Robotic 3D Vision

Lecture 18: Dense Stereo Reconstruction

Prof. Dr. Jörg Stückler
Computer Vision Group, TU Munich
http://vision.in.tum.de

What We Will Cover Today

- Stereo Perception
- Stereo Rectification

- Dense Depth Reconstruction from Two and Multiple Views
  - Dense Correspondence Search
  - Disparity Space Image
  - Regularization, Semi-Global Matching

- Depth Sensors

- Next lecture: From dense depth images to dense 3D maps

Stereo Perception

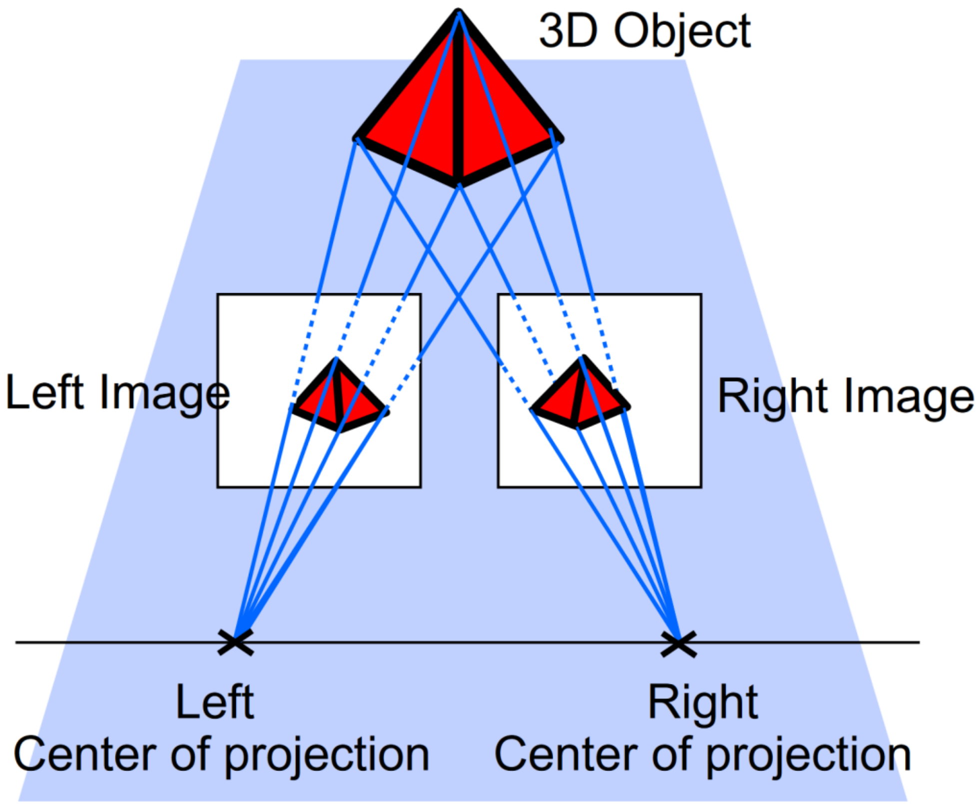
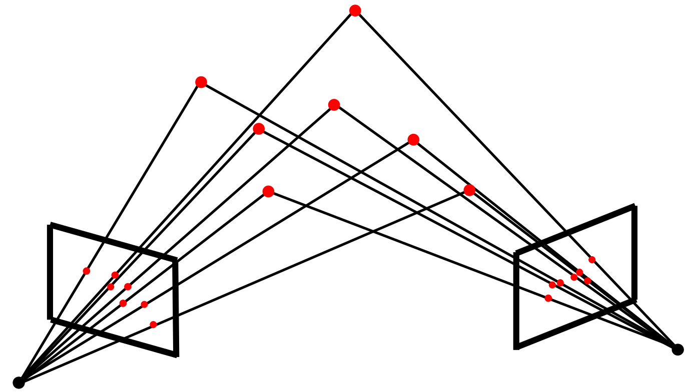


Image credit: D. Scaramuzza

Dense Depth from Two Views

- So far: triangulation of corresponding interest points between two images to find depth
- How can we obtain depth densely for all pixels in an image?
- Assume relative pose between the camera images known
- Assume intrinsic camera calibration known



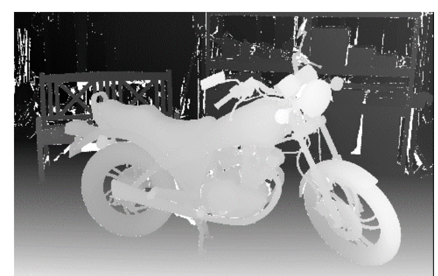
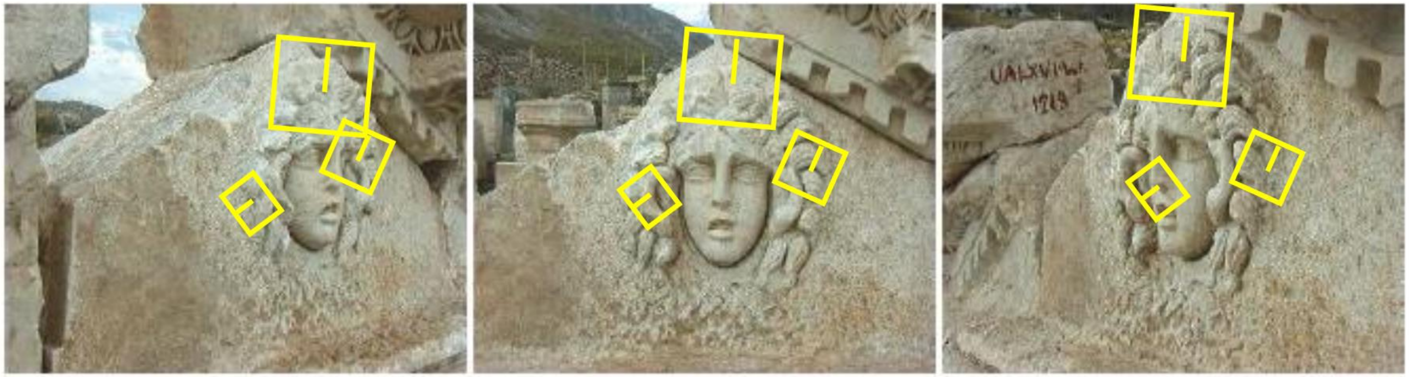


Image source: Scharstein et al., Middlebury stereo benchmark

Sparse 3D Reconstruction



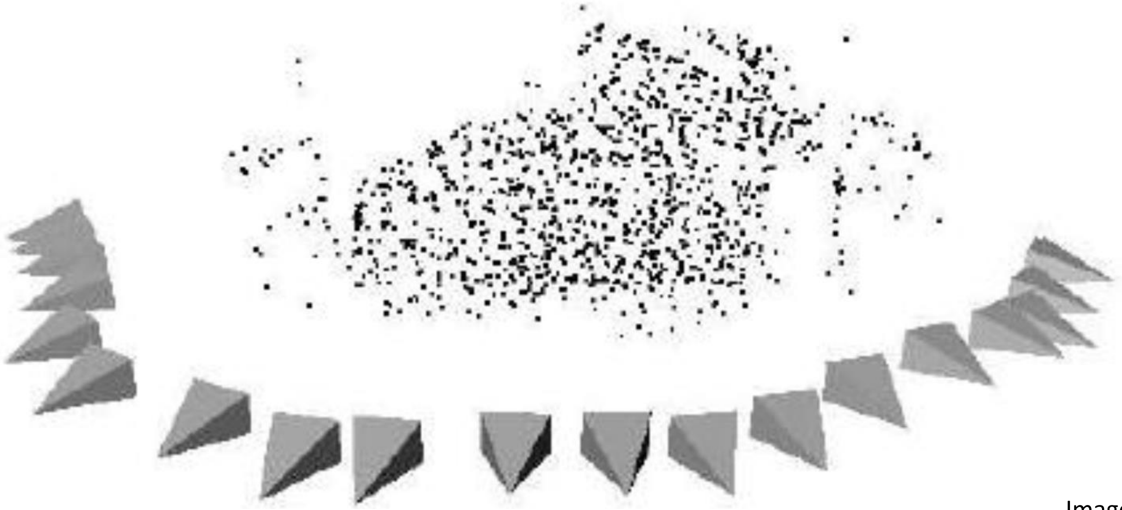
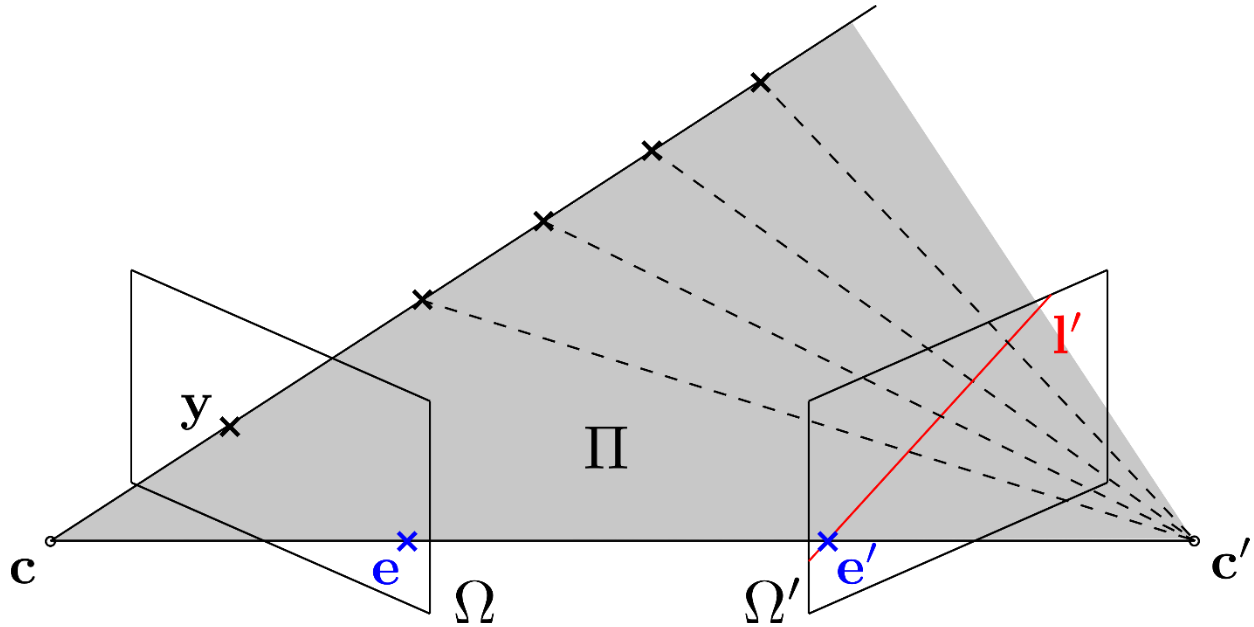


Image credit: D. Scaramuzza

Dense 3D Reconstruction

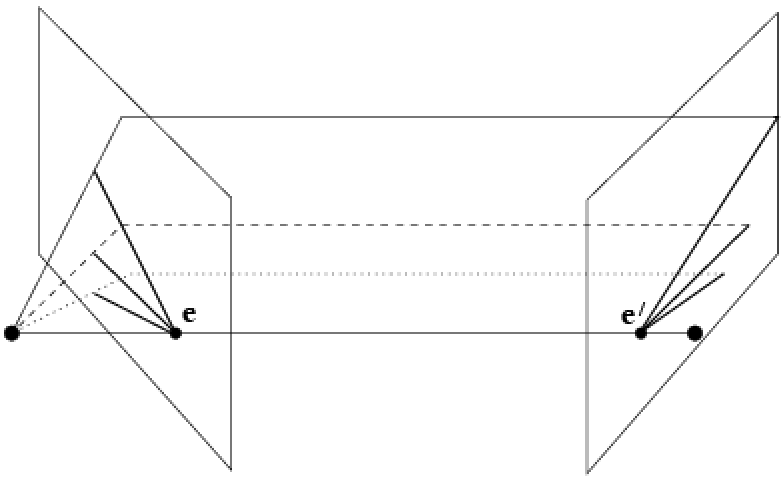
Image credit: D. Scaramuzza

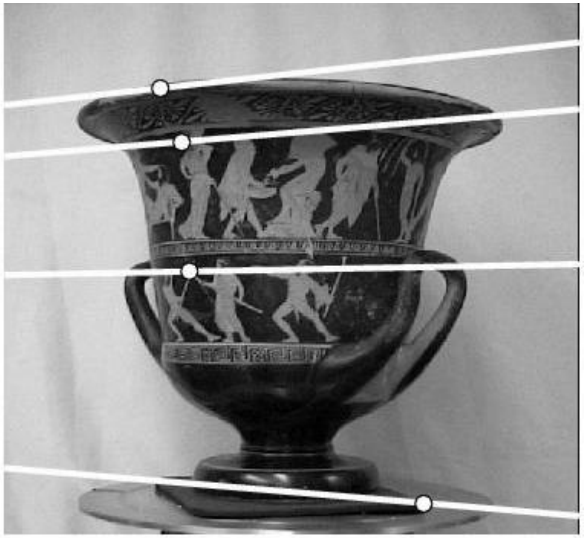
Recap: Epipolar Geometry



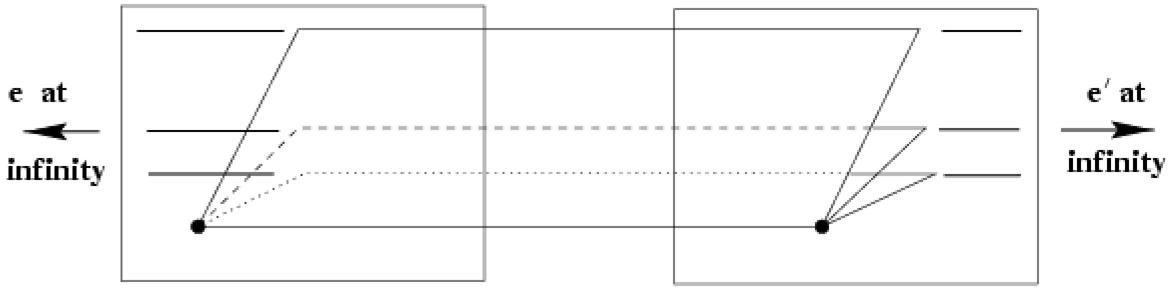
- Camera centers $\mathbf{c}$, $\mathbf{c}'$ and image point $\mathbf{y} \in \Omega$ span the epipolar plane Π
- The ray from camera center $\mathbf{c}$ through point $\mathbf{y}$ projects as the epipolar line $\mathbf{l}'$ in image plane Ω'
- The intersections of the line through the camera centers with the image planes are called epipoles $\mathbf{e}$, $\mathbf{e}'$

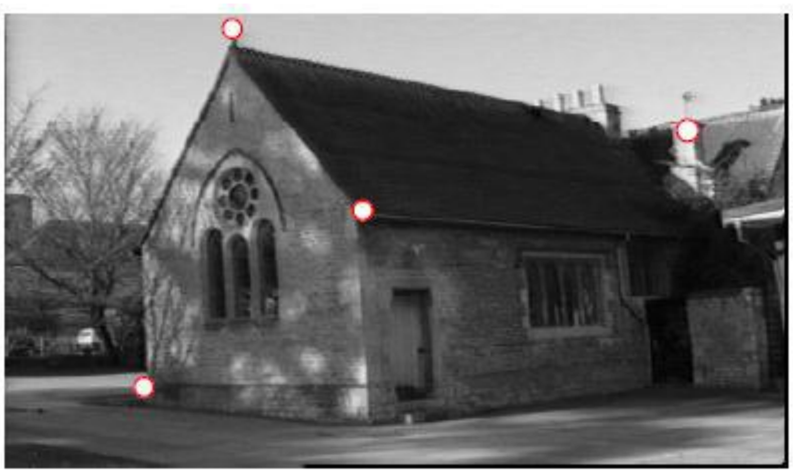
Epipolar Lines, Converging Cameras





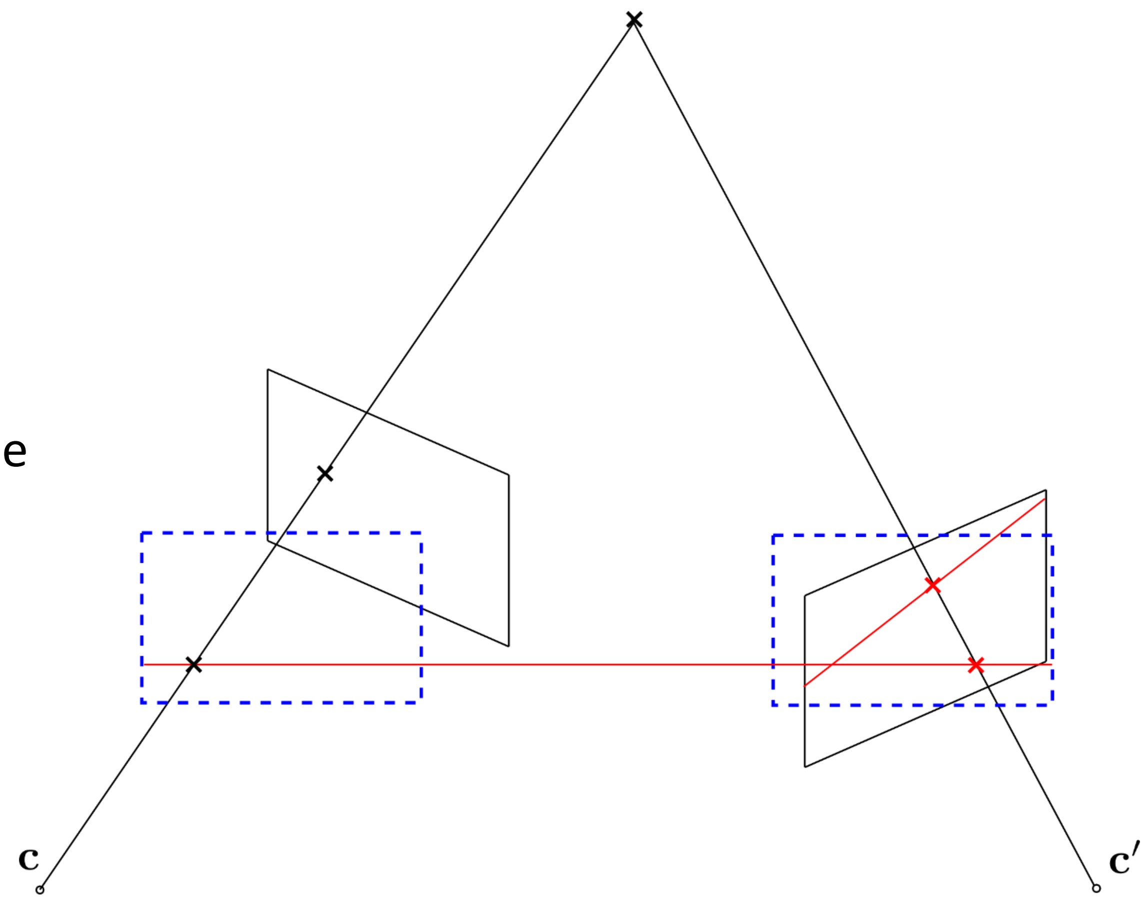
Epipolar Lines, Parallel Cameras





Stereo Image Rectification

- Correspondence search is simplified, if epipolar lines are horizontal (or vertical)
- Idea: Rectify images
  - warp the images onto a common image plane
  - only horizontal or vertical translation between the new cameras
  - Equal intrinsics
  - „minimize“ warping



Stereo Rectification (1)

- In the following for convenience, we will write the perspective projection of a 3D point expressed in the world frame into the camera frame as

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = KR^{-1}\left[\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C\right]$$

camera intrinsics matrix

3D rotation matrix

3D point in world frame

3D translation/ camera center in world frame

Slide adapted from D. Scaramuzza

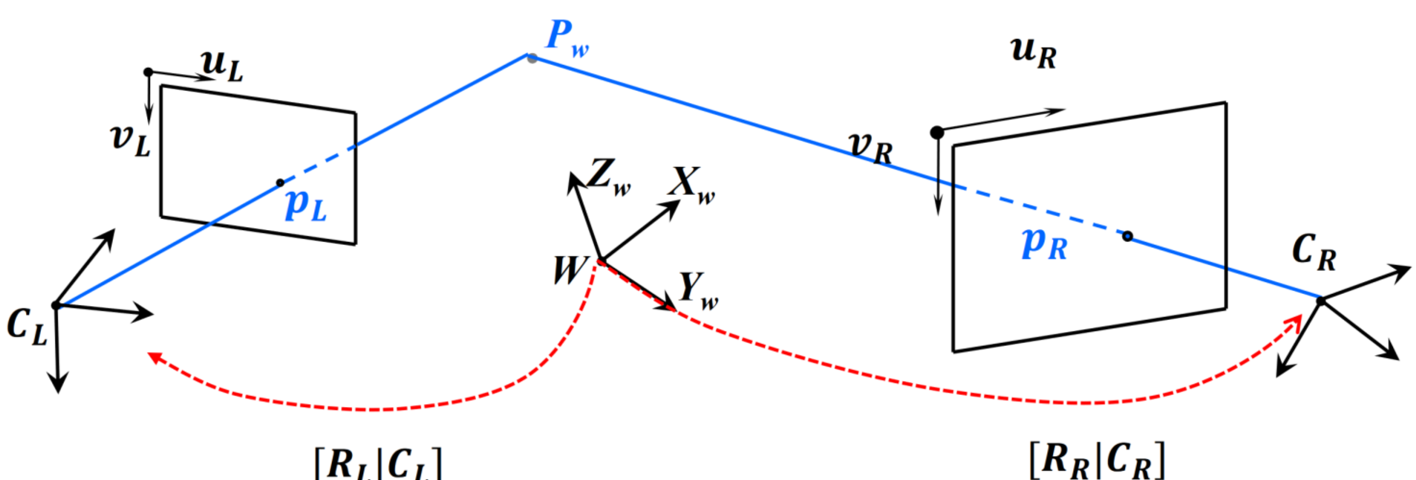
Stereo Rectification (2)

Left camera projection:

$$\lambda_L \begin{bmatrix} u_L \\ v_L \\ 1 \end{bmatrix} = K_L R_L^{-1} \left[\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_L \right]$$

Right camera projection:

$$\lambda_R \begin{bmatrix} u_R \\ v_R \\ 1 \end{bmatrix} = K_R R_R^{-1} \left[\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_R \right]$$



Slide adapted from D. Scaramuzza

Stereo Rectification (3)

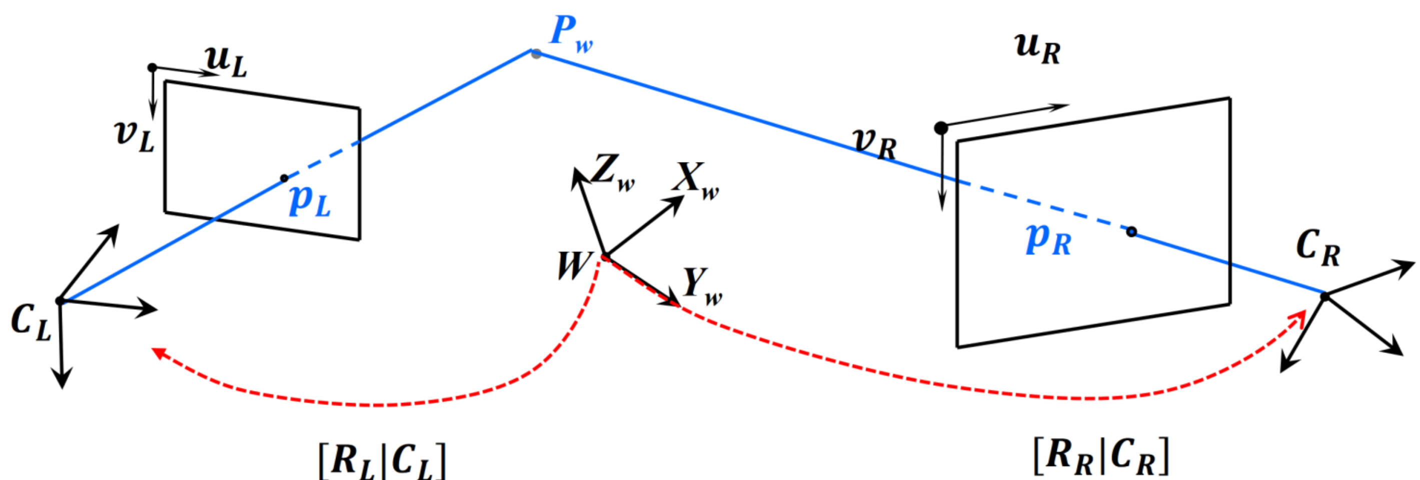
Goal: warp left and right images such that image planes coplanar and intrinsics are equal

Old Left camera → **New Left camera**

$$\lambda_L \begin{bmatrix} u_L \\ v_L \\ 1 \end{bmatrix} = K_L R_L^{-1} \left[\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_L \right] \rightarrow \bar{\lambda}_L \begin{bmatrix} \bar{u}_L \\ \bar{v}_L \\ 1 \end{bmatrix} = \bar{K}\bar{R}^{-1} \left[\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_L \right]$$

Old Right camera → **New Right camera**

$$\lambda_R \begin{bmatrix} u_R \\ v_R \\ 1 \end{bmatrix} = K_R R_R^{-1} \left[\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_R \right] \rightarrow \bar{\lambda}_R \begin{bmatrix} \bar{u}_R \\ \bar{v}_R \\ 1 \end{bmatrix} = \bar{K}\bar{R}^{-1} \left[\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} - C_R \right]$$



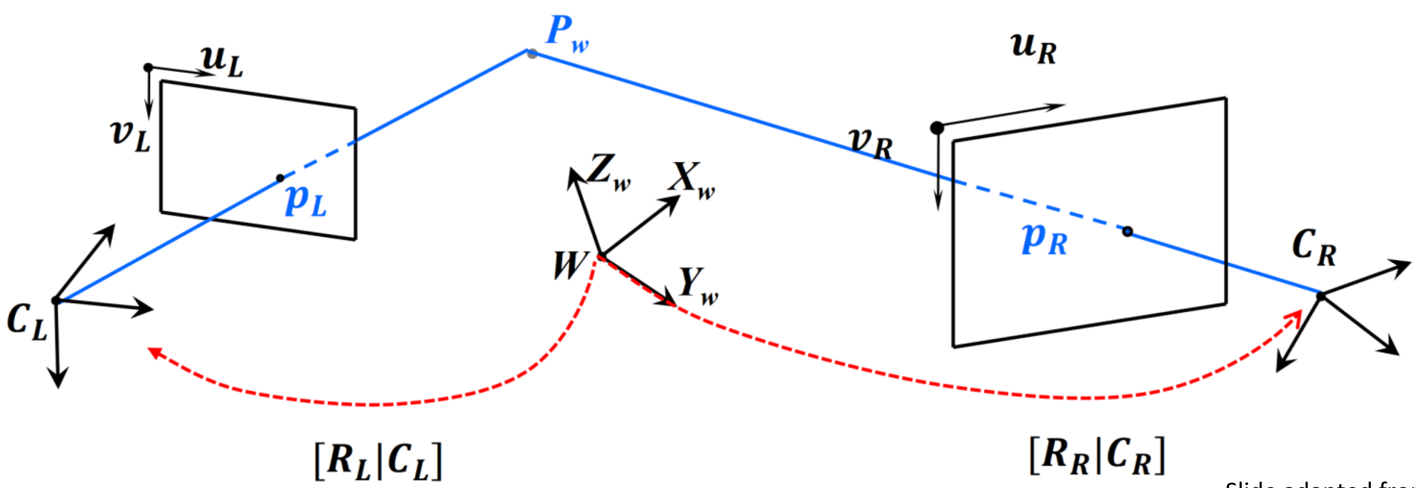
Slide adapted from D. Scaramuzza

Stereo Rectification (4)

Solving for 3D point for each camera yields homographies

$$\bar{\lambda}_L \begin{bmatrix} \bar{u}_L \\ \bar{v}_L \\ 1 \end{bmatrix} = \lambda_L \underbrace{\bar{K}\,\bar{R}^{-1}\,\mathrm{R}_L\,\mathrm{K}_L^{-1}}_{\text{Homography Left Camera}} \begin{bmatrix} u_L \\ v_L \\ 1 \end{bmatrix}$$

$$\bar{\lambda}_R \begin{bmatrix} \bar{u}_R \\ \bar{v}_R \\ 1 \end{bmatrix} = \lambda_R \underbrace{\bar{K}\,\bar{R}^{-1}\,\mathrm{R}_R\,\mathrm{K}_R^{-1}}_{\text{Homography Right Camera}} \begin{bmatrix} u_R \\ v_R \\ 1 \end{bmatrix}$$



Slide adapted from D. Scaramuzza

Stereo Rectification (5)

- How to choose the new intrinsics and rotation ?
- Fusiello et al., A Compact Algorithm for Rectification of Stereo Pairs, Mach. Vision and Appl. 1999
- Choose $\bar{K} = (K_L + K_R)/2$

$$\bar{R} = [\bar{r_1}, \bar{r_2}, \bar{r_3}]$$

where

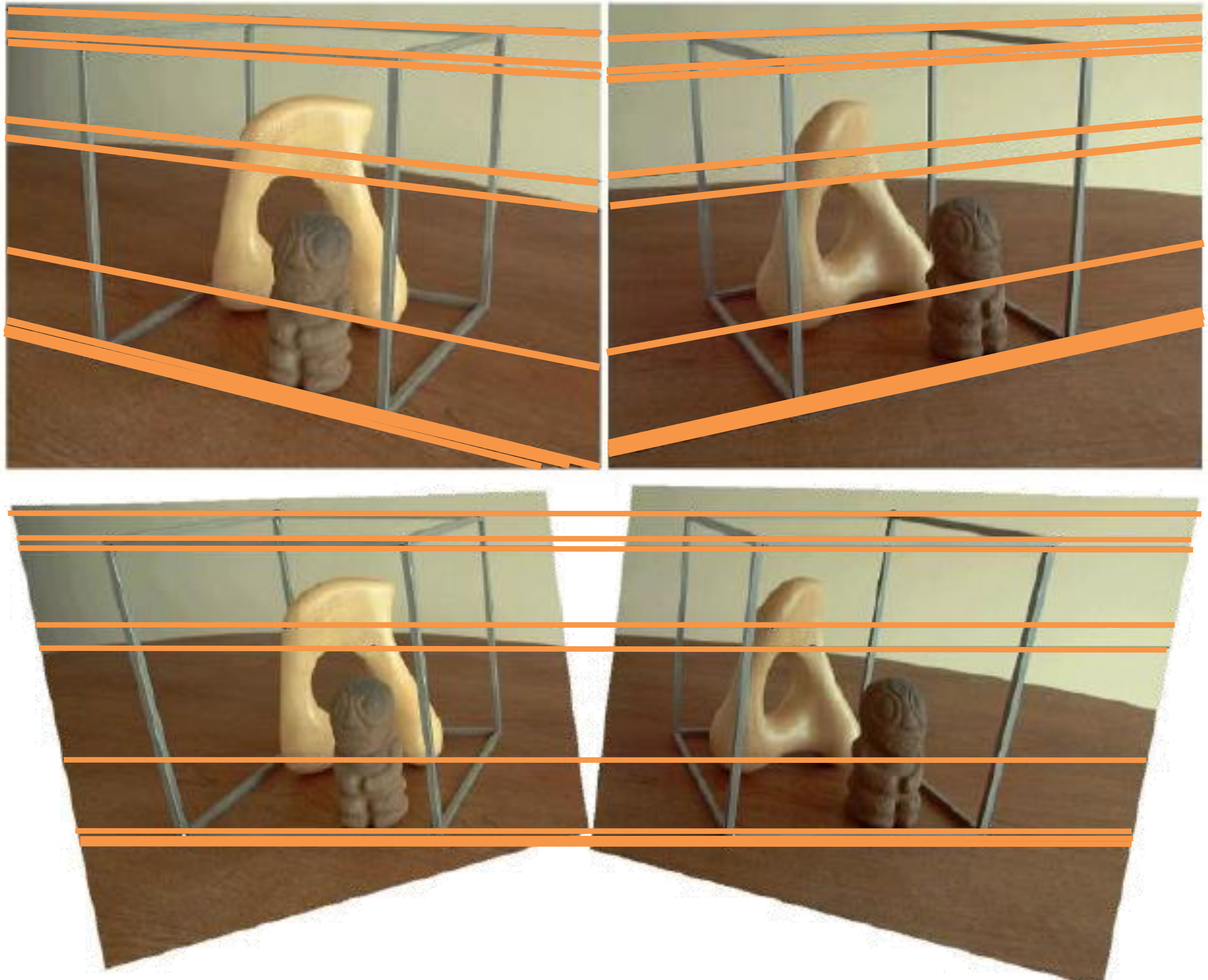
$$\bar{r_1} = \frac{C_2 - C_1}{\|C_2 - C_1\|}$$

$$\bar{r_2} = r_3 \times \bar{r_1}$$, where r_3 is the 3rd column of the rotation matrix of the left camera, i.e., R_L

$$\bar{r_3} = \bar{r_1} \times \bar{r_2}$$

Slide adapted from D. Scaramuzza

Stereo Rectification Example

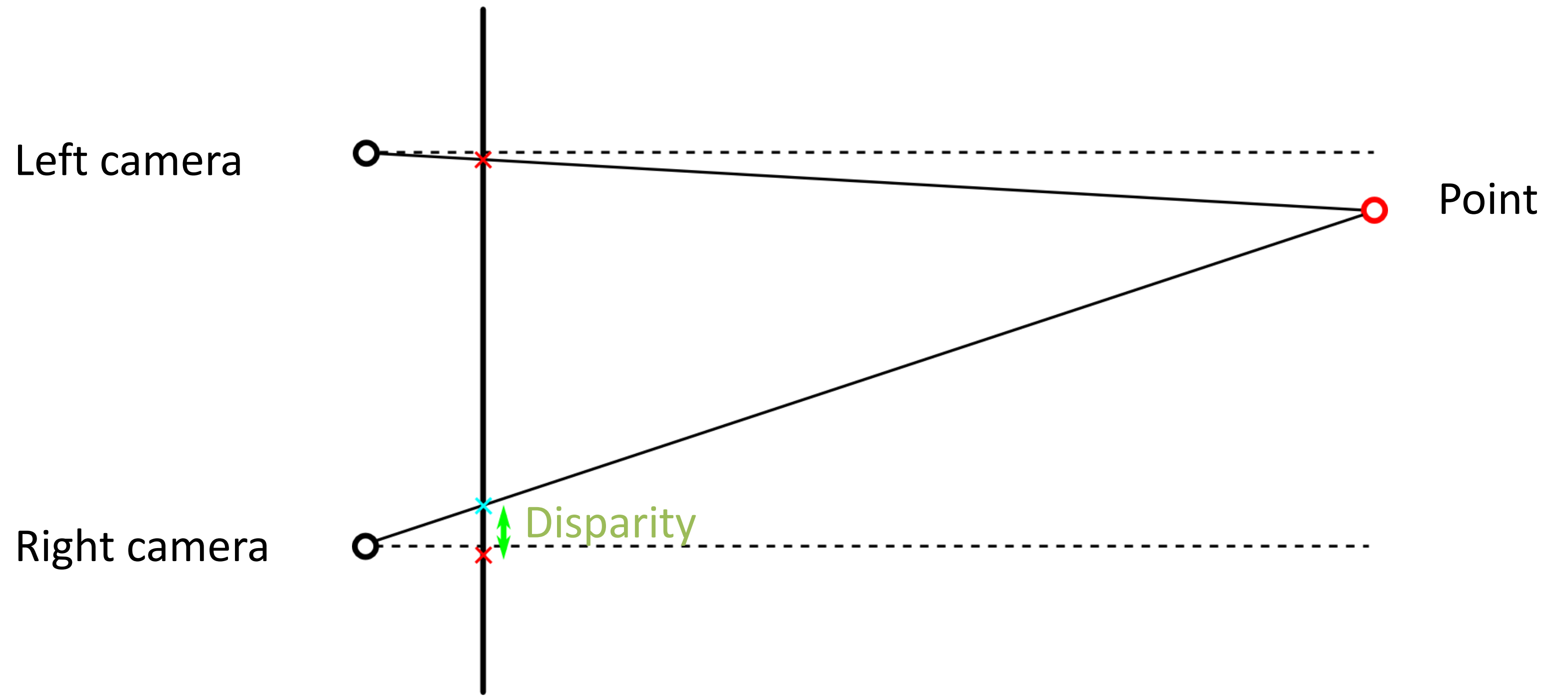


rectified

Image source: Loop and Zhang, 2001

Disparity

- Assume rectified stereo images
- Disparity: (horizontal) pixel difference of corresponding pixels between the two images

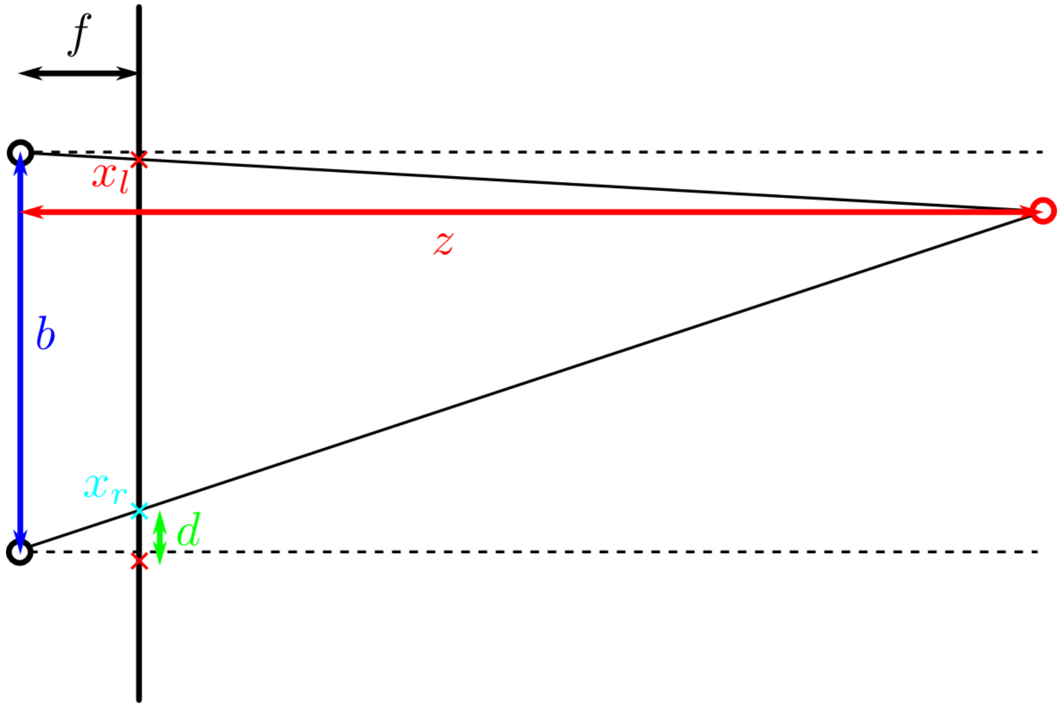


Relation of Disparity and Depth

Similar triangles:

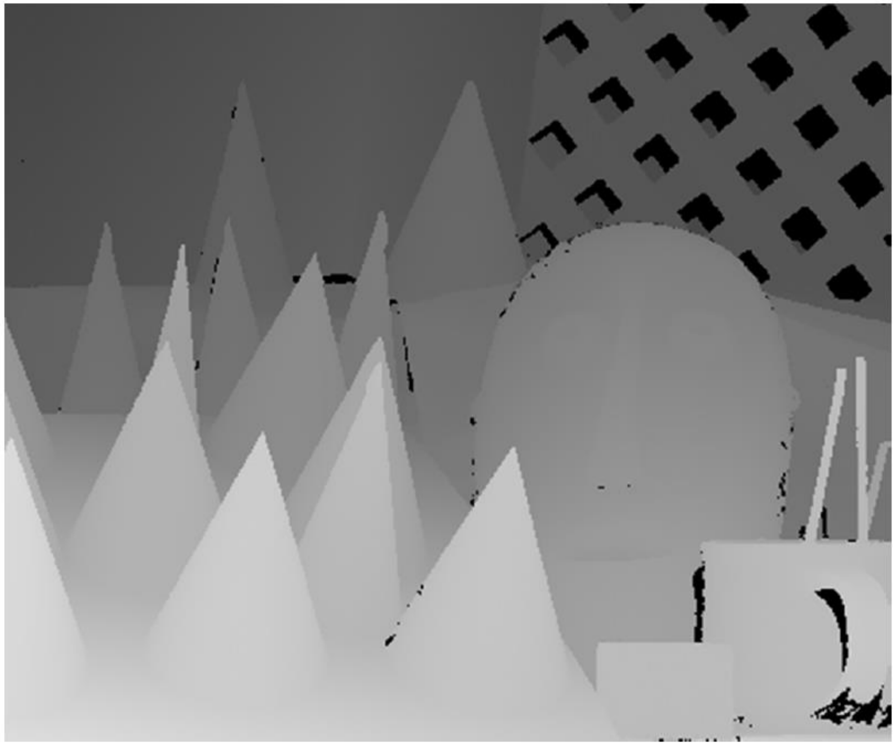
$$\frac{b}{z} = \frac{b-d}{z-f}$$

$$d = \frac{bf}{z}$$



- Disparity is inversely proportional to depth:
  - The larger the depth, the smaller the disparity
- Disparity is proportional to the baseline:
  - The larger the baseline, the larger the disparity

Relation of Disparity and Depth

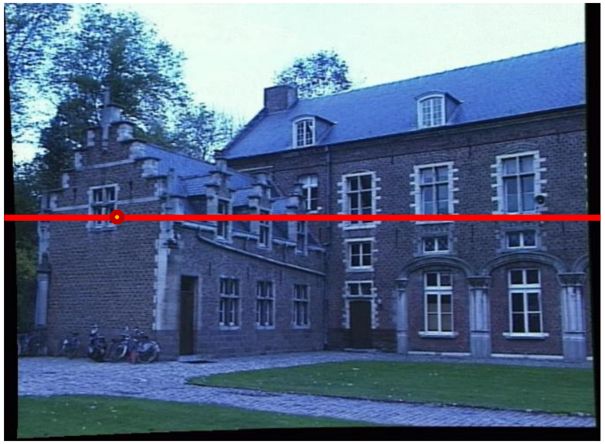


Disparity image

Depth image

Dense Stereo Depth Estimation

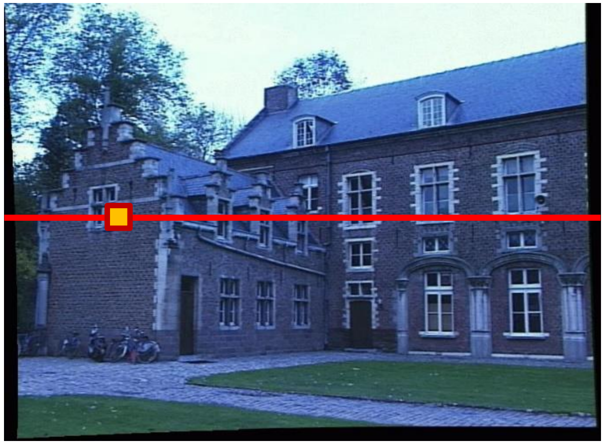
- For each pixel in left image:
  - Compare photoconsistency with every pixel on the corresponding epipolar line in the right image
  - Pick pixel with best similarity

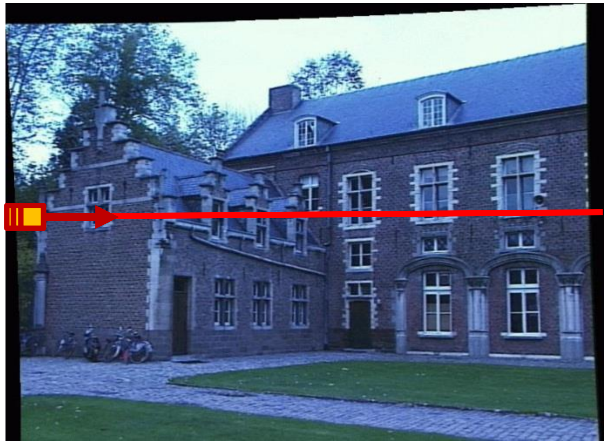



- Problems:
  - Noise
  - Intensity of a single pixel not very distinctive

Dense Stereo Depth Estimation

- Better idea: Compare patches (blocks)





- New questions:
  - What are good patch correlation measures?
  - Patch size?
  - etc.

Block Matching Algorithm

- Input: Two images, intrinsics camera calibration, relative pose
- Output: Disparity image
- Algorithm:
  - Rectify images
  - For each pixel in left image:
    - Compute matching cost along epipolar line using patch comparison
    - Determine minimum in matching cost with sub-pixel accuracy
  - Filter outliers

Patch Correlation Measures

- Sum-of-squared differences:

$$\mathrm{SSD}(B, (\Delta x, \Delta y)) = \sum_{(x,y) \in B} \left(I^l(x,y) - I^r(x + \Delta x, y + \Delta y) \right)^2$$

block/window

- Sum-of-absolute differences:

$$\mathrm{SAD}(B, (\Delta x, \Delta y)) = \sum_{(x,y) \in B} \left| I^l(x,y) - I^r(x + \Delta x, y + \Delta y) \right|$$

Less sensitive to outliers

- Normalized Cross-Correlation:

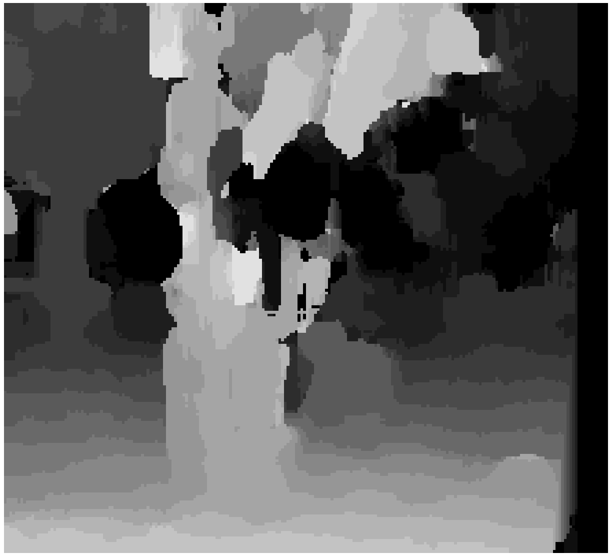
$$\mathrm{NCC}(B, (\Delta x, \Delta y)) = \frac{\sum_{(x,y) \in B} I^l(x,y) I^r(x + \Delta x, y + \Delta y)}{\left(\sum_{(x,y) \in B} I^l(x,y)^2 \right) \left(\sum_{(x,y) \in B} I^r(x + \Delta x, y + \Delta y)^2 \right)}$$

Invariant to illumination changes

Block Size

- Common choices are 5x5, 11x11, …
  - Smaller neighborhood: more details
  - Larger neighborhood: less noise
- Suppress pixels with low confidence (f.e. check ratio best match vs. second best match, examine local behavior of matching cost, etc.)

3x3 block-size



20x20 block-size

Images: R. Szeliski

Probabilistic and Variational Views

- We can formulate stereo disparity estimation as maximum likelihood estimation

$$p(u \mid I^l, I^r, \boldsymbol{\xi}) = \prod_{\mathbf{y} \in \Omega} p(u(\mathbf{y}) \mid I^l, I^r, \boldsymbol{\xi})$$

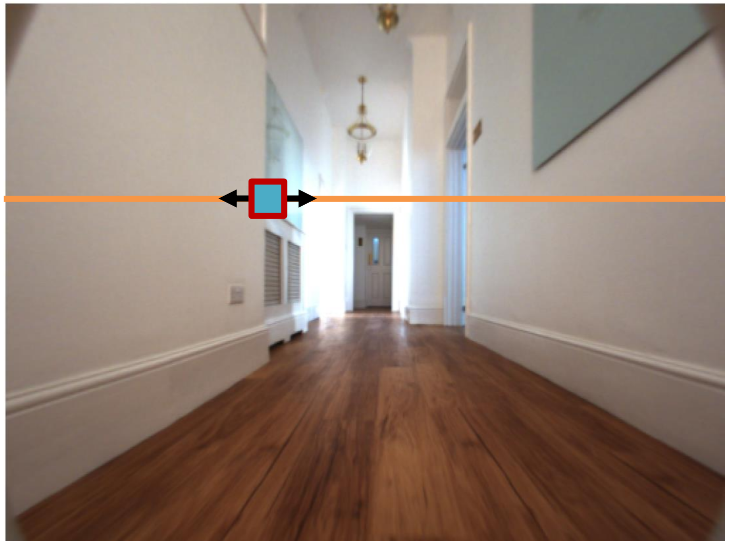
- Variational methods define the true underlying signal as continuous function in the image domain

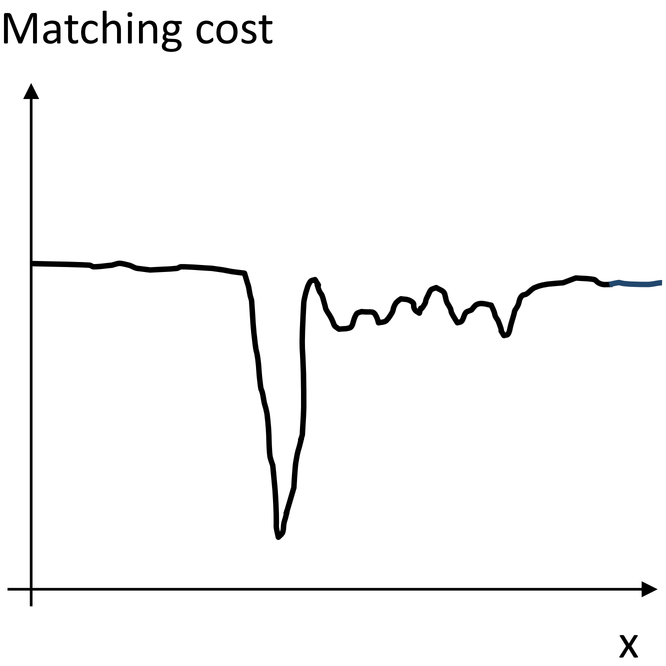
$$E(u) = \int_{\mathbf{y} \in \Omega} \mathcal{F}(u(\mathbf{y})) d\mathbf{y}$$

data fidelity

- Just relying on data fidelity may not work well..

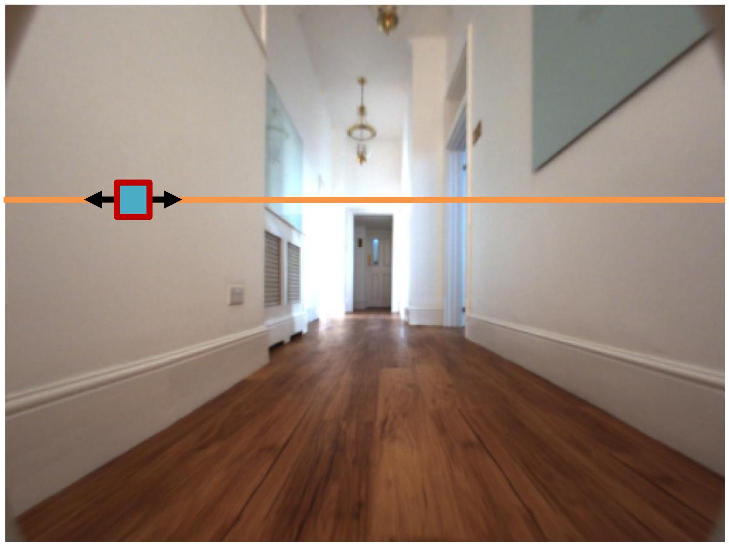
Behavior of the Correspondence Measure

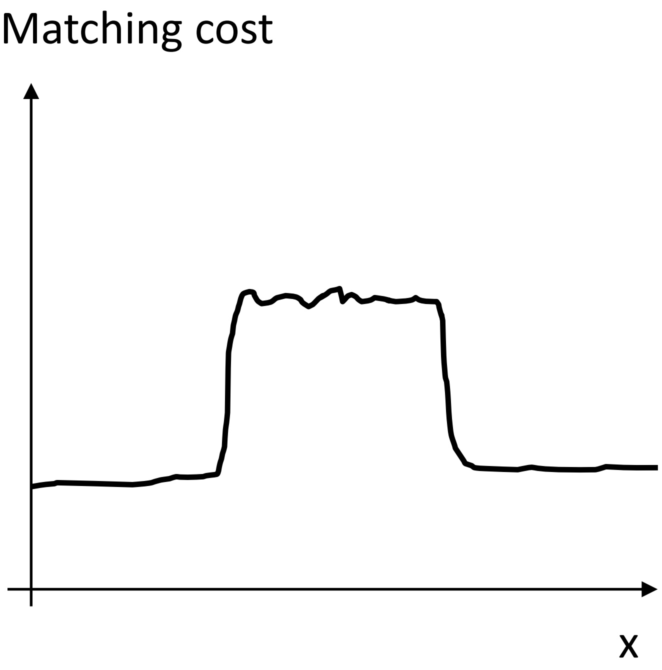




Images: Pinies et al., 2015

Behavior of the Correspondence Measure





Images: Pinies et al., 2015

Challenges for Dense Correspondence Search

- Corresponding patches may differ !

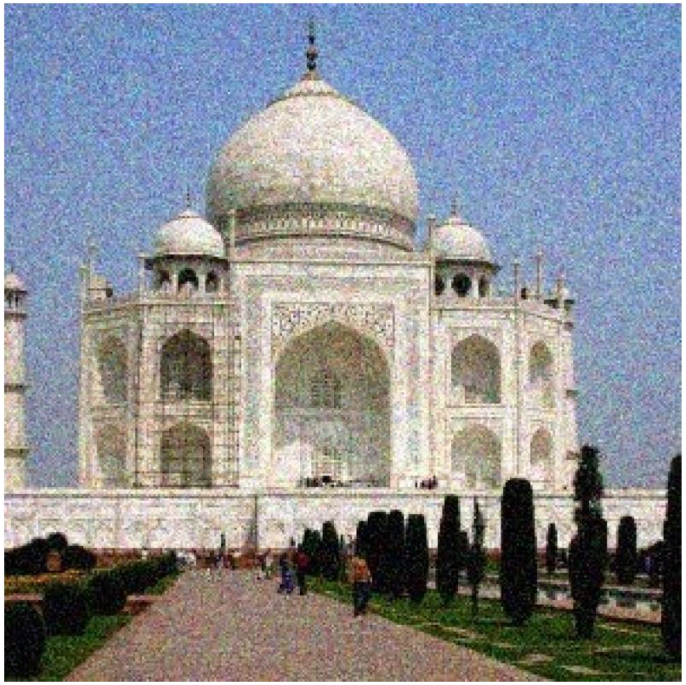


Image Noise
(Camera-related)

Images: C. Gava

Challenges for Dense Correspondence Search

- Corresponding patches may differ !

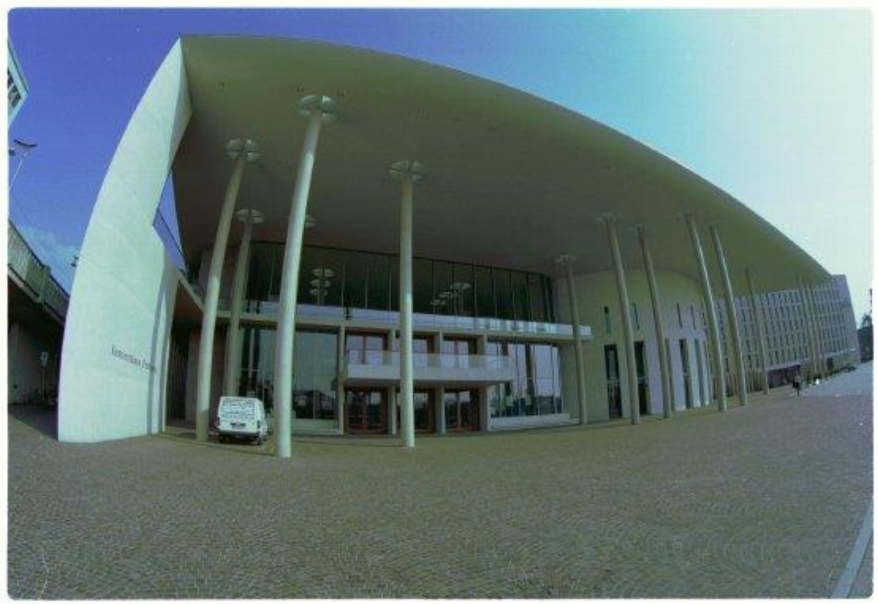


Image Distortion
(Camera-related)

Images: R. Szeliski, H. Dersch

Challenges for Dense Correspondence Search

- Corresponding patches may differ !

Color Abberation
(Camera-related)

Images: C. Gava

Challenges for Dense Correspondence Search

- Corresponding patches may differ !

Perspective Distortion
(Viewpoint-related)

Images: R. Szeliski

Challenges for Dense Correspondence Search

- Corresponding patches may differ !

Occlusions
(Viewpoint-related)

Images: Middlebury benchmark

Challenges for Dense Correspondence Search

- Corresponding patches may differ !

Specular Reflections
(Viewpoint-related)

Images: Weinmann et al., ICCV 2013

Challenges for Dense Correspondence Search

- Corresponding patches may differ !


Illumination changes
(Scene-related)

Images: C. Gava

Challenges for Dense Correspondence Search

- Corresponding patches may differ !

Motion blur
(Scene-related)

Images: C. Gava

Challenges for Dense Correspondence Search

- Correspondence can be ambiguous !

Low Texture
(Scene-related)

Images: Pinies et al., 2015

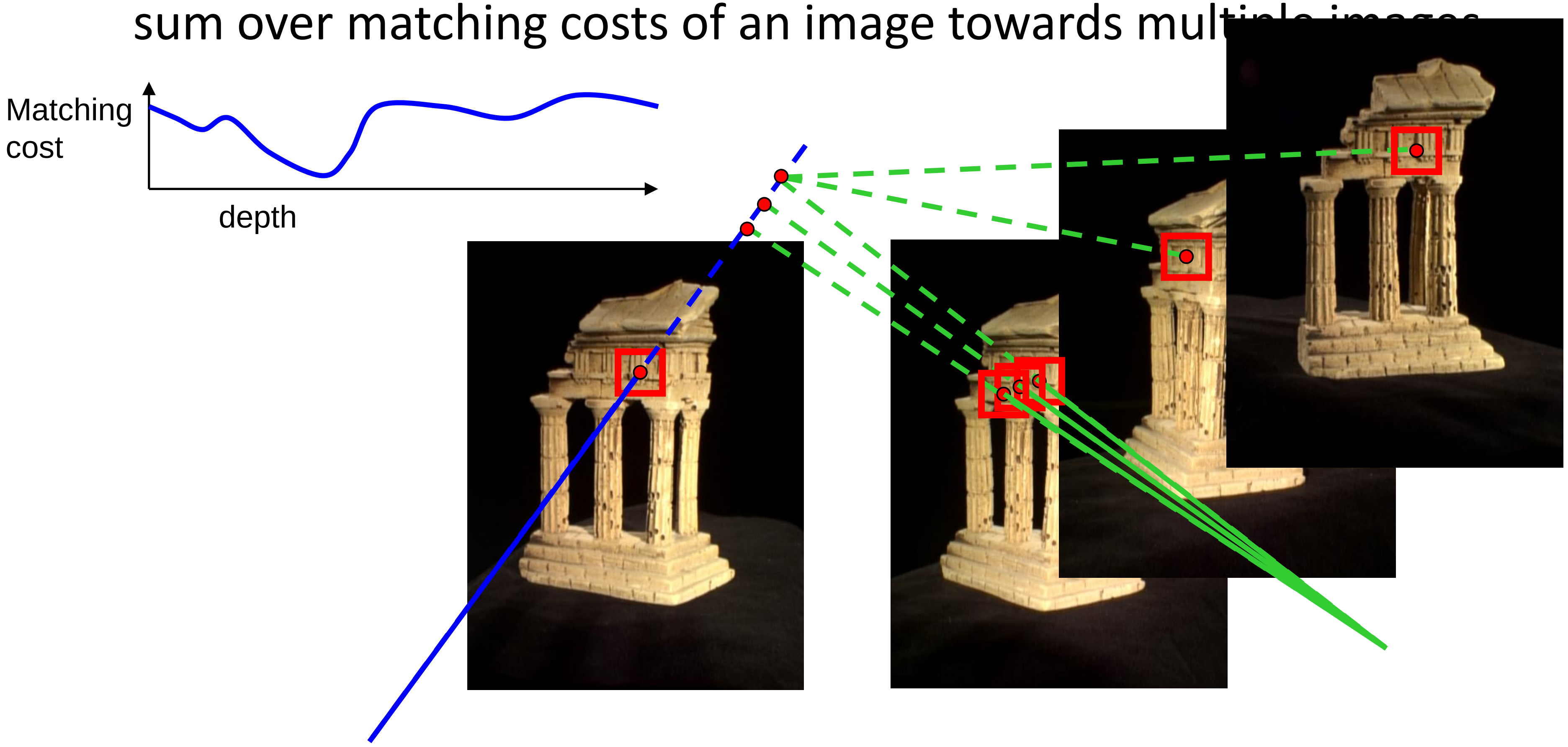
Challenges for Dense Correspondence Search

- Correspondence can be ambiguous !

Repetitive Structure/Texture
(Scene-related)

Dense Depth from Multiple Views

- Straightforward approach: extend two-view matching cost to sum over matching costs of an image towards multiple images



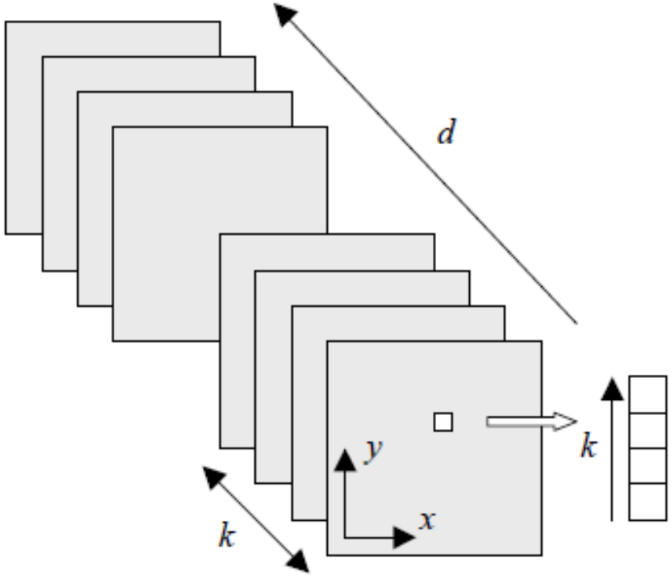
Slide adapted from R. Szeliski

Disparity Space Image / Cost Volumes

- Sum of matching costs between reference and k-th image for discrete depth hypotheses in each pixel

$$C(\mathbf{y}, d) = \sum_k \rho(I_k(\omega(\mathbf{y}, d, \boldsymbol{\xi}_k)) - I_{ref}(\mathbf{y}))$$

- Represent in 3D disparity space image
- Multi-view: inv. depth, „cost volume“



[Szeliski and Golland 1999]

inv.
depth

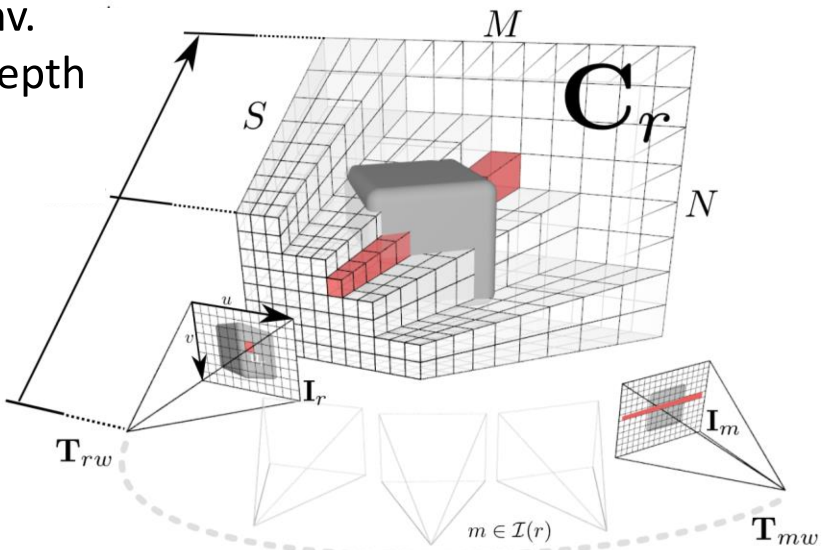
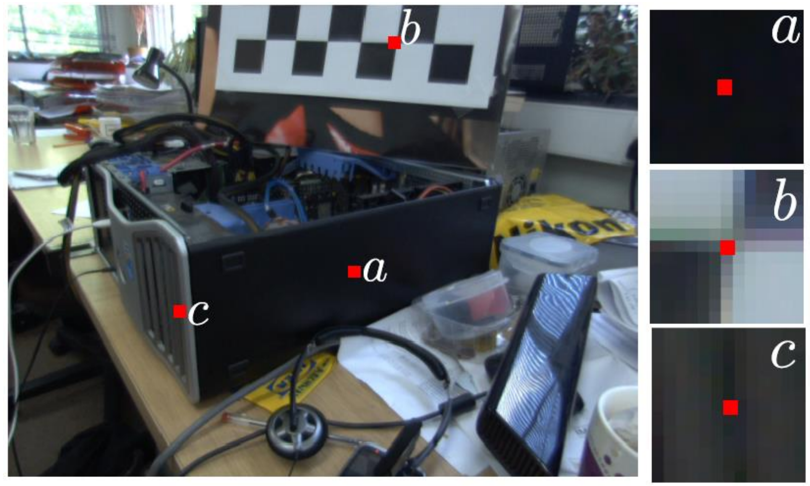
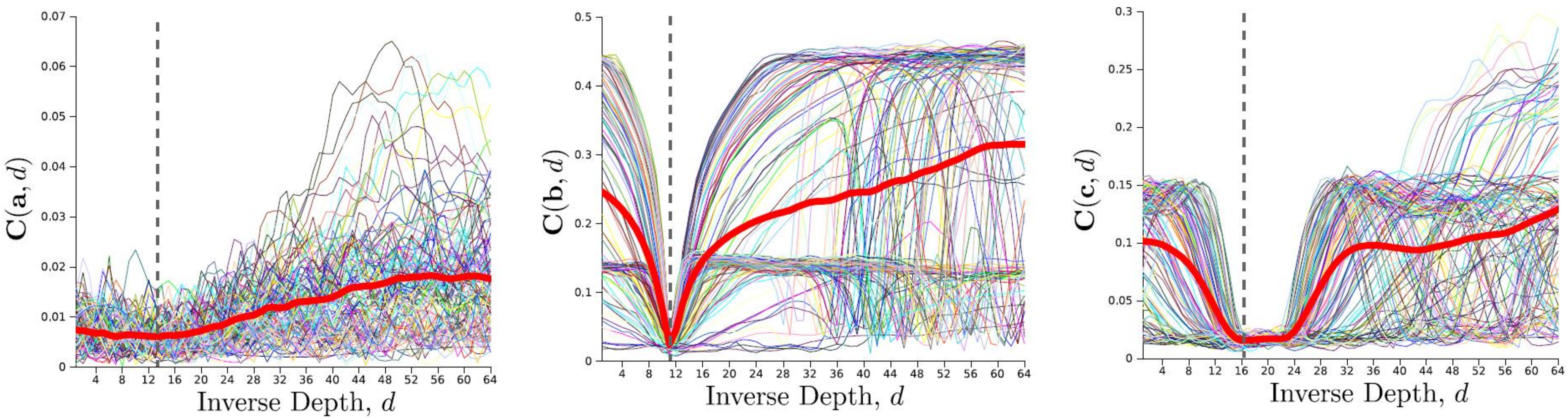


Image from Newcombe et al., 2011

Multi-View Correspondence Measure



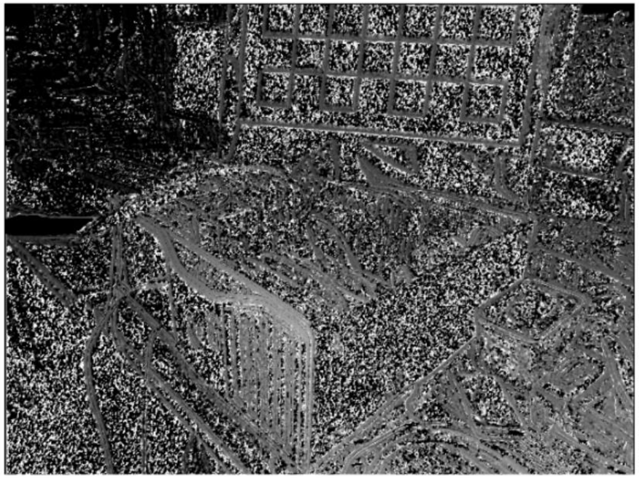


Images: R. Newcombe, 2014

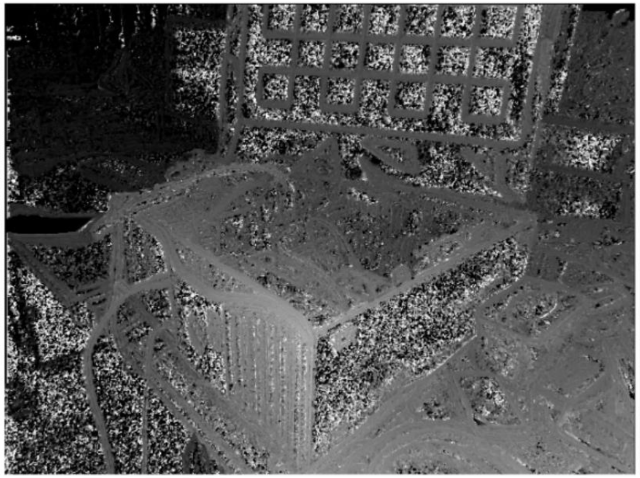
Per-Pixel Max-Likelihood Solution

- Simply choosing the depth with best matching cost at each pixel may not provide a good solution

$$\mathrm{argmin}_d\, C(\mathbf{y}, d)$$



2 comparison frames



10 comparison frames

30 comparison frames

Regularization

- Neighboring pixels should not be treated independently from each others

- How can we incorporate prior knowledge about the observed 3D structures such as smoothness or planarity?

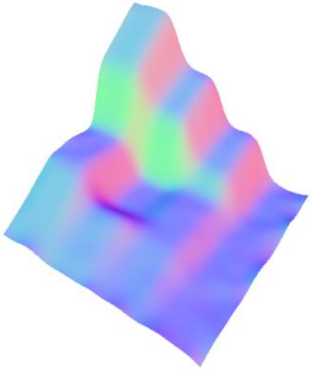
$$E(u) = \int_{\mathbf{y}\in\Omega} \mathcal{F}(u(\mathbf{y}))d\mathbf{y} + \underbrace{\lambda}_{\text{trade-off parameter}} \int_{\mathbf{y}\in\Omega} \underbrace{\mathcal{R}}_{\text{regularizer}}(u(\mathbf{y}))d\mathbf{y}$$

- Idea: add regularizing prior term to the optimization problem

Smoothness Regularizers

- Quadratic regularizers oversmooth at discontinuities

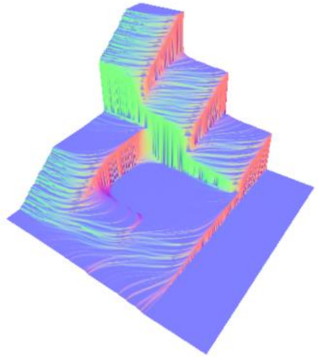
over-
smooth!



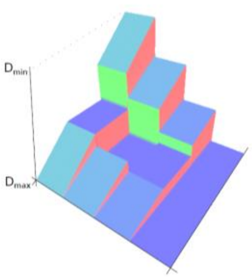
$$E(u) = \frac{1}{2}\int_{\mathbf{y}\in\Omega} \|u(\mathbf{y}) - z(\mathbf{y})\|_2^2 \, d\mathbf{y} + \lambda \frac{1}{2}\int_{\mathbf{y}\in\Omega} \|\nabla u(\mathbf{y})\|_2^2 \, d\mathbf{y}$$

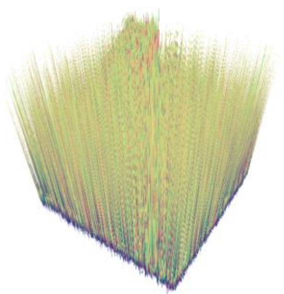
- Total variation (TV) favors piece-wise constant i.e. fronto-parallel solutions

Stair-
casing!



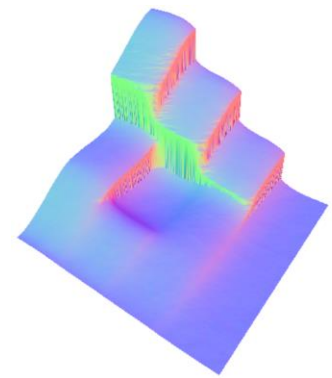
$$E(u) = \int_{\mathbf{y}\in\Omega} \|u(\mathbf{y}) - z(\mathbf{y})\|_1 \, d\mathbf{y} + \lambda \int_{\mathbf{y}\in\Omega} \|\nabla u(\mathbf{y})\|_1 \, d\mathbf{y}$$





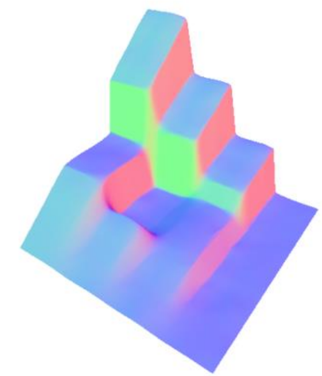
Images: R. Newcombe, 2014

Smoothness Regularizers



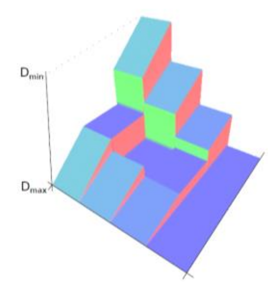
- Huber-norm regularizer as a trade-off between quadratic and TV

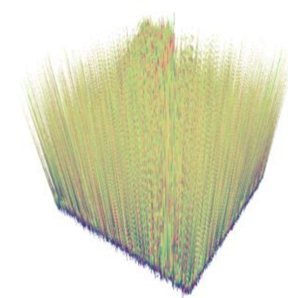
$$E(u) = \int_{\mathbf{y}\in\Omega} \|u(\mathbf{y}) - z(\mathbf{y})\|_{\delta_{\mathcal{F}}} \, d\mathbf{y} + \lambda \int_{\mathbf{y}\in\Omega} \|\nabla u(\mathbf{y})\|_{\delta_{\mathcal{R}}} \, d\mathbf{y}$$



- Total generalized variation (TGV) for piecewise affine or higher-order smooth polynomial surfaces

$$E(u) = \int_{\mathbf{y}\in\Omega} \|u(\mathbf{y}) - z(\mathbf{y})\|_{\delta_{\mathcal{F}}} \, d\mathbf{y} + \alpha_1 \int_{\mathbf{y}\in\Omega} \|\nabla u(\mathbf{y}) - v(\mathbf{y})\|_1 \, d\mathbf{y}$$
$$+\alpha_0 \int_{\mathbf{y}\in\Omega} \left\| \frac{1}{2}\left(\nabla v(\mathbf{y}) + \nabla v(\mathbf{y})^\top\right) \right\|_1 d\mathbf{y} \qquad (\mathrm{TGV}_\alpha^2)$$





Images: R. Newcombe, 2014

Optimization with Regularizers

- Depending on the formulation, different techniques for optimization can be applied
  - Variational methods (f.e. primal-dual optimization) for continuous depth
  - Discrete energy optimization methods (f.e. graph-cuts)

Semi-Global Matching

- Approximate discrete inference of disparities in Markov Random Field
- Define aggregated cost along scanlines

$$L^{\mathbf{r}}(d_{\mathbf{p}}) = \varphi(d_{\mathbf{p}}) + \min_{d_{\mathbf{p}-\mathbf{r}} \in \Sigma} \left\{ L^{\mathbf{r}}(d_{\mathbf{p}-\mathbf{r}}) + \varphi(d_{\mathbf{p}-\mathbf{r}}, d_{\mathbf{p}}) \right\}$$

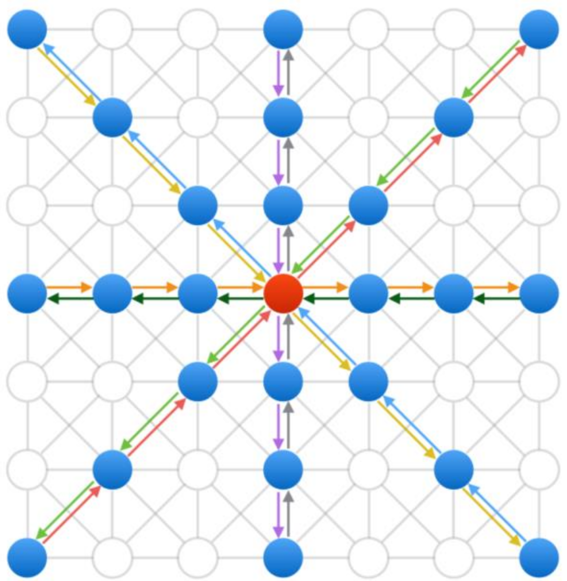
matching cost

disparity range

pairwise terms

pixel $\mathbf{p} = (x, y)$

scanline direction $\mathbf{r} = (dx, dy)$



8 scanlines

- Determine disparity that minimizes sum of aggregated costs over scanlines

$$\operatorname{argmin}_{d_{\mathbf{p}} \in \Sigma} \sum_{\mathbf{r} \in \mathcal{R}} L^{\mathbf{r}}(d_{\mathbf{p}})$$

- Typically 8 or 16 scanlines

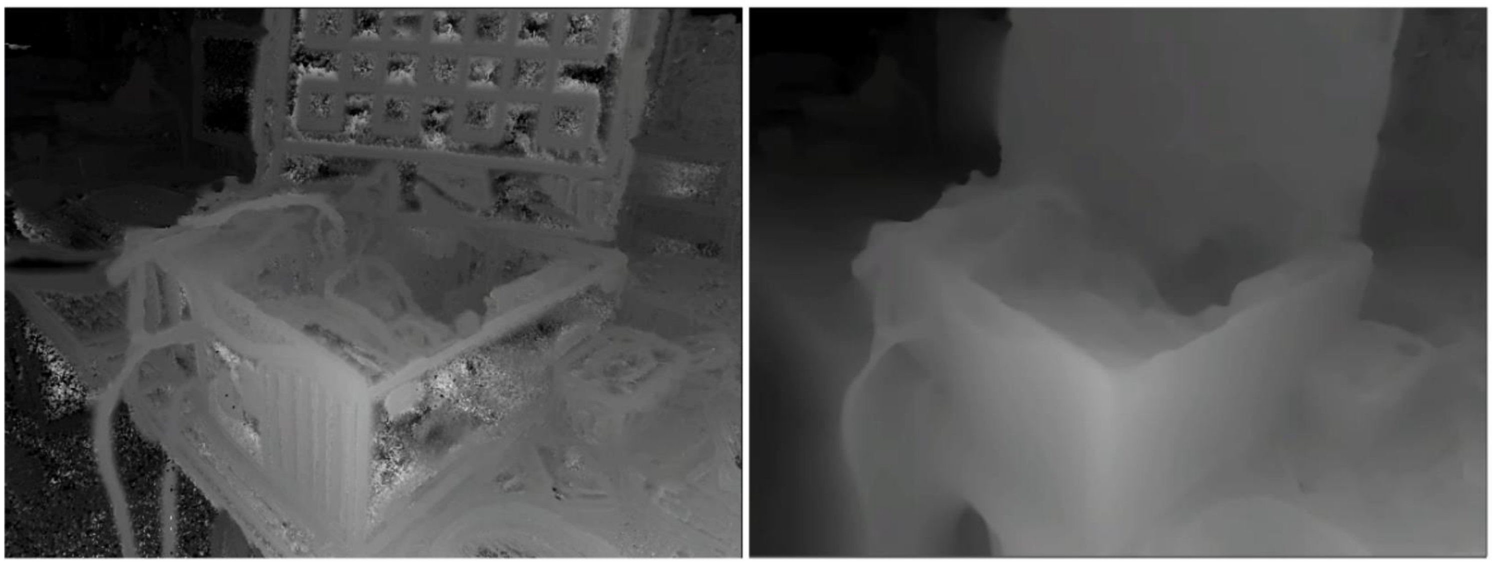
Popular choice of pairwise term:

$$\varphi(d_{\mathbf{p}-\mathbf{r}}, d_{\mathbf{p}}) = \begin{cases} 0, & \text{if } d_{\mathbf{p}-\mathbf{r}} = d_{\mathbf{p}} \\ P_1, & \text{if } |d_{\mathbf{p}-\mathbf{r}} - d_{\mathbf{p}}| = 1 \\ P_2, & \text{if } |d_{\mathbf{p}-\mathbf{r}} - d_{\mathbf{p}}| > 1 \end{cases}$$

Effect of Regularization

Data term: cost volume over L1-norm on photometric residuals

Regularizer: Huber-norm on inverse depth gradient

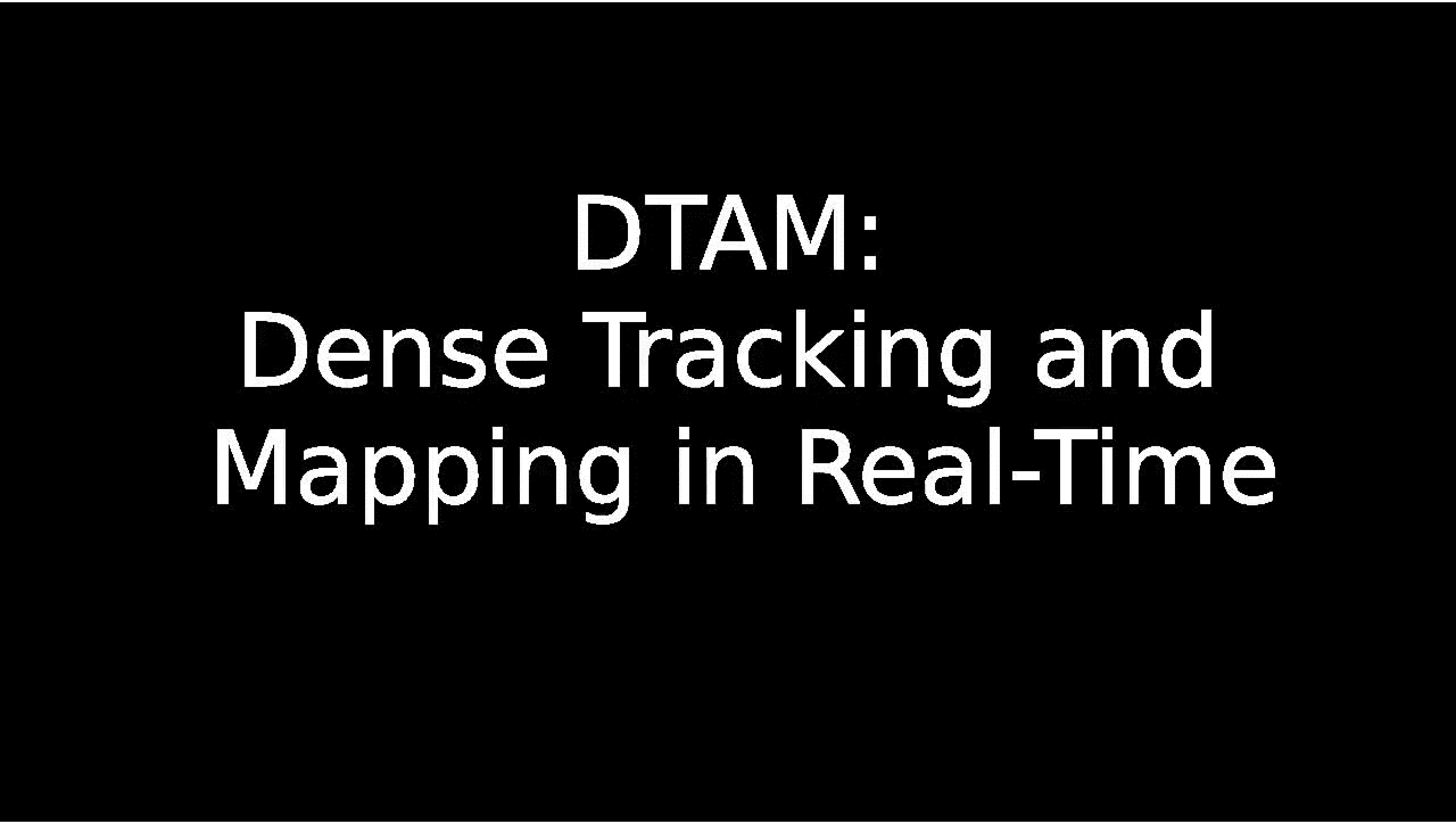


Images: R. Newcombe et al., 2011

Dense Depth from Multiple Views

- Some problems:
  - How to select reference image?
  - How to select comparison images?
  - How to handle varying image overlap?
  - How to handle varying occlusions in each image pair?
  - How to perform optimization efficiently?

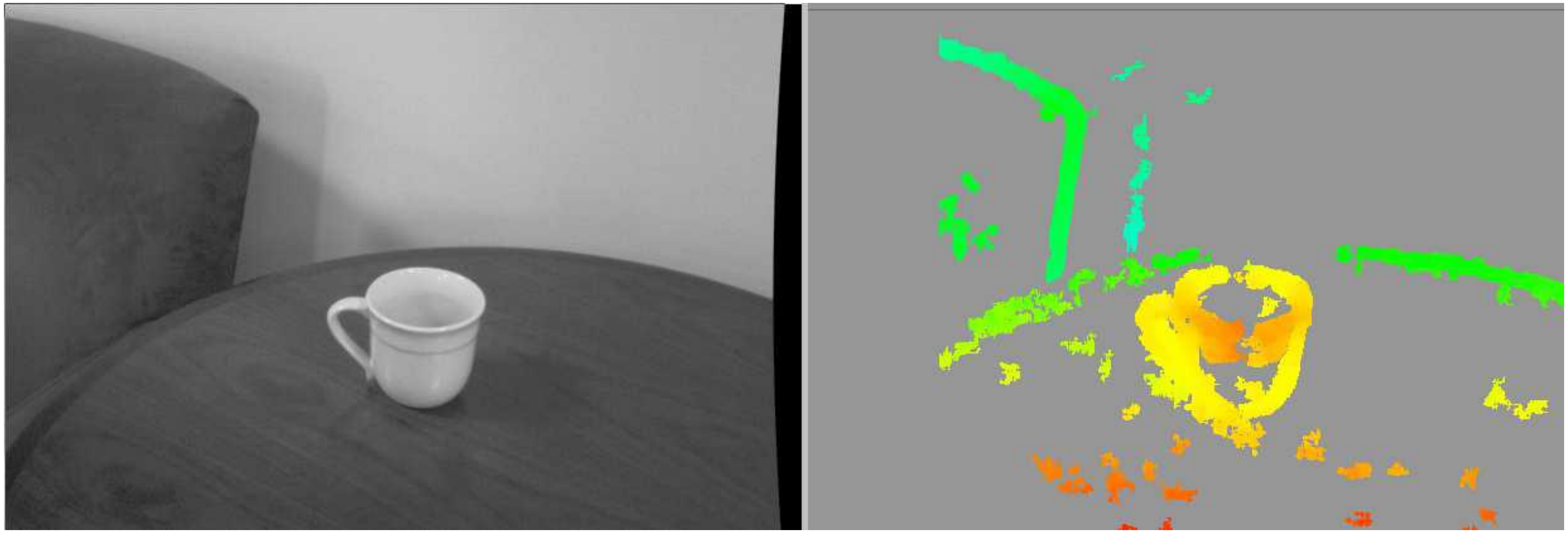
Dense Tracking and Mapping



Newcombe et al., DTAM: Dense Tracking and Mapping in Real-time, ICCV 2011

Active Depth Sensing

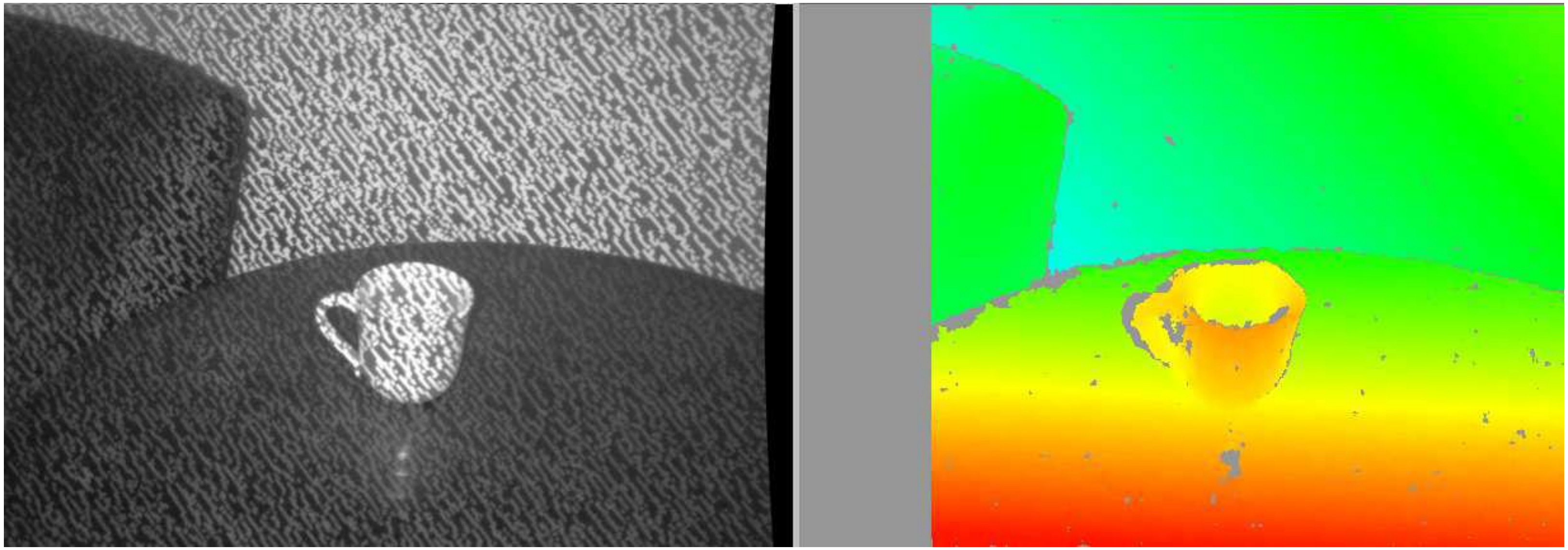
- What can we do about textureless scenes?



Images: J. Sturm

Active Depth Sensing

- Idea: Project light/texture



Images: J. Sturm

Depth Cameras

...

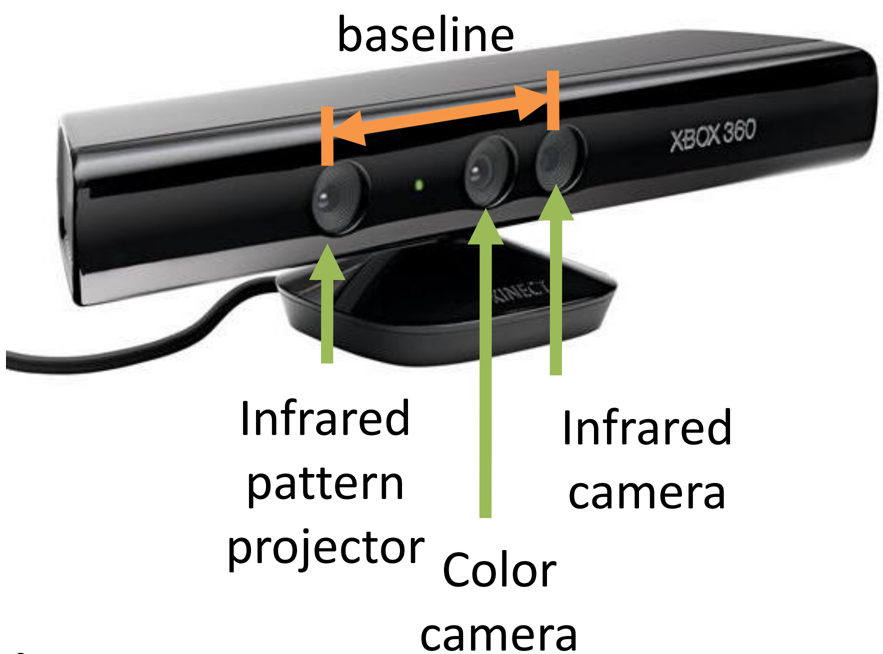
Time-of-Flight

...

Structured Light

Structured Light Measurement Principle

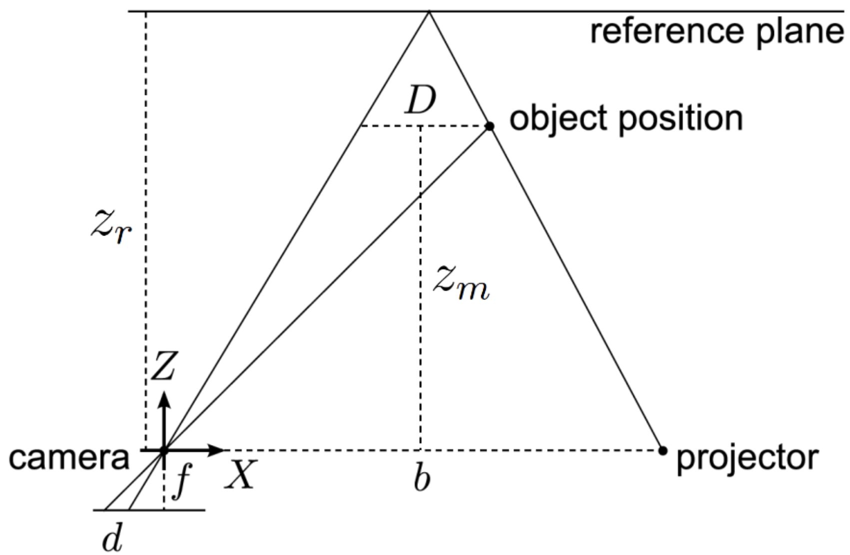
- Project speckle pattern using infrared laser and diffraction element
- Measure infrared speckles using infrared camera
- Measure corresponding RGB image using color camera



Slide adapted from J. Sturm

Structured Light Measurement Principle

- Use known baseline and reference pattern for depth measurement

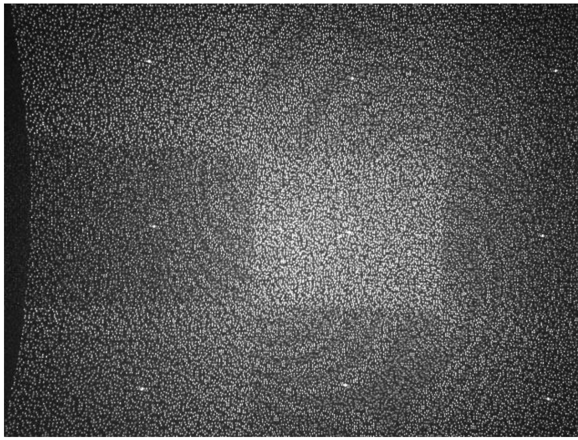


$$\frac{d}{f} = \frac{D}{z_m}$$

$$\frac{D}{z_r - z_m} = \frac{b}{z_r}$$

$$\rightarrow \quad z_m = \frac{z_r}{\frac{d z_r}{b f} + 1}$$

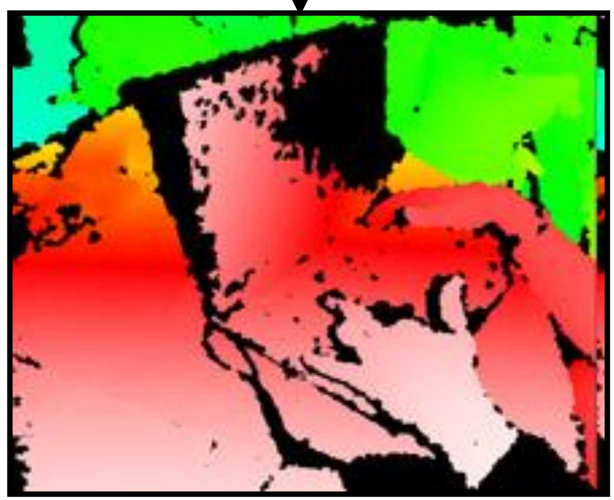
Structured Light Measurement Principle



IR reference pattern

Block matching (9x9)

IR pattern
in actual scene

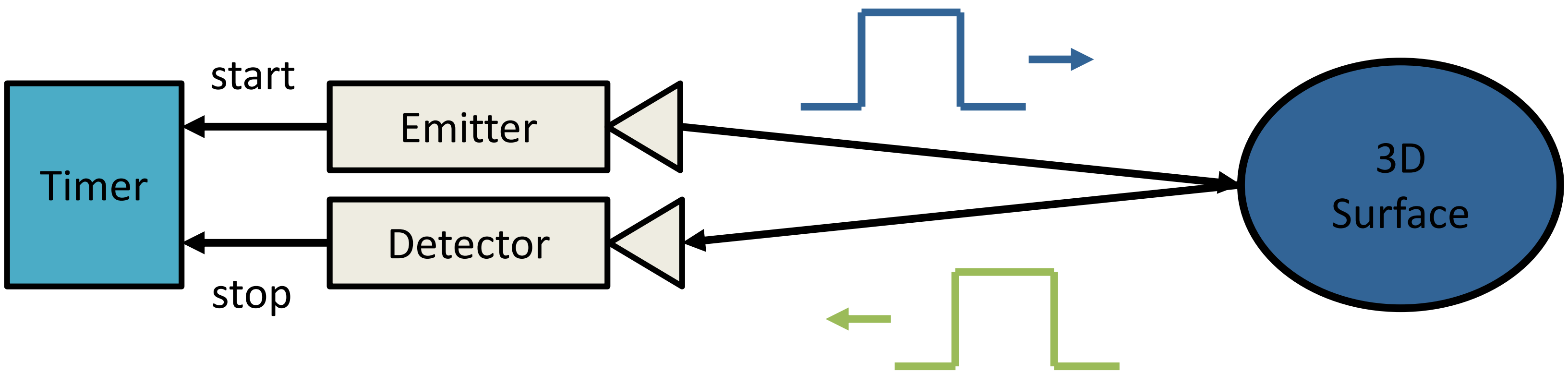


Depth image

Slide adapted from J. Sturm

Time-of-Flight Measurement Principle

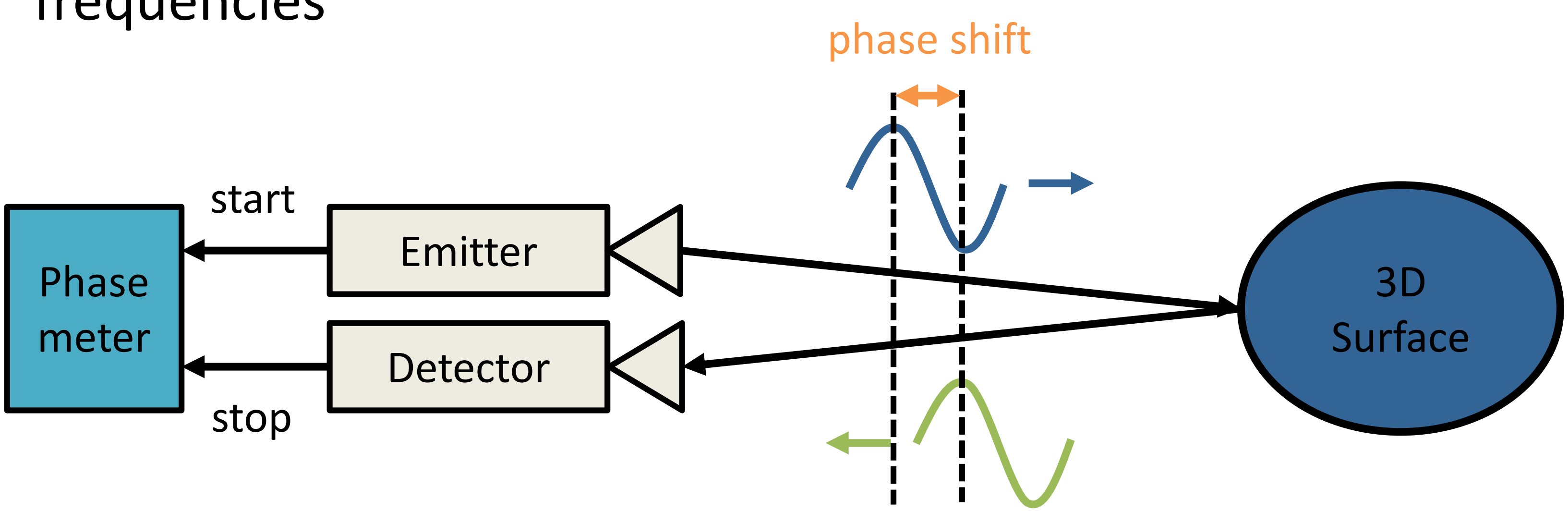
- Idea: emit timed IR pulse and measure its time of return
- Difficult to create pulses and measure time precisely



Slide adapted from N. Navab

Time-of-Flight Measurement Principle

- Idea: emit continous modulated IR wave signal and measure phase shift
- Signal periodicity creates phase ambiguities: use multiple frequencies



Slide adapted from N. Navab

Active vs. Passive Sensors

- Active Sensors
  - Surfaces do not need to be textured
  - Bring their own light, also work in low-light scenarios
  - But: Diffuse IR sunlight typically overrides emitted light
  - Difficulties for IR-absorbing or reflective materials

- Passive Sensors (f.e. RGB-only)
  - Do not rely on measuring emitted light
  - Are not limited by the resolution of the projection or ToF measurement principle
    - Distance
    - Multi-path noise (ToF)

Lessons Learned Today

- Stereo depth reconstruction from two and multiple views
  - Stereo rectification simplifies correspondence search for two views
  - Dense correspondence search using block matching
  - Correspondences can be ambiguous
  - Regularization with priors to help with noisy and ambiguous data terms

- Depth cameras
  - Structured light principle
  - Time-of-flight principle

Thanks for your attention!