E(\boldsymbol{\xi}) = \mathbf{r}(\boldsymbol{\xi})^\top \mathbf{W} \mathbf{r}(\boldsymbol{\xi})$

$$r(\mathbf{y}, \boldsymbol{\xi}) = I_1(\mathbf{y}) - I_2\left(\pi\left(\mathbf{T}(\boldsymbol{\xi}) Z_1(\mathbf{y}) \overline{\mathbf{y}}\right)\right)$$

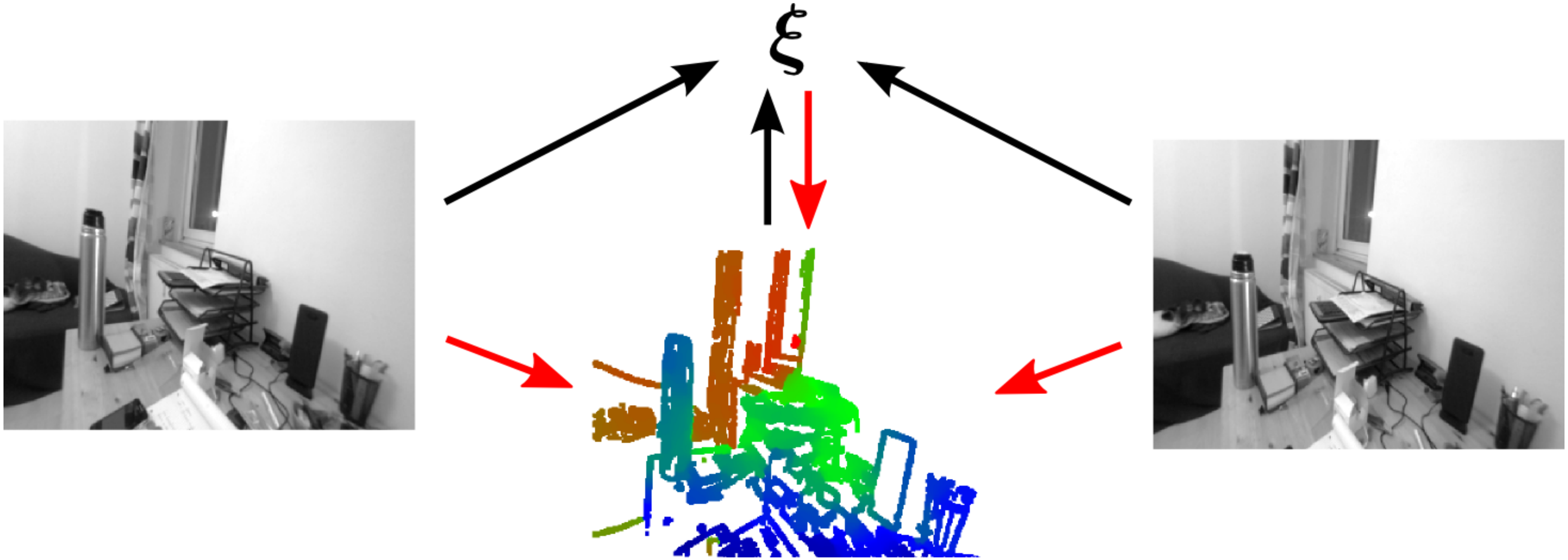
- Non-linear least squares problem can be efficiently optimized using standard second-order tools (Gauss-Newton, Levenberg-Marquardt)

Direct Visual Odometry

Direct RGB-D Odometry	Direct Monocular Odometry
Dense depth from sensor	Semi-dense depth estimated concurrently from short-baseline stereo comparisons and filtering
Only tracking of camera pose	Alternating, interdependent camera pose and depth map estimation
Track on keyframe	Track/depth estimation on keyframe
Metric scale from measured depth	No metric scale

Monocular Direct Visual Odometry

- Estimate motion and depth concurrently

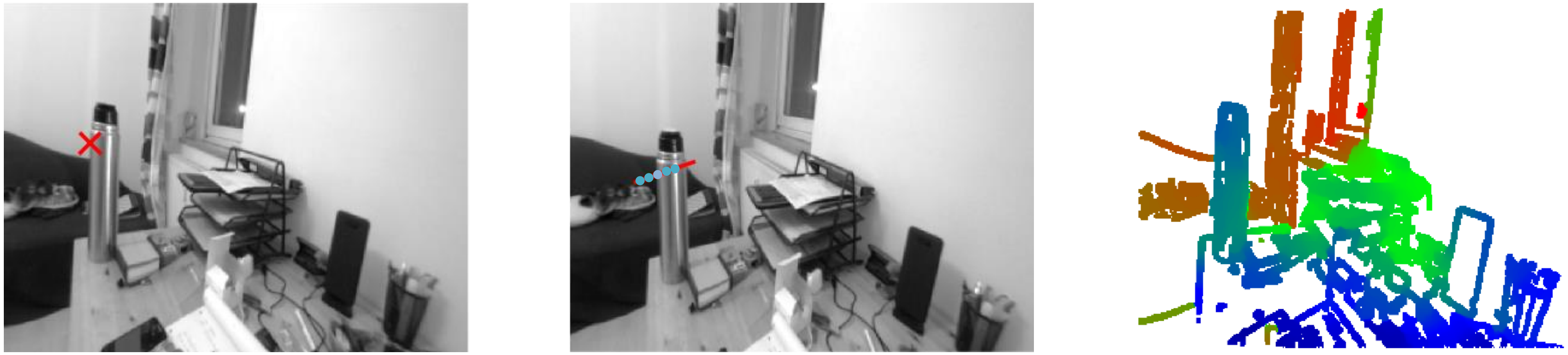


- Alternating optimization: **Tracking** and **Mapping**

Images from: Engel et al., ICCV 2013

Semi-Dense Mapping

- Estimate inverse depth and variance at high gradient pixels
- Correspondence search along epipolar line (5-pixel intensity SSD)



- Kalman-filtering of depth map:
  - Propagate depth map & variance from previous frame
  - Update depth map & variance with new depth observations

Images from: Engel et al., ICCV 2013

Visual-Inertial Fusion

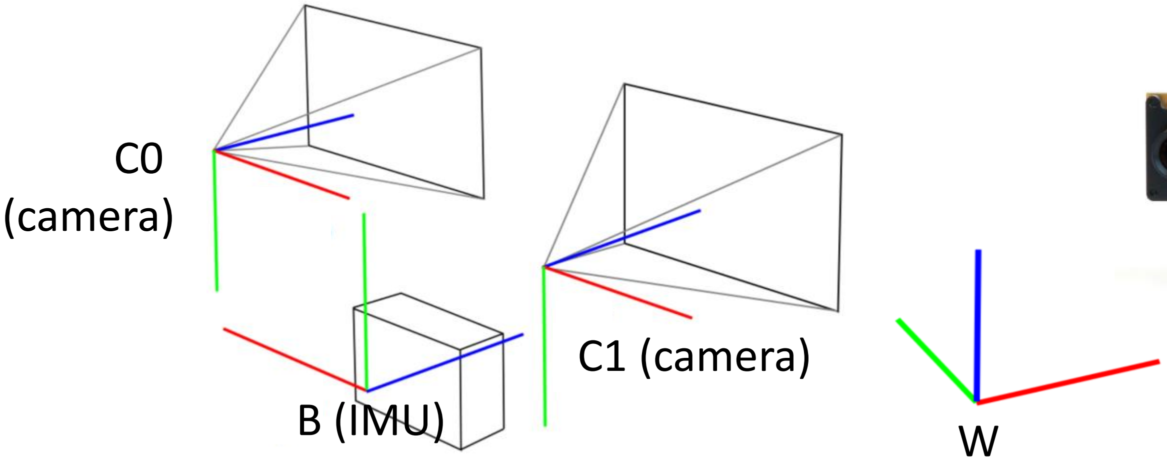
- Vision and IMU are complementary!

Visual sensing	Inertial sensing
+ Accurate at small to medium motion	- Large relative uncertainty for low acceleration/angular velocity
+ Rich information for other purposes	
- Limited output rate (~100Hz)	+ High output rate (~1000Hz)
- Scale ambiguity for monocular camera	+ Scale directly observable
- Lack of robustness for rapid motion, textureless areas, low illumination	+ Independent of environmental conditions

- Odometry using both sensor types is still prone to drift!

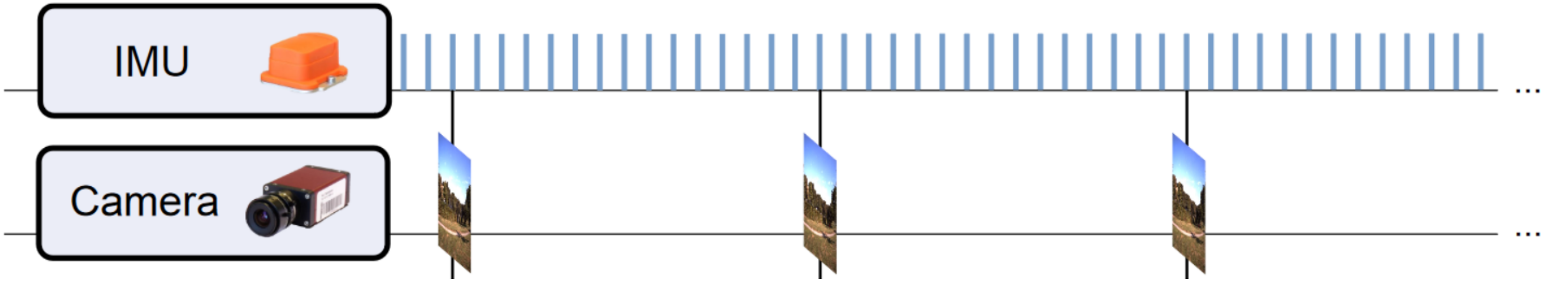
Camera-IMU System

- Extrinsic calibration between camera(s) and IMU frame



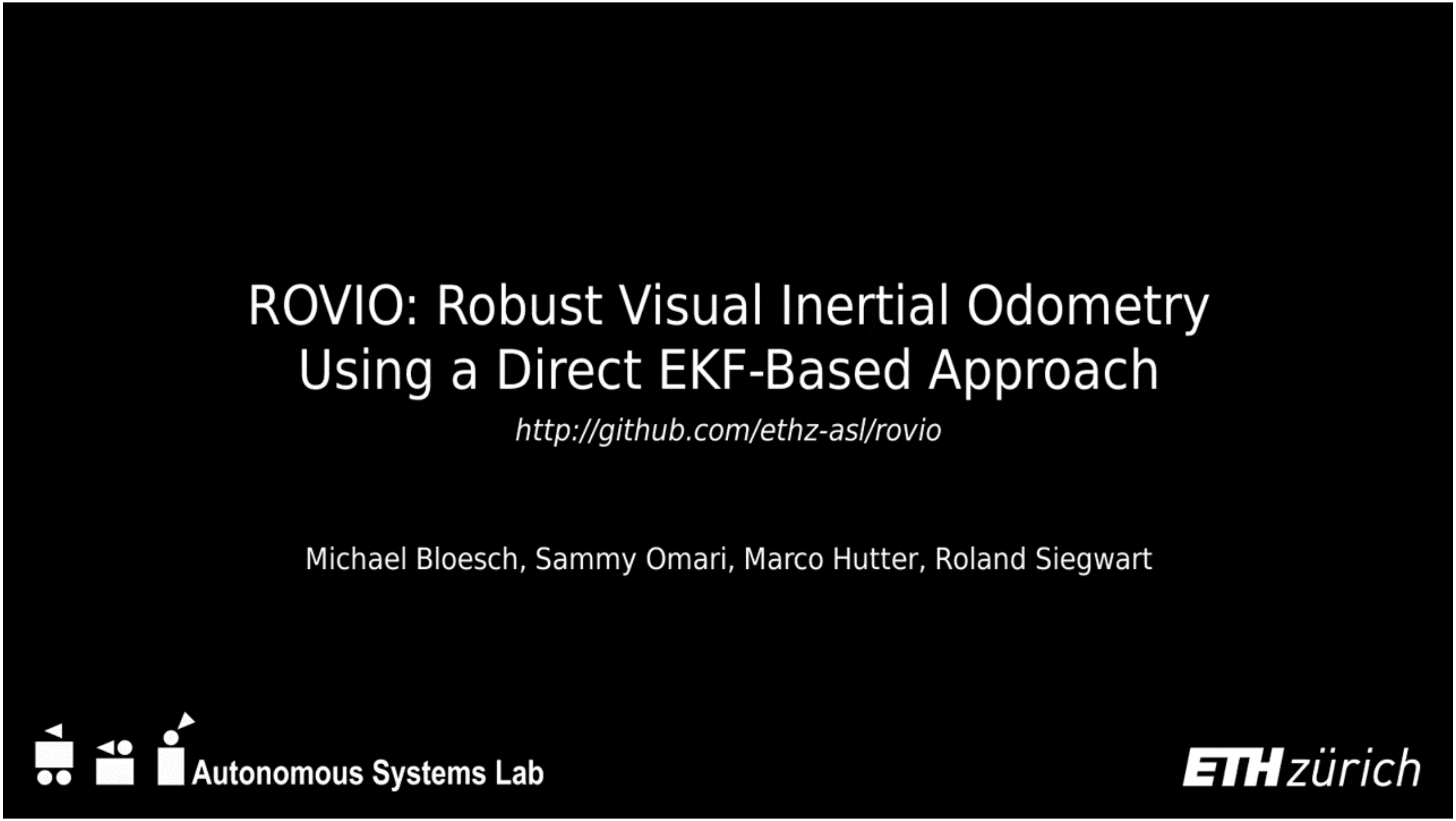
Skybotix VI-Sensor

- Time synchronization



Tightly-Coupled Filter for Visual-Inertial Fusion

- Photoconsistency measurements of landmark patch projections



ROVIO: Robust Visual Inertial Odometry
Using a Direct EKF-Based Approach

http://github.com/ethz-asl/rovio

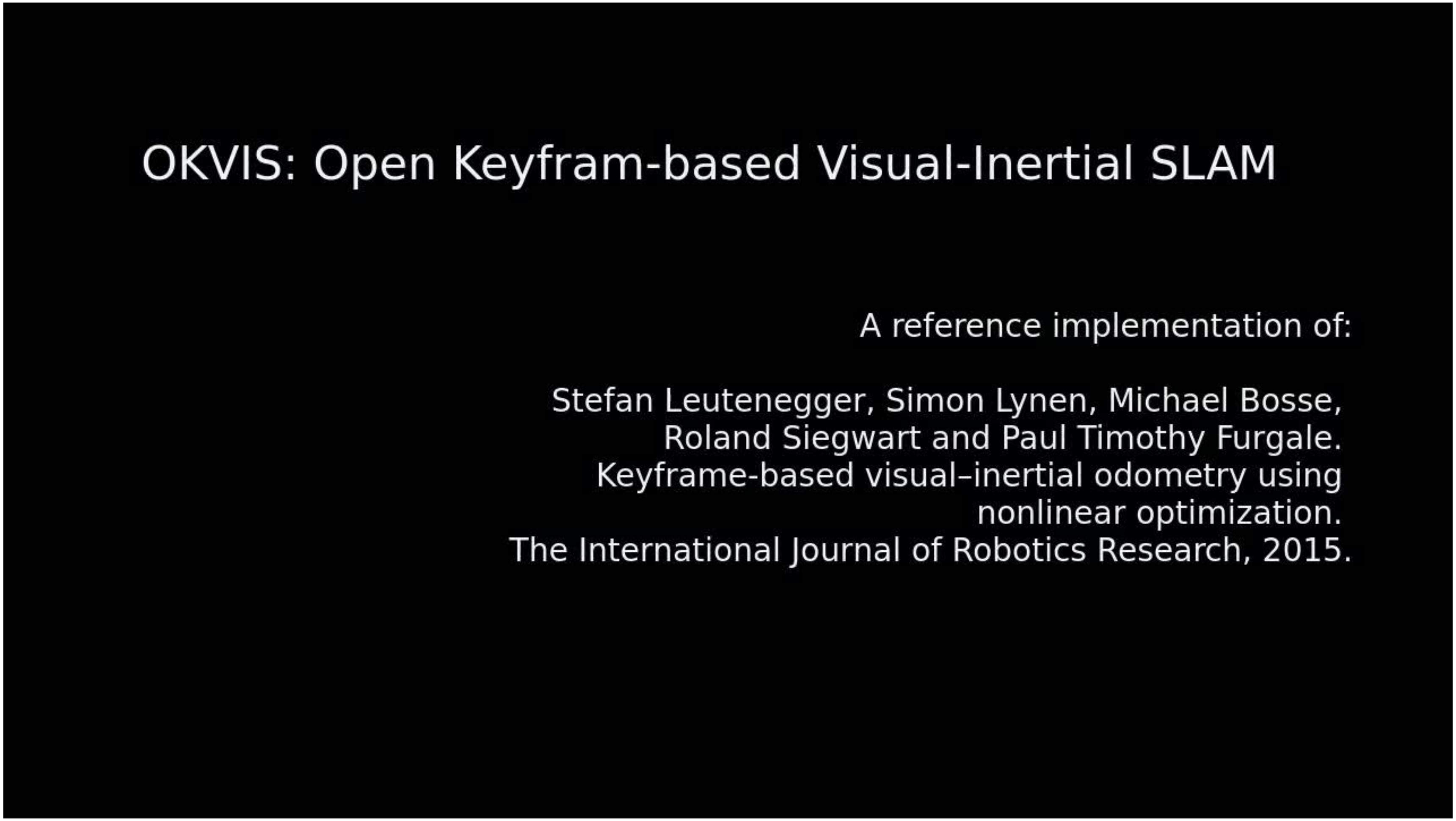
Michael Bloesch, Sammy Omari, Marco Hutter, Roland Siegwart

Autonomous Systems Lab

ETH zürich

Indirect Fixed-Lag Smoothing Example

- OKVIS: Keyframe-based indirect fixed-lag smoothing VIO



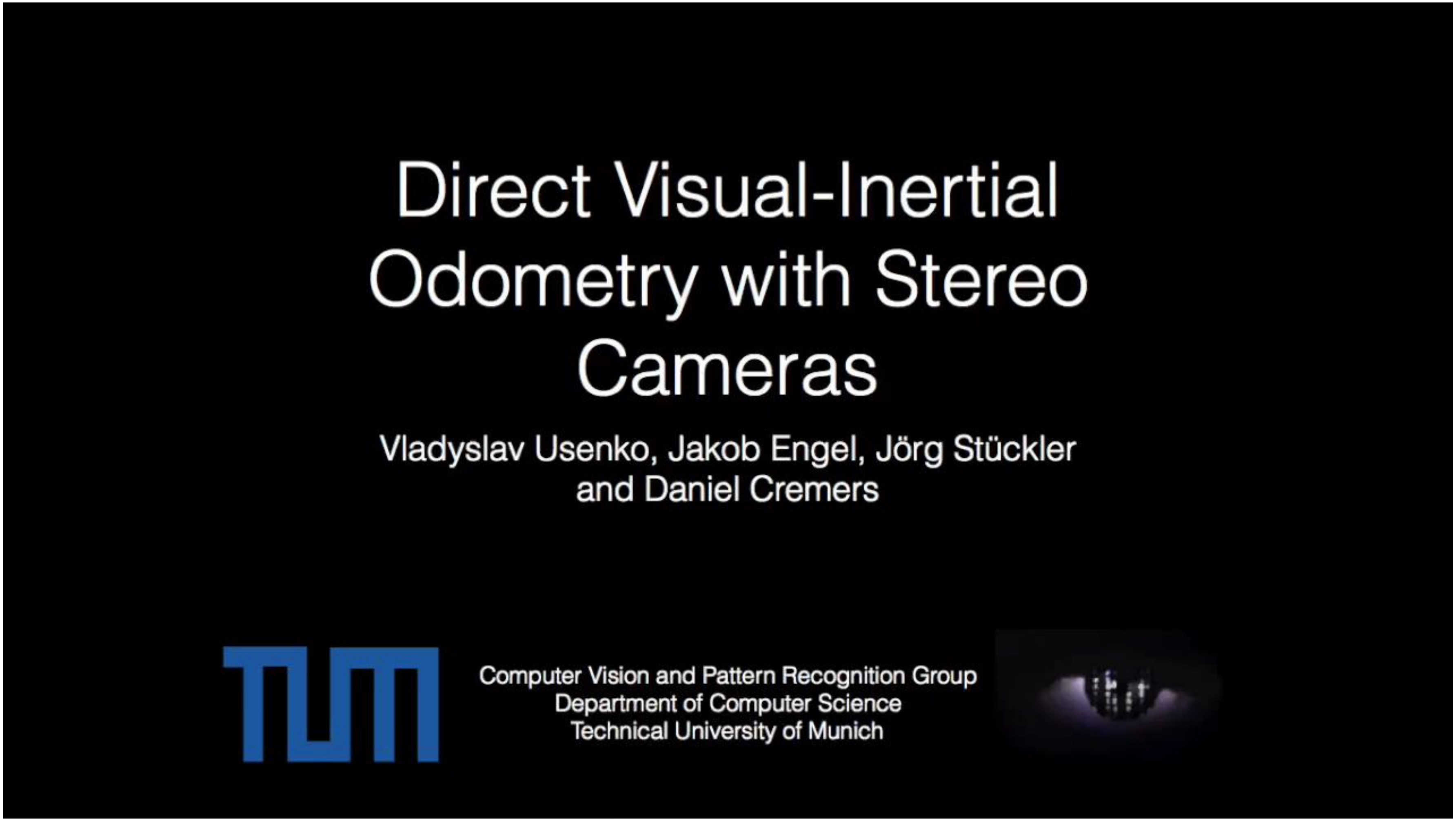
OKVIS: Open Keyfram-based Visual-Inertial SLAM

A reference implementation of:

Stefan Leutenegger, Simon Lynen, Michael Bosse,
Roland Siegwart and Paul Timothy Furgale.
Keyframe-based visual–inertial odometry using
nonlinear optimization.
The International Journal of Robotics Research, 2015.

Direct Fixed-Lag Smoothing Example

- Direct Fixed-Lag Smoothing VIO



Direct Visual-Inertial Odometry with Stereo Cameras

Vladyslav Usenko, Jakob Engel, Jörg Stückler
and Daniel Cremers

Computer Vision and Pattern Recognition Group
Department of Computer Science
Technical University of Munich

What is Visual SLAM?

- Visual simultaneous localization and mapping (VSLAM)…
  - Tracks the pose of the camera in a map, and simultaneously
  - Estimates the parameters of the environment map (f.e. reconstruct the 3D positions of interest points in a common coordinate frame)
- Loop-closure: Revisiting a place allows for drift compensation
  - How to detect a loop closure?

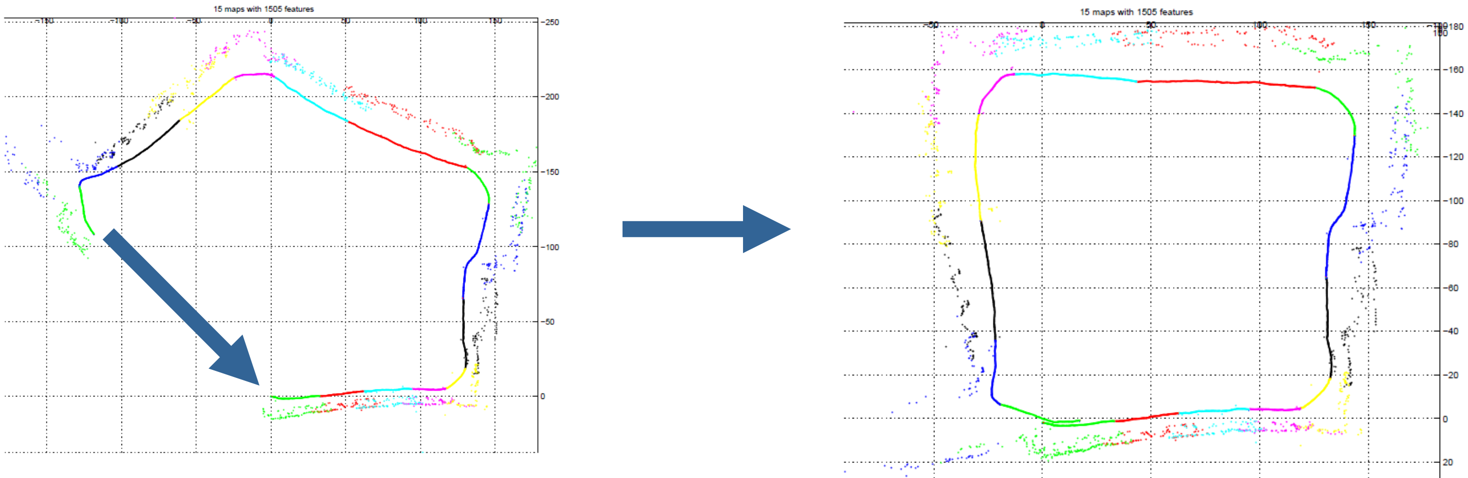
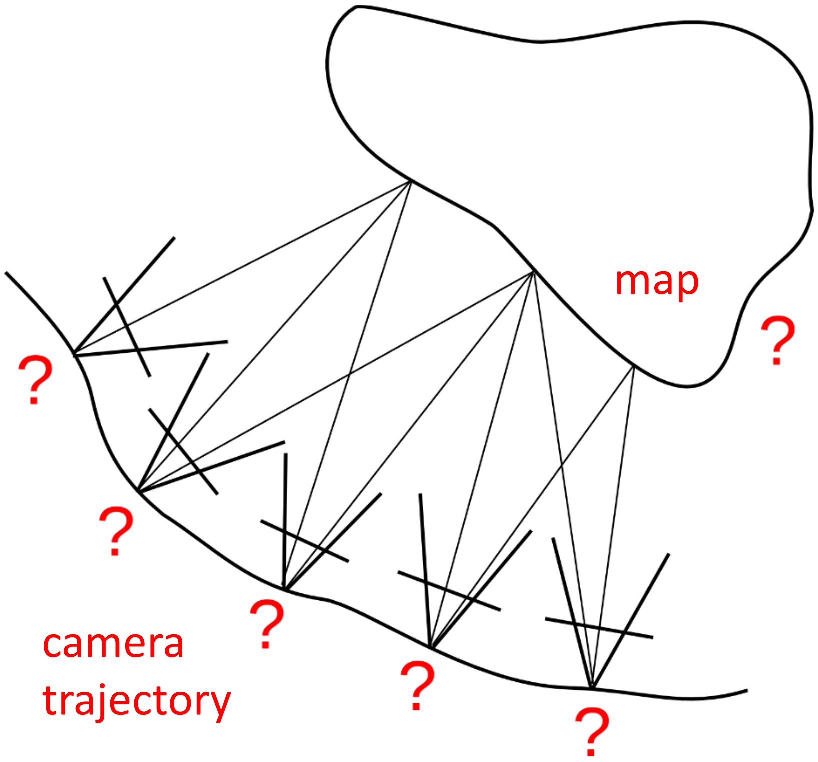


Image credit: Clemente et al., RSS 2007

Why is SLAM difficult?

- Chicken-or-egg problem
  - Camera trajectory and map are unknown and need to be estimated from observations
  - Accurate localization requires an accurate map
  - Accurate mapping requires accurate localization

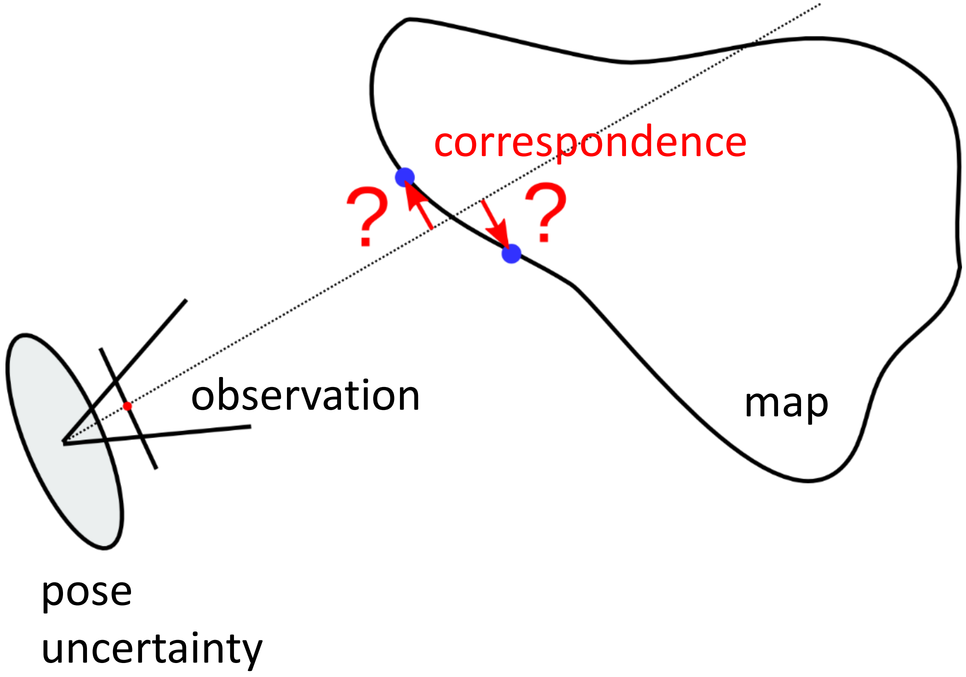


Why is SLAM difficult?

- Correspondences between observations and the map are unknown

- Wrong correspondences can lead to divergence of trajectory/map estimates

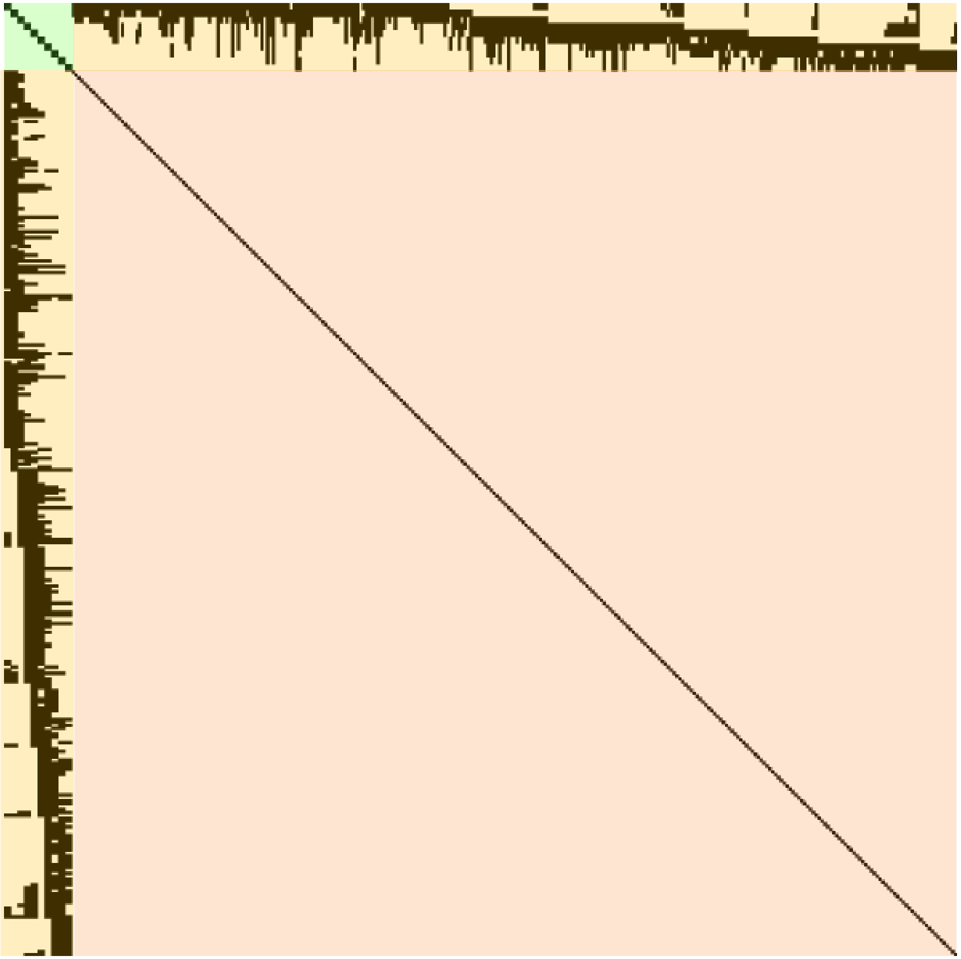
- Important to model uncertainties of observations and estimates in a probabilistic formulation of the SLAM problem



Example Hessian of a BA Problem

Pose dimensions

(10 poses)



Landmark dimensions

(982 landmarks)

Image source: Manolis Lourakis (CC BY 3.0)

Exploiting the Sparse Structure

- Idea:
  Apply the Schur complement to solve the system in a partitioned way

$$\mathbf{H}_k \Delta \mathbf{x} = -\mathbf{b}_k \quad \longrightarrow \quad \begin{pmatrix} \mathbf{H}_{\xi\xi} & \mathbf{H}_{\xi\mathrm{m}} \\ \mathbf{H}_{\mathrm{m}\xi} & \mathbf{H}_{\mathrm{mm}} \end{pmatrix} \begin{pmatrix} \Delta \mathbf{x}_{\xi} \\ \Delta \mathbf{x}_{\mathrm{m}} \end{pmatrix} = - \begin{pmatrix} \mathbf{b}_{\xi} \\ \mathbf{b}_{\mathrm{m}} \end{pmatrix}$$

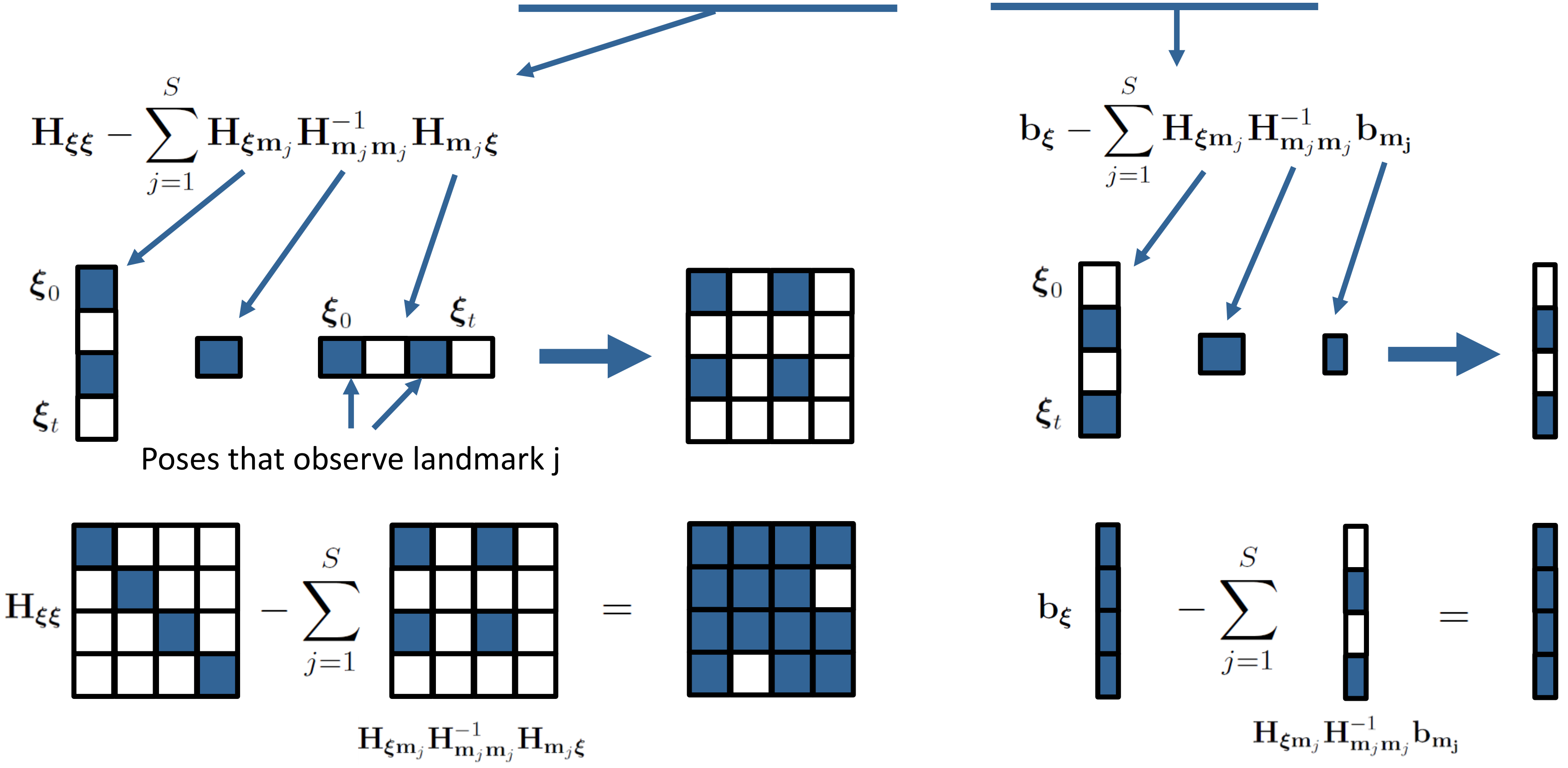
$$\longrightarrow \quad \Delta \mathbf{x}_{\xi} = - \left(\mathbf{H}_{\xi\xi} - \mathbf{H}_{\xi\mathrm{m}} \mathbf{H}_{\mathrm{mm}}^{-1} \mathbf{H}_{\mathrm{m}\xi} \right)^{-1} \left(\mathbf{b}_{\xi} - \mathbf{H}_{\xi\mathrm{m}} \mathbf{H}_{\mathrm{mm}}^{-1} \mathbf{b}_{\mathrm{m}} \right)$$

$$\longrightarrow \quad \Delta \mathbf{x}_{\mathrm{m}} = - \mathbf{H}_{\mathrm{mm}}^{-1} \left(\mathbf{b}_{\mathrm{m}} + \mathbf{H}_{\mathrm{m}\xi} \Delta \mathbf{x}_{\xi} \right)$$

- Is this any better?

Exploiting the Sparse Structure

- What is the structure of the two sub-problems ?
- Poses: $\Delta \mathbf{x}_\xi = -\left(\underline{\mathbf{H}_{\xi\xi} - \mathbf{H}_{\xi\mathbf{m}}\mathbf{H}_{\mathbf{mm}}^{-1}\mathbf{H}_{\mathbf{m}\xi}}\right)^{-1}\left(\underline{\mathbf{b}_\xi - \mathbf{H}_{\xi\mathbf{m}}\mathbf{H}_{\mathbf{mm}}^{-1}\mathbf{b}_\mathbf{m}}\right)$



$$\mathbf{H}_{\xi\xi} - \sum_{j=1}^{S} \mathbf{H}_{\xi\mathbf{m}_j}\mathbf{H}_{\mathbf{m}_j\mathbf{m}_j}^{-1}\mathbf{H}_{\mathbf{m}_j\xi}$$

$$\mathbf{b}_\xi - \sum_{j=1}^{S} \mathbf{H}_{\xi\mathbf{m}_j}\mathbf{H}_{\mathbf{m}_j\mathbf{m}_j}^{-1}\mathbf{b}_{\mathbf{m_j}}$$

Poses that observe landmark j

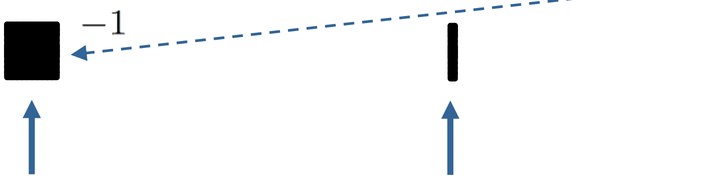
$$\mathbf{H}_{\xi\xi} \quad - \sum_{j=1}^{S} \quad = $$

$$\mathbf{H}_{\xi\mathbf{m}_j}\mathbf{H}_{\mathbf{m}_j\mathbf{m}_j}^{-1}\mathbf{H}_{\mathbf{m}_j\xi}$$

$$\mathbf{b}_\xi \quad - \sum_{j=1}^{S} \quad = $$

$$\mathbf{H}_{\xi\mathbf{m}_j}\mathbf{H}_{\mathbf{m}_j\mathbf{m}_j}^{-1}\mathbf{b}_{\mathbf{m_j}}$$

Exploiting the Sparse Structure



$$\Delta \mathbf{x}_\xi = -\left(\mathbf{H}_{\xi\xi} - \mathbf{H}_{\xi \mathrm{m}} \mathbf{H}_{\mathrm{mm}}^{-1} \mathbf{H}_{\mathrm{m}\xi}\right)^{-1} \left(\mathbf{b}_\xi - \mathbf{H}_{\xi \mathrm{m}} \mathbf{H}_{\mathrm{mm}}^{-1} \mathbf{b}_{\mathrm{m}}\right)$$

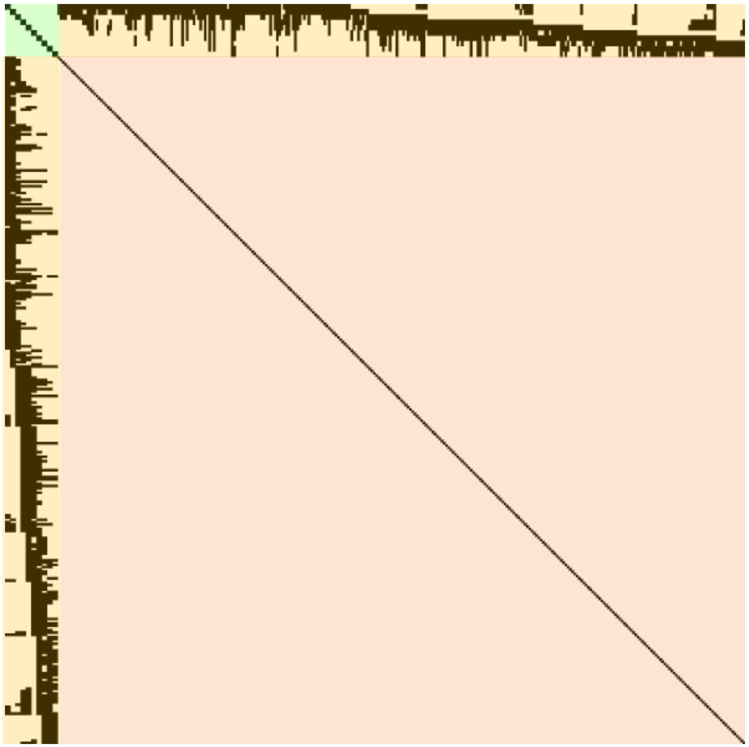
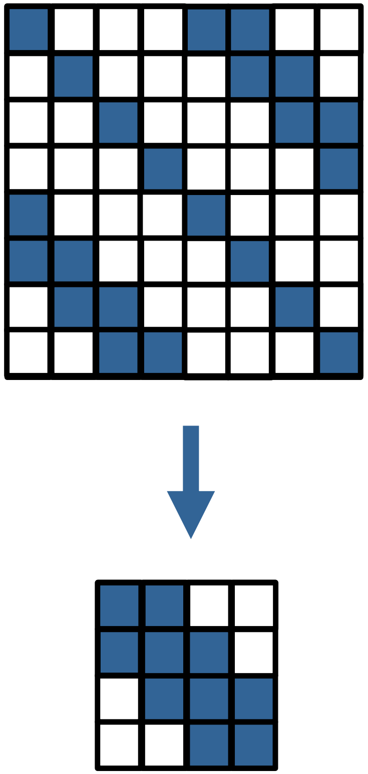
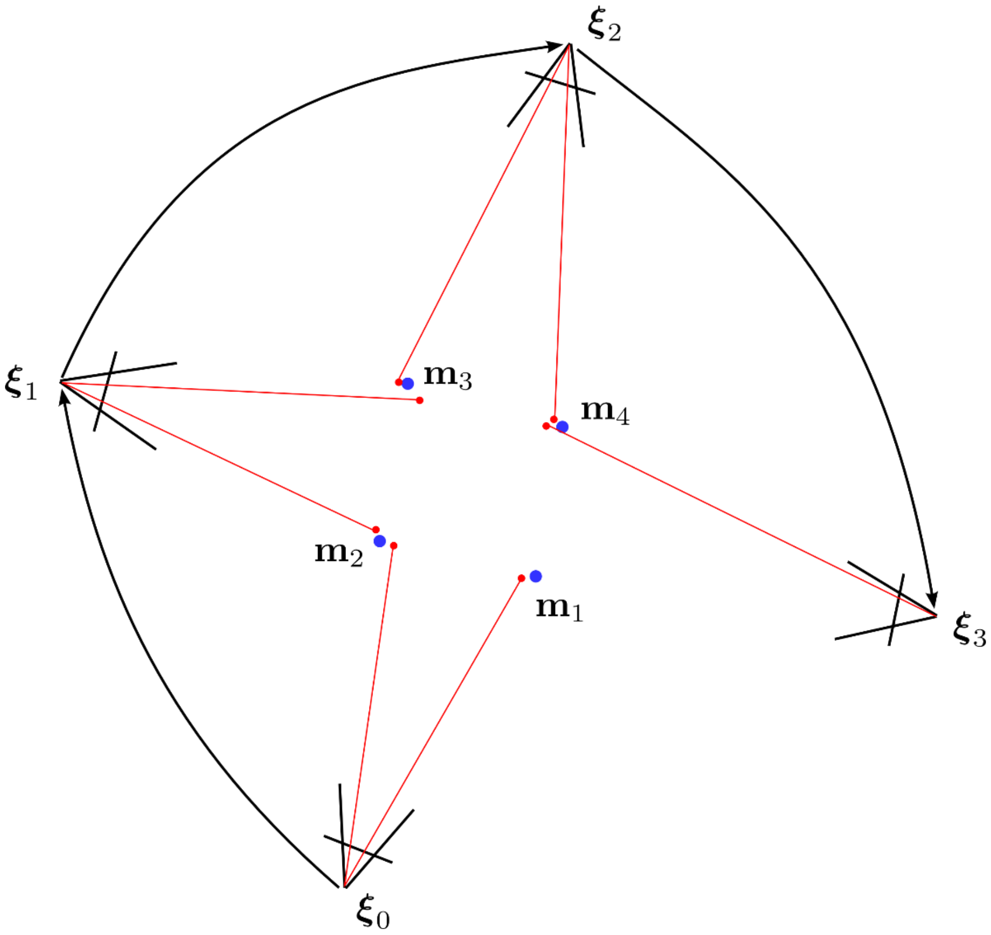


Image source: Manolis Lourakis (CC BY 3.0)

Effect of Loop-Closures on the Hessian



Band matrix



Effect of Loop-Closures on the Hessian

🟦	⬜	⬜	⬜	🟦	🟦	⬜	⬜
⬜	🟦	⬜	⬜	⬜	🟦	🟦	⬜
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⬜	🟦	🟦	⬜	⬜	⬜	🟦	⬜
⬜	⬜	🟦	🟦	⬜	⬜	⬜	🟦

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🟦	🟦	⬜	🟧
🟦	🟦	🟦	⬜
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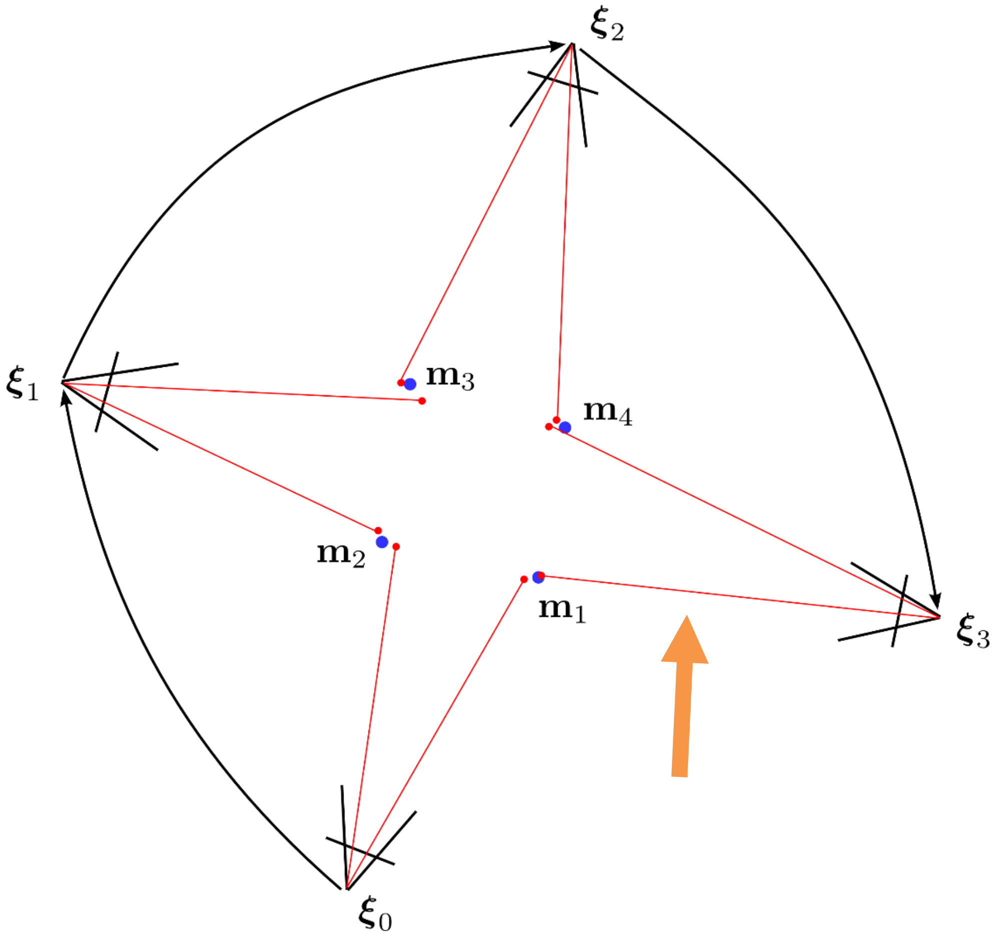
Not band matrix: costlier to solve

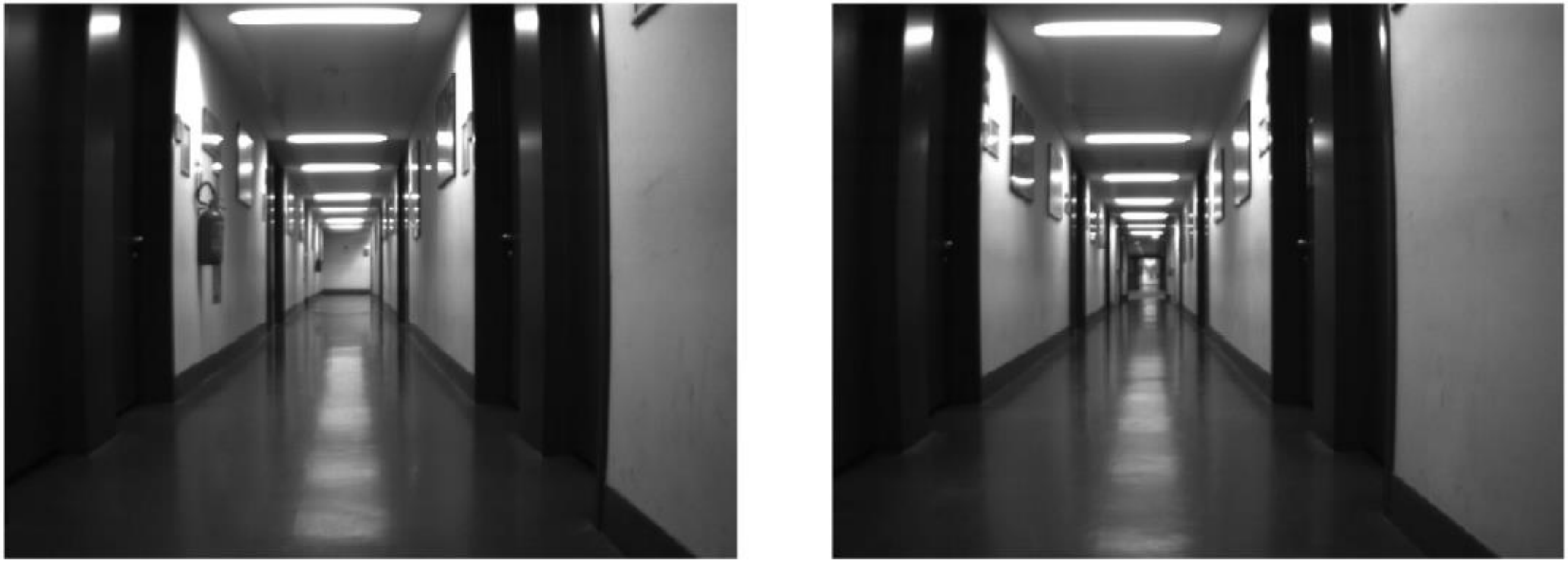


Loop Closing by Place Recognition

- Idea: use image retrieval techniques
- Popular approach for place recognition is to use bag-of-visual-words based image retrieval in conjunction with geometric verification (f.e. 8-point with RANSAC)

Images: Cummins and Newman, Highly Scalable Appearance-Only SLAM – FAB-MAP 2.0, RSS 2009

Loop Closing is Difficult!



Perceptual Aliasing

Image credit: Juan D. Tardós

Overview on SLAM Approaches from the Lecture

Tracking-and-Mapping	Filtering	Fixed-Lag Smoothing	Pose Graph Optimization	Bundle Adjustment
Indirect: - PTAM	Indirect: - MonoSLAM		Indirect: - ORB-SLAM 1 & 2	Indirect: - ORB-SLAM 2
Direct: - Semi-dense monocular visual odometry		Direct: - Direct sparse odometry	Direct: - DVO-SLAM - LSD-SLAM	
No loop closing	Loop closing	No loop closing	Loop closing	Loop closing

Thanks for your attention!