Practical Course:
Vision-based Navigation
Winter Term 2017/2018

Lecture 1: Basics

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What we will cover today

- Linear algebra, notation
- 3D geometry
- Projective geometry
  - Camera intrinsics
  - Epipolar geometry

- Robot Operating System (ROS)

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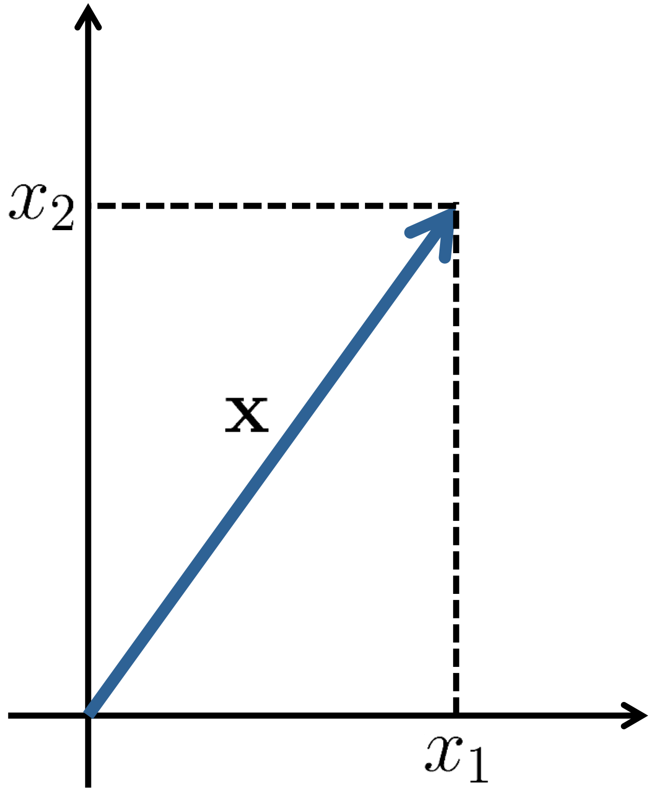
Vectors

- Vector and its coordinates

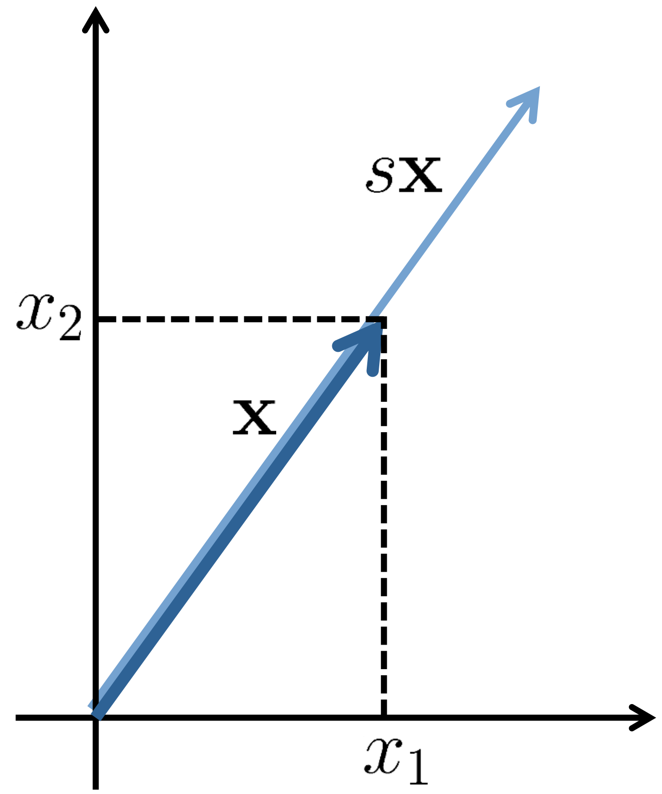
$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$$



- Vectors represent points in an n-dimensional space

Vector Operations

- **Scalar multiplication**
- Addition/subtraction
- Length
- Normalized vector
- Dot product
- Cross product



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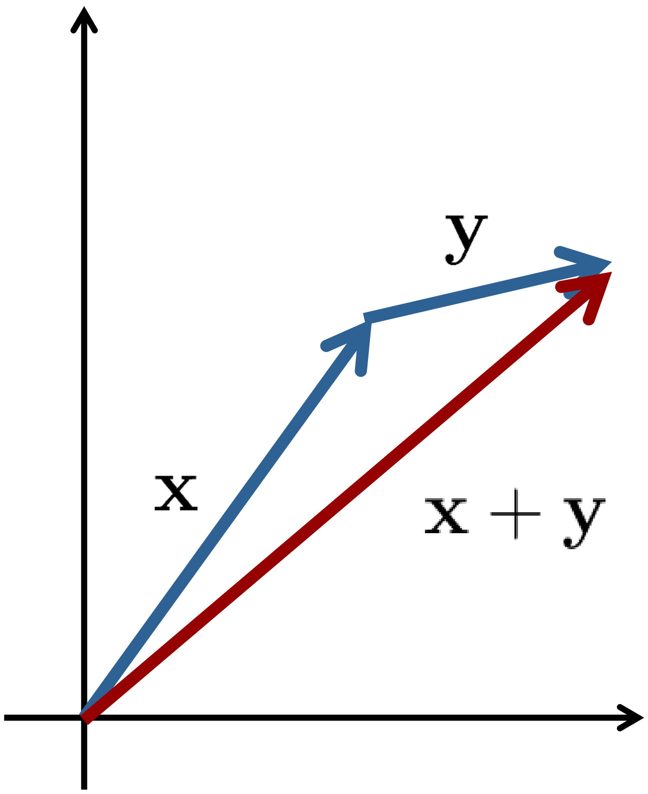

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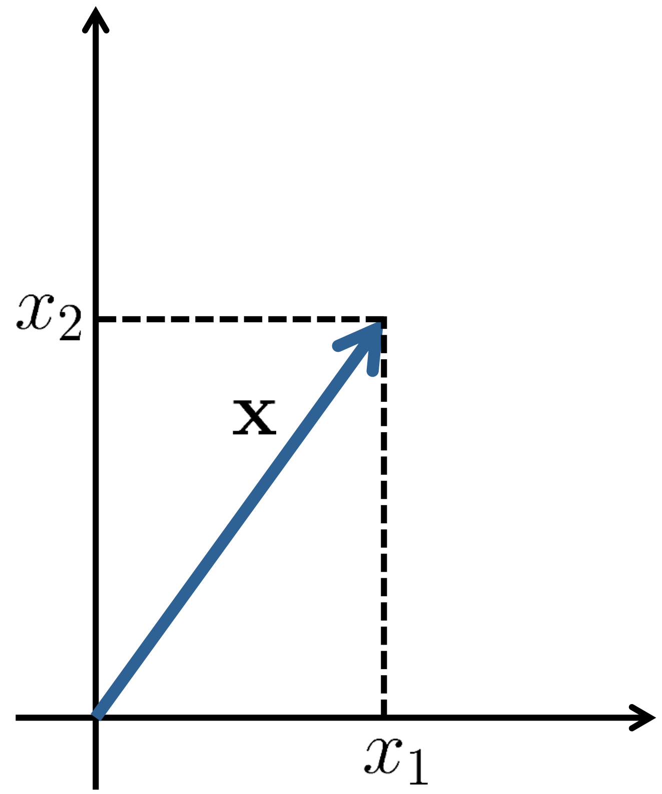

$$\|x\|_2 = \|x\| = \sqrt{x_1^2 + x_2^2 + \ldots}$$

Vector Operations

- Scalar multiplication
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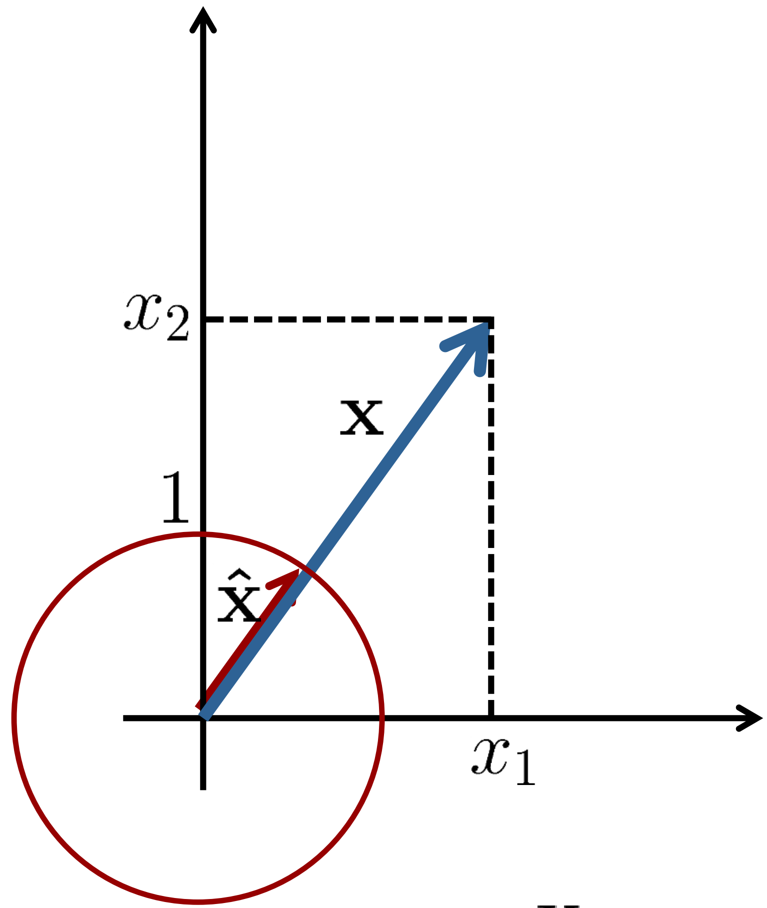
- Length
- **Normalized vector**
- Dot product
- Cross product



$$\hat{\mathbf{x}} = \frac{\mathbf{x}}{\|\mathbf{x}\|}$$

Vector Operations

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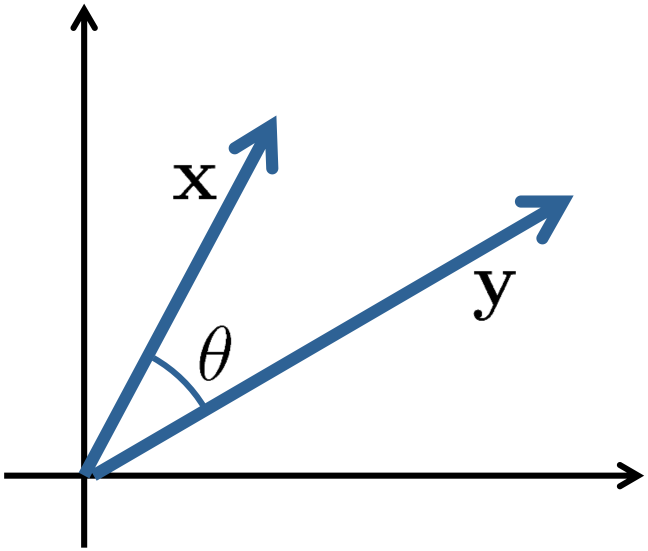
- **Dot product**
- Cross product



$$\mathbf{x} \cdot \mathbf{y} = \|\mathbf{x}\| \|\mathbf{y}\| \cos\theta$$

$\mathbf{x}, \mathbf{y}$ are orthogonal if $\mathbf{x} \cdot \mathbf{y} = 0$

$\mathbf{y}$ is lin. dependent from $\{\mathbf{x}_1, \mathbf{x}_2, \ldots\}$ if $\mathbf{y} = \sum_i k_i \mathbf{x}_i$

Vector Operations

- Scalar multiplication
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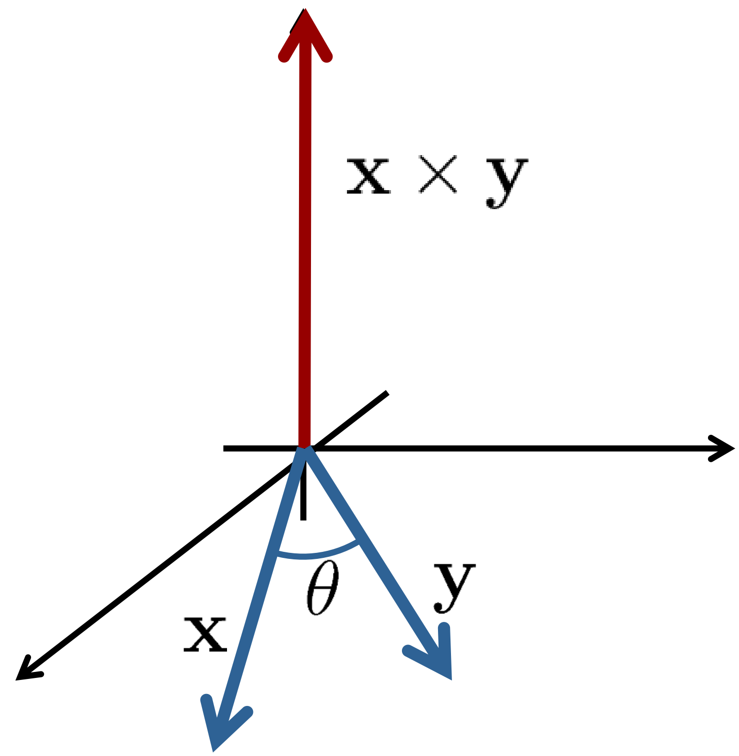
- Length
- Normalized vector
- Dot product
- **Cross product**



$$\mathbf{x} \times \mathbf{y} = \|\mathbf{x}\| \, \|\mathbf{y}\| \sin(\theta)\mathbf{n}$$

Cross Product

- Definition
$$\mathbf{x} \times \mathbf{y} = \begin{pmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - x_2 y_1 \end{pmatrix}$$

- Matrix notation for the cross product
$$[\mathbf{x}]_\times = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}$$

- Verify that $\mathbf{x} \times \mathbf{y} = [\mathbf{x}]_\times \mathbf{y}$

Matrices

- Rectangular array of numbers

$$X = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & & & \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}$$

rows columns

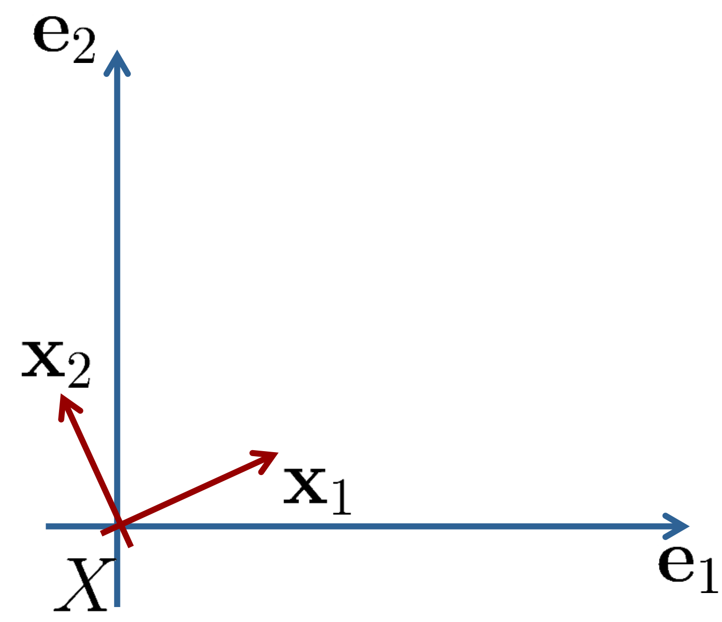
- First index refers to row
- Second index refers to column

Matrices

- Column vectors of a matrix

$$X = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & & & \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{x}_{*1} & \mathbf{x}_{*2} & \dots & \mathbf{x}_{*m} \end{pmatrix}$$



- Geometric interpretation: for example, column vectors can form basis of a coordinate system

Matrices

- Row vectors of a matrix

$$X = \begin{pmatrix} \boxed{x_{11} \quad x_{12} \quad \dots \quad x_{1m}} \\ \boxed{x_{21} \quad x_{22} \quad \dots \quad x_{2m}} \\ \vdots \\ \boxed{x_{n1} \quad x_{n2} \quad \dots \quad x_{nm}} \end{pmatrix} = \begin{pmatrix} \mathbf{x}_{1*}^\top \\ \mathbf{x}_{2*}^\top \\ \vdots \\ \mathbf{x}_{n*}^\top \end{pmatrix}$$

Matrices

- Square matrix
- Diagonal matrix
- Upper and lower triangular matrix
- Symmetric matrix
- Skew-symmetric matrix
- (Semi-)positive definite matrix
- Invertible matrix
- Orthonormal matrix
- Matrix rank

Matrices

- Square matrix
- Diagonal matrix
- Upper and lower triangular matrix
- Symmetric matrix $X = X^\top$
- Skew-symmetric matrix $X = -X^\top (= \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix})$
- (Semi-)positive definite matrix $\mathbf{a}^\top X \mathbf{a} \geq 0$
- Invertible matrix
- Orthonormal matrix
- Matrix rank

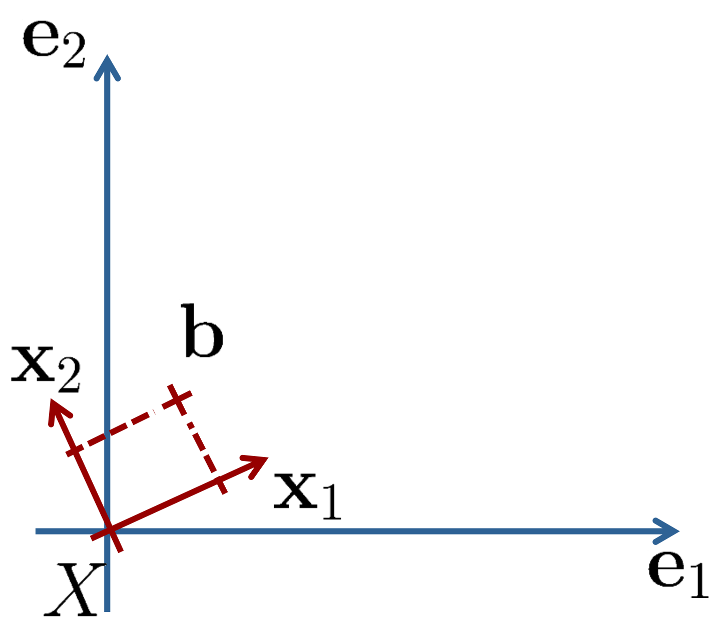
Matrix Operations

- Scalar multiplication
- Addition/subtraction
- Transposition
- Matrix-vector multiplication
- Matrix-matrix multiplication
- Inversion

Matrix-Vector Multiplication

$$X \cdot \mathbf{b} = \sum_{k=1}^{n} \mathbf{x}_{*k} \cdot b_k$$

column vectors



- Geometric interpretation:
  A linear combination of the columns of X scaled by the coefficients of **b**
  → coordinate transf. from local to global frame

Matrix-Matrix Multiplication

- Operator $\mathbb{R}^{n\times m} \times \mathbb{R}^{m\times p} \to \mathbb{R}^{n\times p}$
- Definition

$$
\begin{aligned}
C =& AB \\
=& A \left(\mathbf{b}_{*1} \quad \mathbf{b}_{*2} \quad \cdots \mathbf{b}_{*p} \right)
\end{aligned}
$$

- Interpretation: transformation of coordinate systems
- Can be used to concatenate transforms

Matrix-Matrix Multiplication

- Not commutative (in general) $AB \neq BA$
- Associative $A(BC) = (AB)C$
- Transpose $(AB)^\top = B^\top A^\top$

Matrix Inversion

- If A is a square matrix of full rank, then there is a unique matrix $B = A^{-1}$ such that $AB = I$.

- Different ways to compute, e.g., Gauss-Jordan elimination, LU decomposition, …

- When A is orthonormal, then $A^{-1} = A^\top$

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Geometric Primitives in 3D

- 3D point
  (same as before)

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$$

- Augmented vector

$$\bar{\mathbf{x}} = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \in \mathbb{R}^4$$

- Homogeneous coordinates

$$\tilde{\mathbf{x}} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ \tilde{w} \end{pmatrix} \in \mathbb{P}^3$$

3D Transformations

- Translation

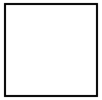
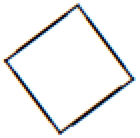
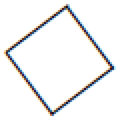
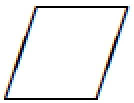
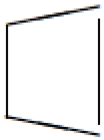
$$\bar{\mathbf{x}}' = \underbrace{\begin{pmatrix} \mathbf{I} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{pmatrix}}_{4\times 4} \bar{\mathbf{x}}$$

- Euclidean transform (translation + rotation), (also called the Special Euclidean group SE(3))

$$\bar{\mathbf{x}}' = \begin{pmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{pmatrix} \bar{\mathbf{x}}$$

- Scaled rotation, affine transform, projective transform…

3D Transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{3\times 4}$	3	orientation	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{3\times 4}$	6	lengths	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{3\times 4}$	7	angles	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{3\times 4}$	12	parallelism	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{4\times 4}$	15	straight lines	

3D Euclidean Transformtions

- Translation $\mathbf{t}$ has 3 degrees of freedom
- Rotation R has 3 degrees of freedom

$$X = \begin{pmatrix} R & \mathbf{t} \\ \mathbf{0} & 1 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3D Rotations

- Rotation matrix R
  (also called the special orthogonal group SO(3))

Alternative representations

- Euler angles
- Axis/angle
- Unit quaternion

Rotation Matrix

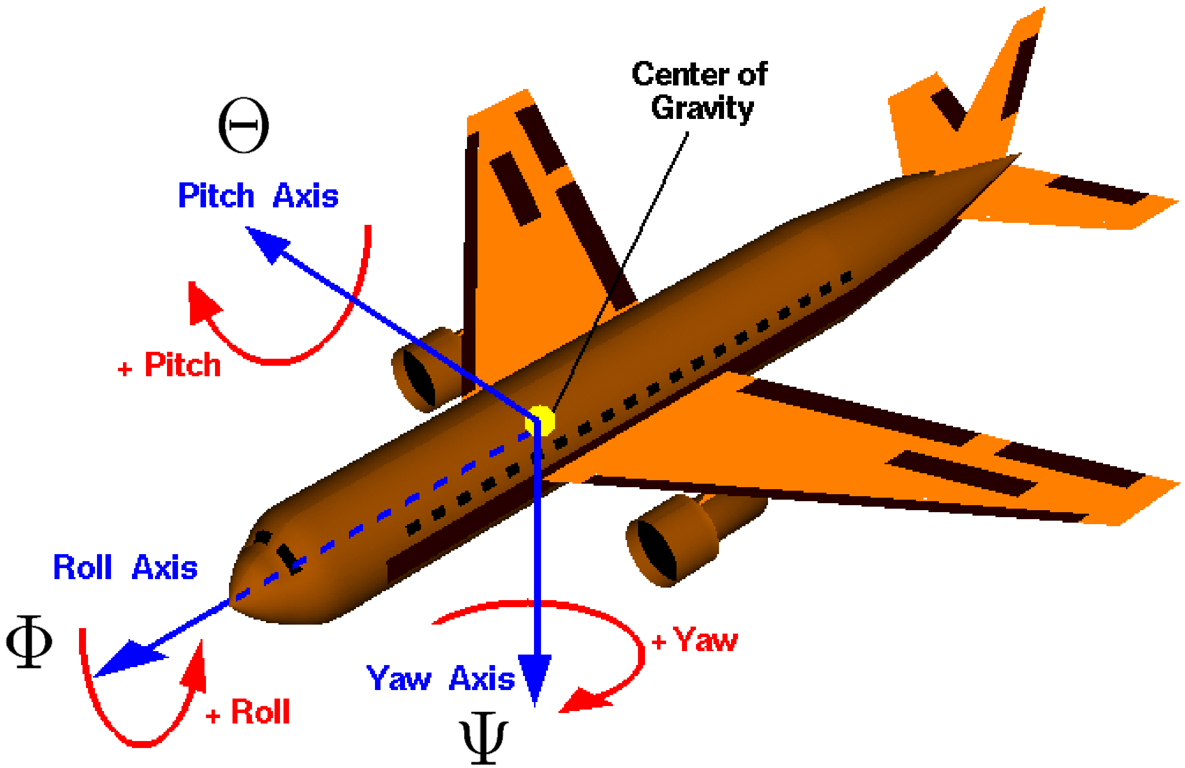
- Orthonormal 3x3 matrix

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

- Advantage: Can be easily concatenated and inverted (how?)
- Disadvantage: Over-parameterized
  (9 parameters instead of 3)

Euler Angles

- Product of 3 consecutive rotations (e.g., around X-Y-Z axes)
- Roll-pitch-yaw convention is very common in aerial navigation (DIN 9300)



Roll-Pitch-Yaw Convention

- Yaw Ψ, Pitch Θ, Roll Φ to rotation matrix

$$R = R_Z(\Psi) R_Y(\Theta) R_X(\Phi)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & \sin\Phi \\ 0 & -\sin\Phi & \cos\Phi \end{pmatrix} \begin{pmatrix} \cos\Theta & 0 & -\sin\Theta \\ 0 & 1 & 0 \\ \sin\Theta & 0 & \cos\Theta \end{pmatrix} \begin{pmatrix} \cos\Psi & \sin\Psi & 0 \\ -\sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\Theta\cos\Psi & \cos\Theta\sin\Psi & -\sin\Theta \\ \sin\Phi\sin\Theta\cos\Psi - \cos\Phi\sin\Psi & \sin\Phi\sin\Theta\sin\Psi + \cos\Phi\cos\Psi & \sin\Phi\cos\Theta \\ \cos\Phi\sin\Theta\cos\Psi + \sin\Phi\sin\Psi & \cos\Phi\sin\Theta\sin\Psi - \sin\Phi\cos\Psi & \cos\Phi\cos\Theta \end{pmatrix}$$

- Rotation matrix to Yaw-Pitch-Roll

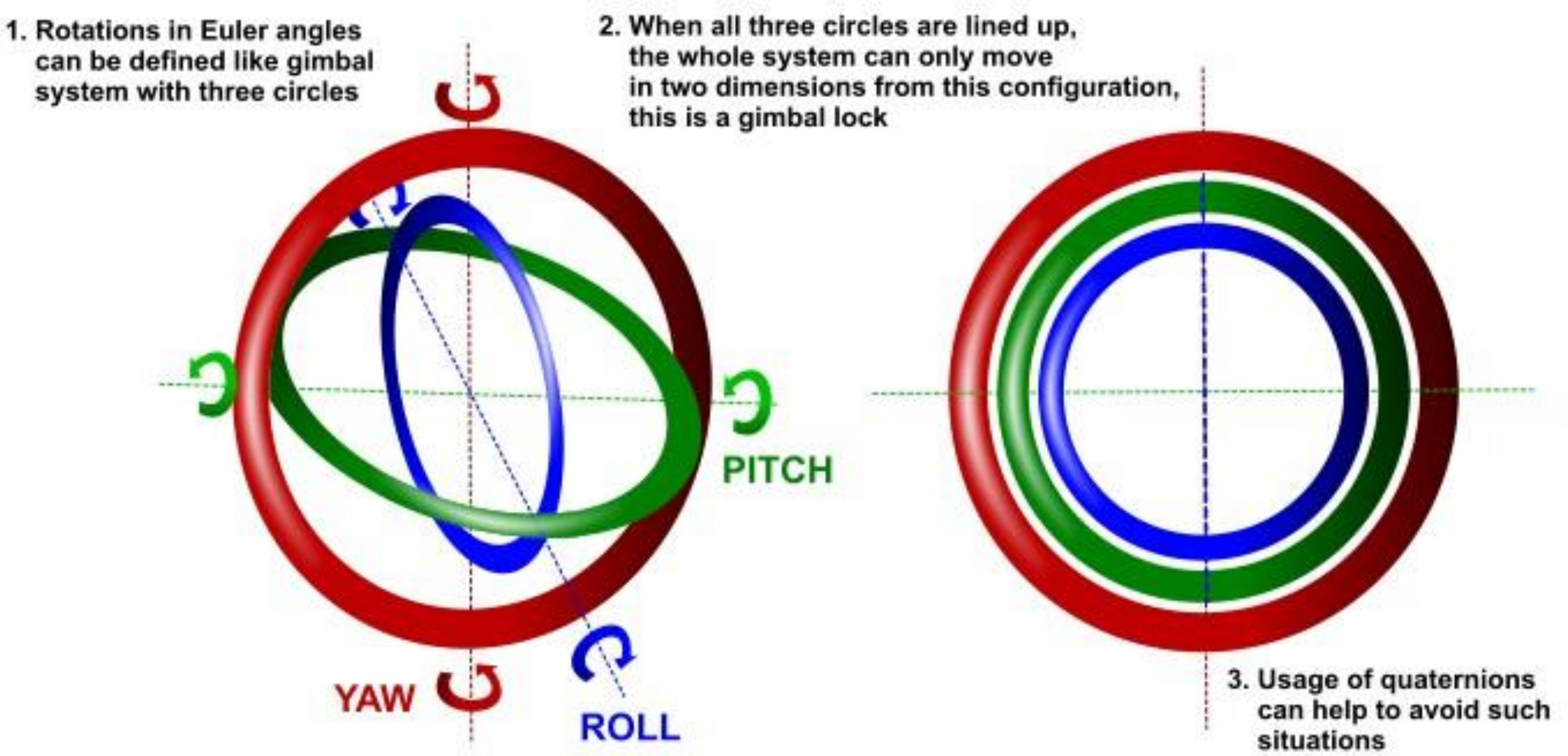
$$\phi = \operatorname{Atan2}\left(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}\right)$$

$$\psi = -\operatorname{Atan2}\left(\frac{r_{21}}{\cos(\phi)}, \frac{r_{11}}{\cos(\phi)}\right)$$

$$\theta = \operatorname{Atan2}\left(\frac{r_{32}}{\cos(\phi)}, \frac{r_{33}}{\cos(\phi)}\right)$$

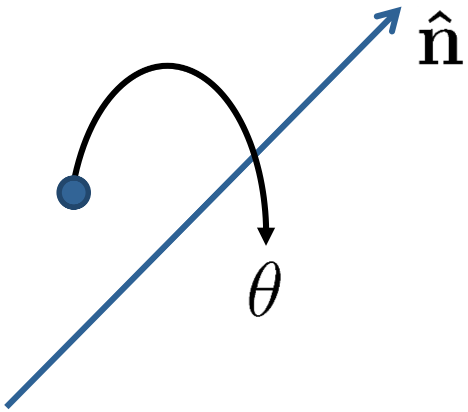
Gimbal Lock

- When the axes align, one degree-of-freedom (DOF) is lost…



Axis/Angle

- Represent rotation by
  - rotation axis $\hat{\mathbf{n}}$ and
  - rotation angle θ
- 4 parameters $(\hat{\mathbf{n}}, \theta)$
- 3 parameters $\boldsymbol{\omega} = \theta\hat{\mathbf{n}}$
  - length is rotation angle
  - also called the angular velocity
  - minimal but not unique (why?)



Conversion

- Rodriguez' formula

$$R(\hat{\mathbf{n}}, \theta) = I + \sin\theta[\hat{\mathbf{n}}]_\times + (1 - \cos\theta)[\hat{\mathbf{n}}]_\times^2$$

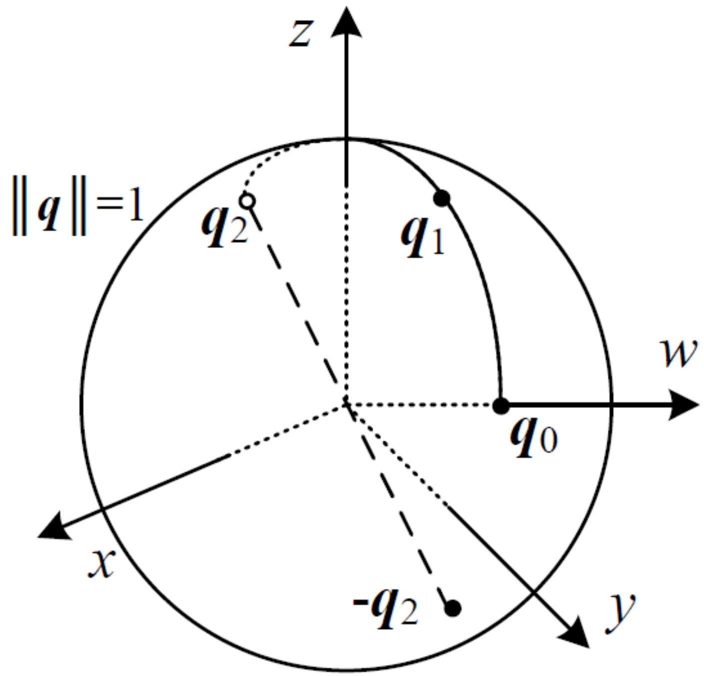
- Inverse

$$\theta = \cos^{-1}\left(\frac{\text{trace}(R) - 1}{2}\right), \hat{\mathbf{n}} = \frac{1}{2\sin\theta}\begin{pmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{pmatrix}$$

- see: An Invitation to 3D Vision, Y. Ma, S. Soatto, J. Kosecka, S. Sastry, Chapter 2 (available online)

Unit Quaternions

- Quaternion $\mathbf{q} = (q_x, q_y, q_z, q_w)^\top \in \mathbb{R}^4$
- Unit quaternions have $\|\mathbf{q}\| = 1$
- Opposite sign quaternions represent the same rotation
- Otherwise unique $\mathbf{q} = -\mathbf{q}$



Unit Quaternions

- Advantage: multiplication and inversion operations are efficient
- Quaternion-Quaternion Multiplication

$$
\begin{aligned}
\mathbf{q}_0\mathbf{q}_1 &= (\mathbf{v}_0, w_0)(\mathbf{v}_1, w_1) \\
&= (\mathbf{v}_0 \times \mathbf{v}_1 + w_0\mathbf{v}_1 + w_1\mathbf{v}_0, w_0w_1 - \mathbf{v}_0\mathbf{v}_1)
\end{aligned}
$$

- Inverse (flip sign of v or w)

$$
\begin{aligned}
\mathbf{q}^{-1} &= (\mathbf{v}, w)^{-1} \\
&= (\mathbf{v}, -w)
\end{aligned}
$$

Unit Quaternions

- Quaternion-Vector multiplication (rotate point p with rotation q)

$$\mathbf{p}' = \mathbf{q}\,\bar{\mathbf{p}}\mathbf{q}^{-1}$$

with

$$\bar{\mathbf{p}} = (x, y, z, 0)^\top$$

- Relation to Axis/Angle representation

$$\mathbf{q} = (\mathbf{v}, w) = (\sin\frac{\theta}{2}\hat{\mathbf{n}}, \cos\frac{\theta}{2})$$

3D Orientations

- **Note:** In general, it is very hard to "read" 3D orientations/rotations, no matter in what representation

- **Observation:** They are usually easy to visualize and can then be intuitively interpreted

- **Advice:** Use 3D visualization tools for debugging (RVIZ, libqglviewer, …)

C++ Libraries for Lin. Alg./Geometry

- Many C++ libraries exist for linear algebra and 3D geometry
- Typically conversion necessary
- Examples:
  - C arrays, std::vector (no linear alg. functions)
  - gsl (gnu scientific library, many functions, plain C)
  - boost::array (used by ROS messages)
  - Bullet library (3D geometry, used by ROS tf)
  - **Eigen (both linear algebra and geometry)**
  - **Sophus (Eigen extensions, SE(3)/se(3), see Lecture 2)**

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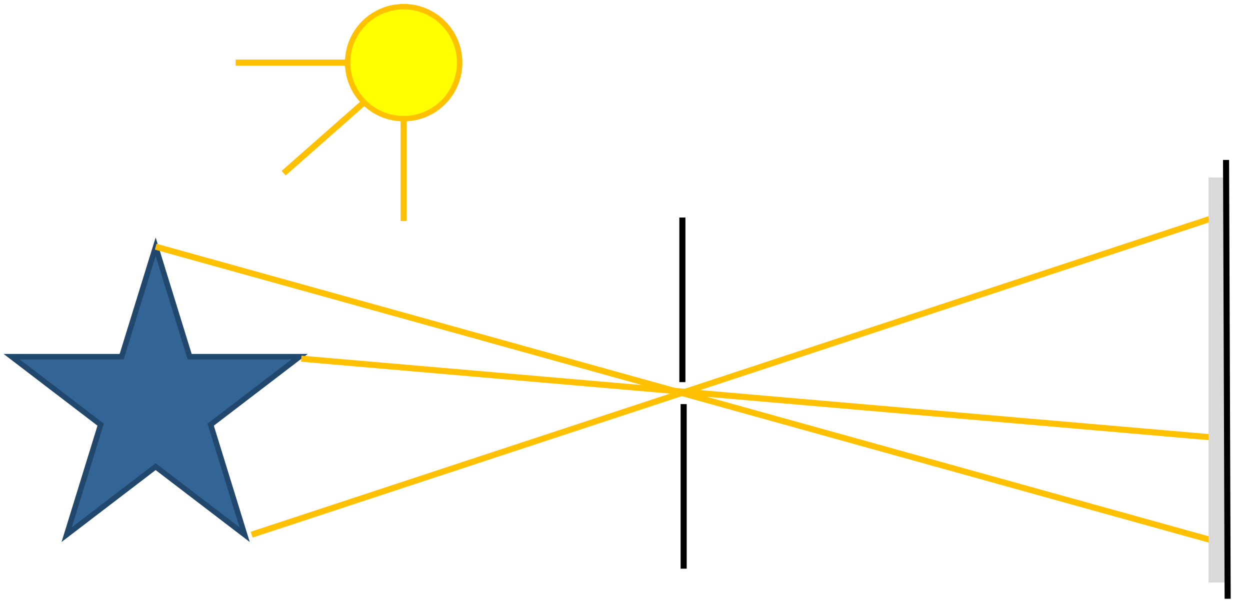
  - **Camera intrinsics**
  - **Epipolar geometry**

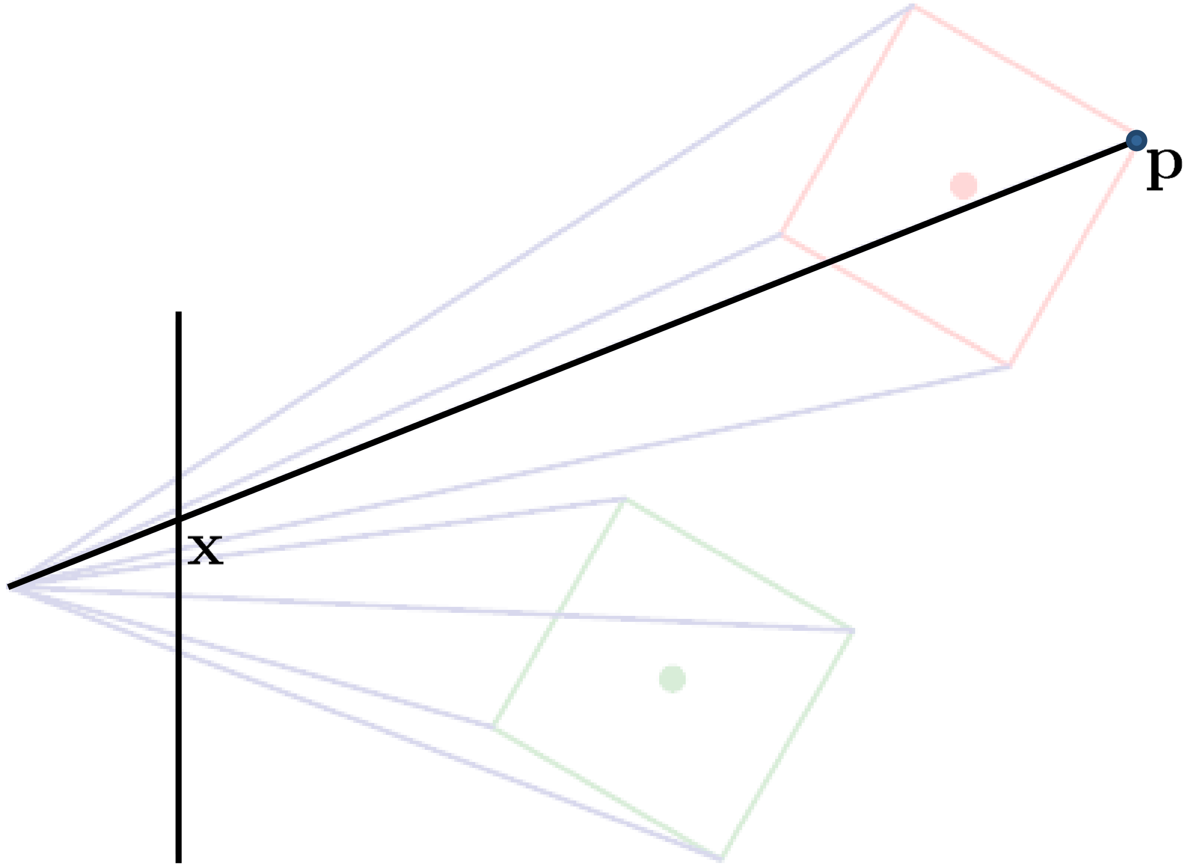
- Robot Operating System (ROS)

Pin-hole Camera

- Lit scene emits light
- Film/sensor is light sensitive



3D to 2D Perspective Projection



3D to 2D Perspective Projection

- 3D point $\mathbf{p}$ (in the camera frame)
- 2D point $\mathbf{x}$ (on the image plane)
- Pin-hole camera model

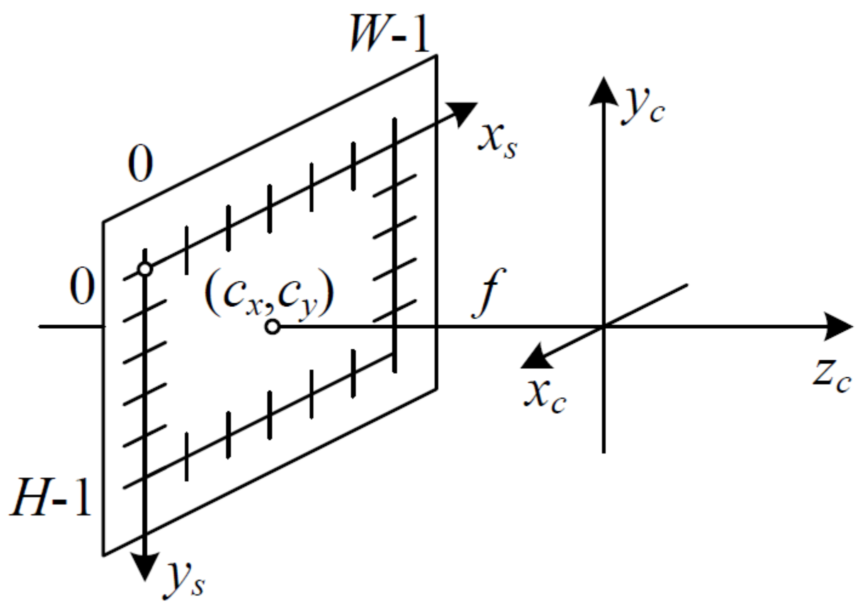
$$\tilde{\mathbf{x}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tilde{\mathbf{p}}$$

- Remember, $\tilde{\mathbf{x}}$ is homogeneous, need to normalize

$$\tilde{\mathbf{x}} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} \quad \Rightarrow \quad \mathbf{x} = \begin{pmatrix} \tilde{x}/\tilde{z} \\ \tilde{y}/\tilde{z} \end{pmatrix}$$

Camera Intrinsics

- So far, 2D point is given in meters on image plane
- But: we want 2D point be measured in pixels (as the sensor does)



Camera Intrinsics

- Need to apply some scaling/offset

$$\tilde{\mathbf{x}} = \underbrace{\begin{pmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}}_{\text{intrinsics } K} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{projection}} \tilde{\mathbf{p}}$$

- Focal length f_x, f_y
- Camera center c_x, c_y
- Skew s

Camera Extrinsics

- Assume $\tilde{\mathbf{p}}_w$ is given in world coordinates
- Transform from world to camera (also called the camera extrinsics)

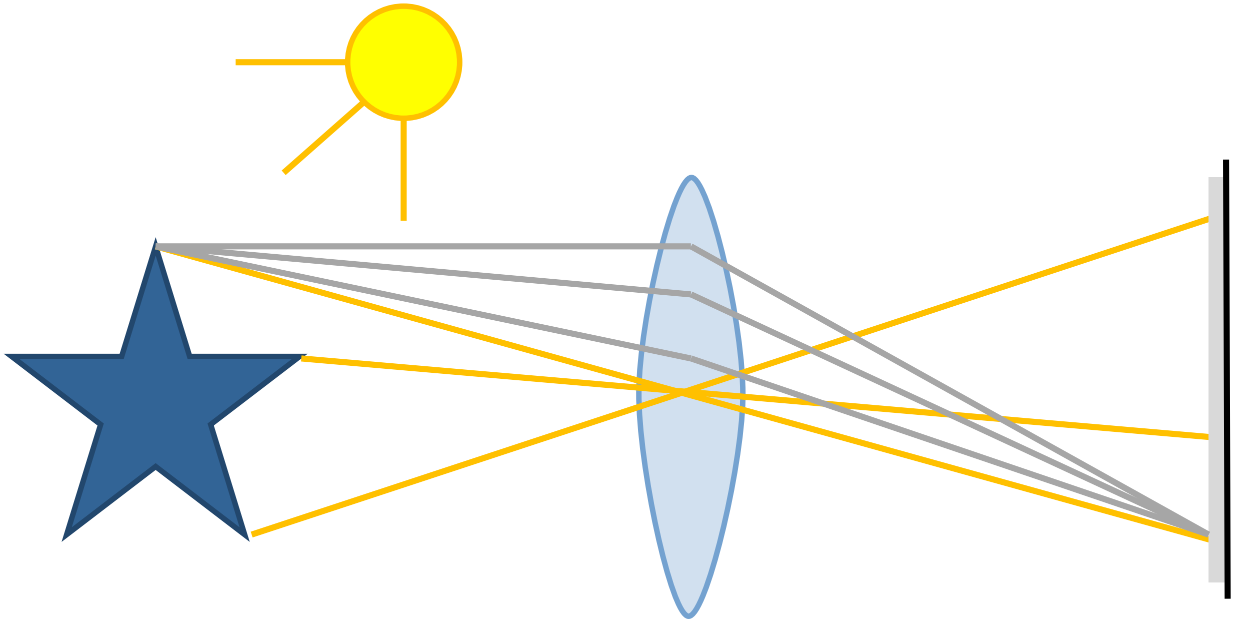
$$\tilde{\mathbf{p}} = \begin{pmatrix} R & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{pmatrix} \tilde{\mathbf{p}}_w$$

- Full camera matrix

$$\tilde{\mathbf{x}} = \begin{pmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} R & \mathbf{t} \end{pmatrix} \tilde{\mathbf{p}}_w$$

Real Camera Lenses

- Lit scene emits light
- Film/sensor is light sensitive
- A lens focuses rays onto the film/sensor



Real Cameras

- Radial distortion of the image
  - Caused by imperfect lenses
  - Deviations are most noticeable for rays that pass through the edge of the lens

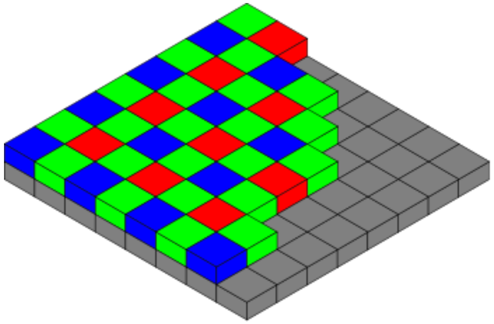
Radial Distortion

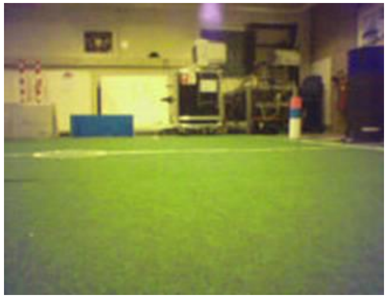
- Radial distortion of the image
  - Caused by imperfect lenses
  - Deviations are most noticeable for rays that pass through the edge of the lens
- Typically compensated with a low-order polynomial

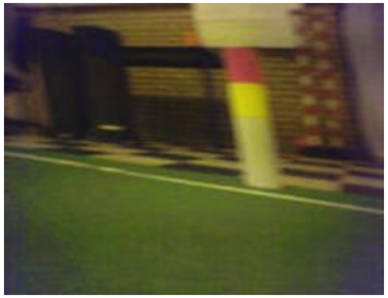
$$\hat{x}_c = x_c(1 + \kappa_1 r_c^2 + \kappa_2 r_c^4)$$
$$\hat{y}_c = y_c(1 + \kappa_1 r_c^2 + \kappa_2 r_c^4)$$

Digital Cameras

- Vignetting
- De-bayering
- Rolling shutter and motion blur
- Compression (JPG)
- Noise

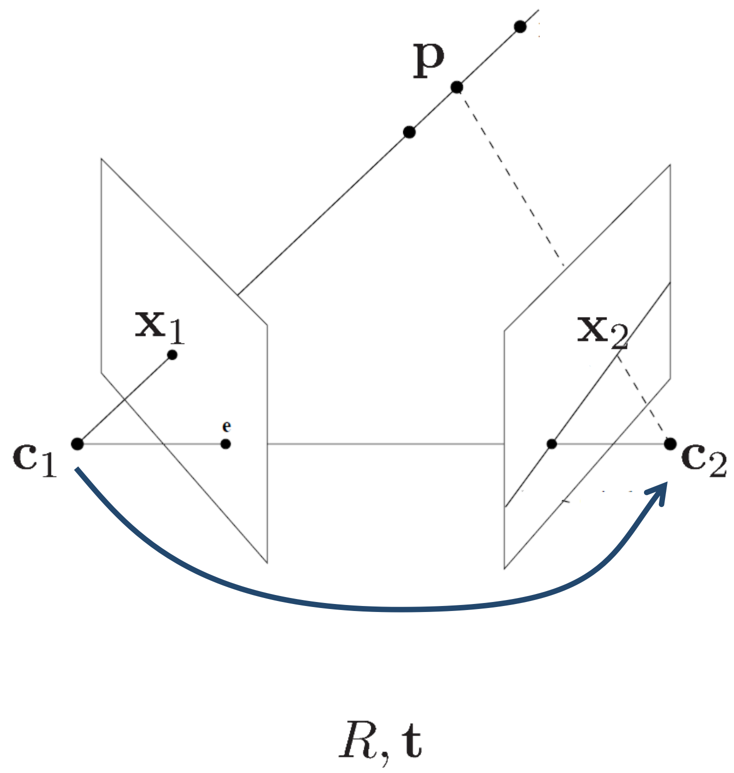






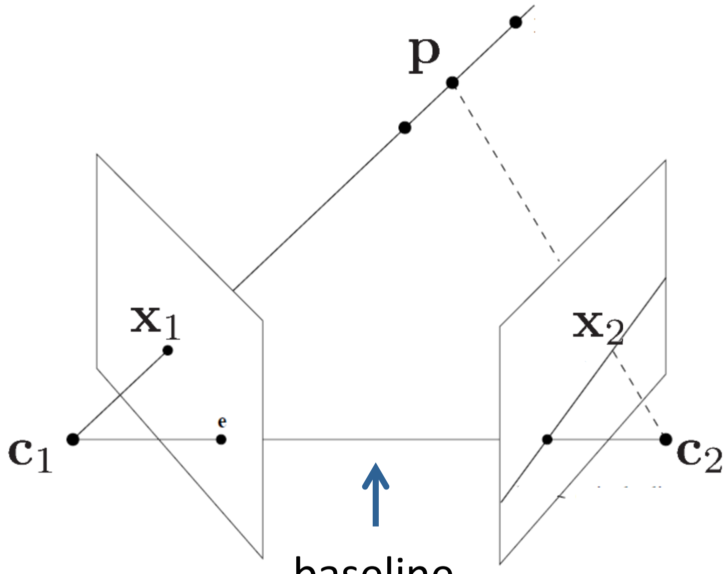
Epipolar Geometry

- Let’s consider two cameras that observe a 3D world point



Epipolar Geometry

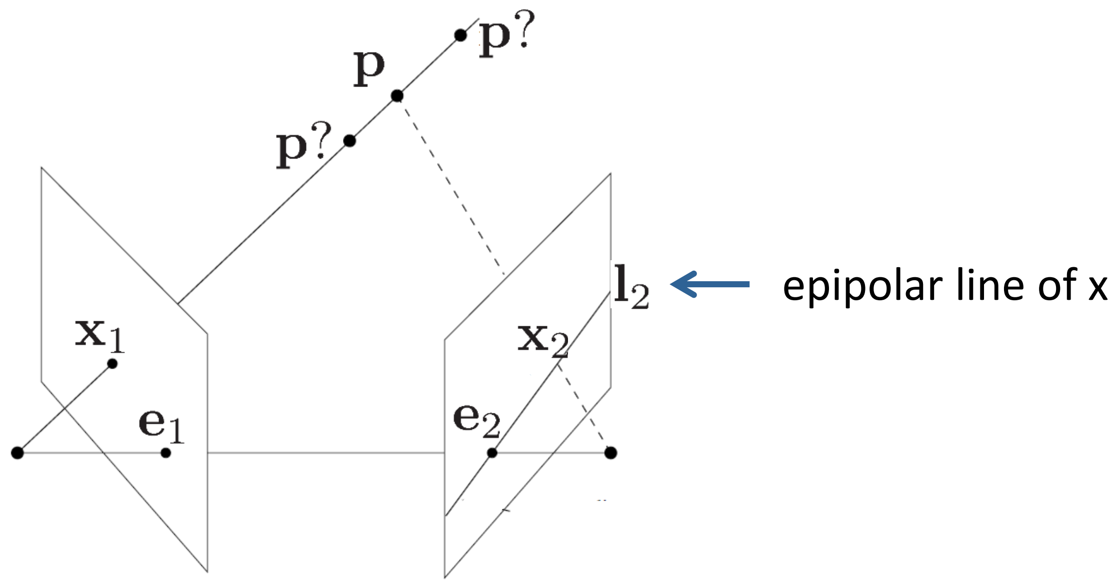
- The line connecting both camera centers is called the **baseline**



baseline
(line joining both camera centers)

Epipolar Geometry

- Given the image of a point in one view, what can we say about its position in another?



- A point in one image "generates" a line in another image (called the **epipolar line**)

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- **Robot Operating System (ROS)**

Modern Robot Architectures

- Robots became rather complex systems
- Often, a large set of individual capabilities is needed
- Flexible composition of different capabilities for different tasks

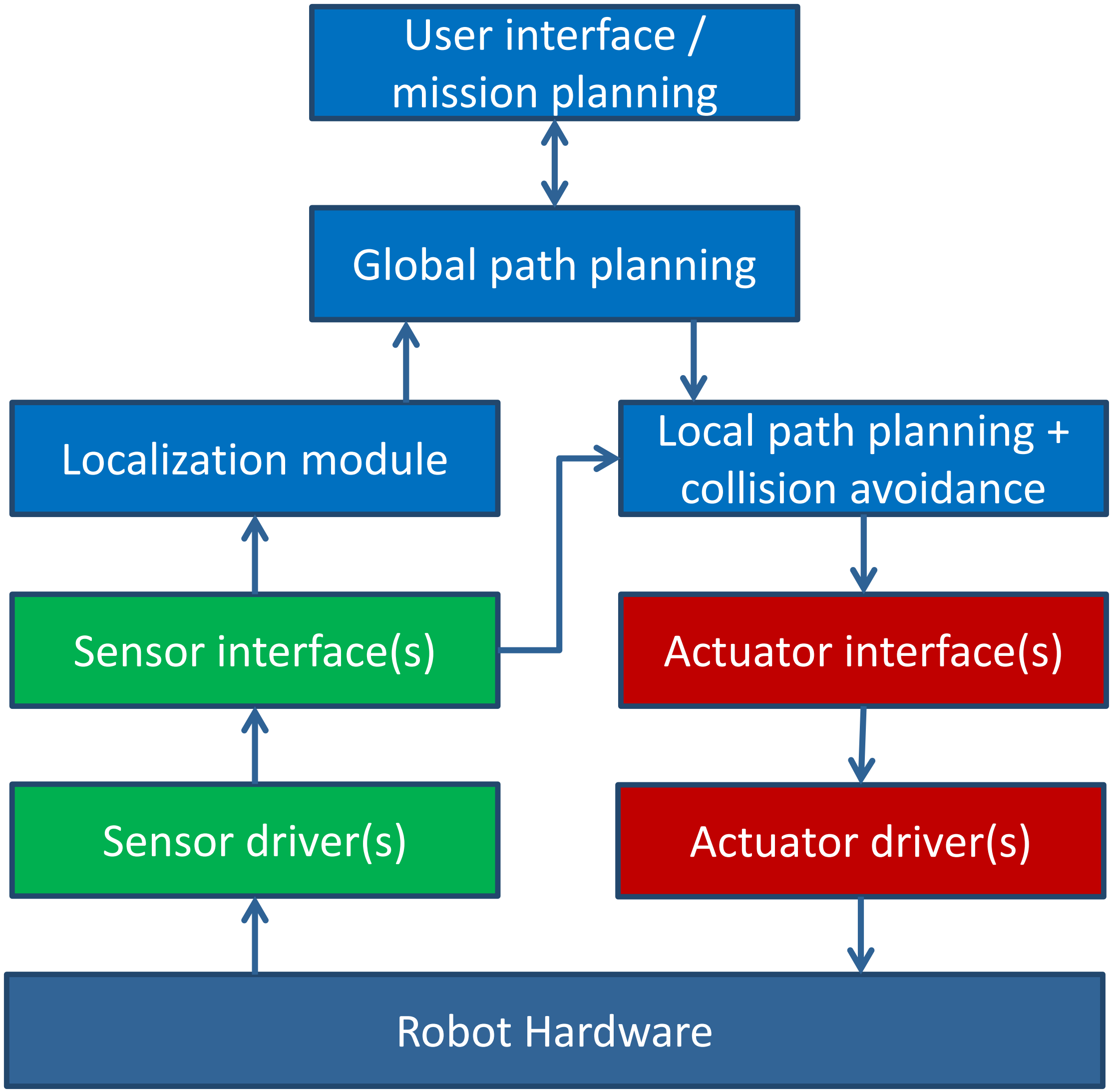
Best Practices for Robot Architectures

- Modular
- Robust
- De-centralized
- Facilitate software re-use
- Hardware and software abstraction
- Provide introspection
- Data logging and playback
- Easy to learn and to extend

Robotic Middleware

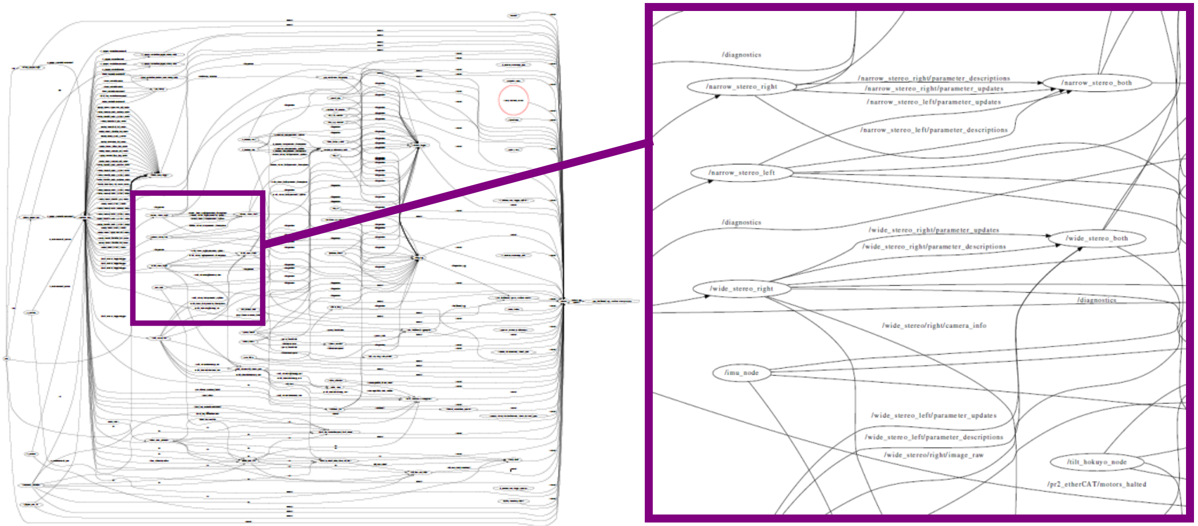
- Provides infrastructure
- Communication between modules
- Data logging facilities
- Tools for visualization
- Several open-source systems available
  - **ROS (Robot Operating System)**,
  - Player/Stage,
  - CARMEN,
  - YARP,
  - OROCOS

Example Architecture for Navigation



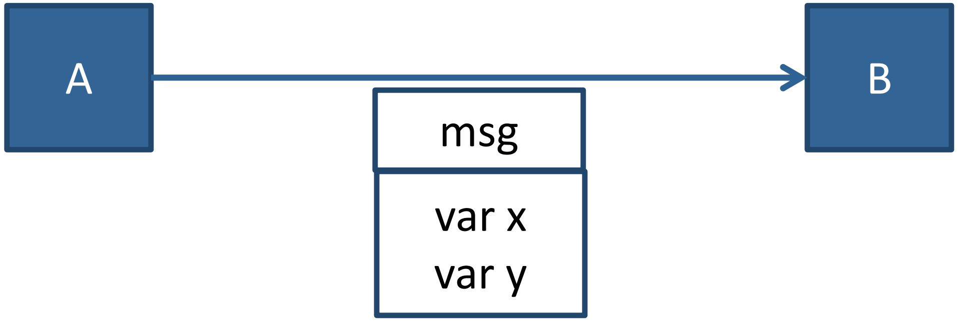
PR2 Software Architecture

- Two 7-DOF arms, grippers, torso, 2-DOF head
- 7 cameras, 2 laser scanners
- Two 8-core CPUs, 3 network switches
- 73 nodes, 328 message topics, 174 services

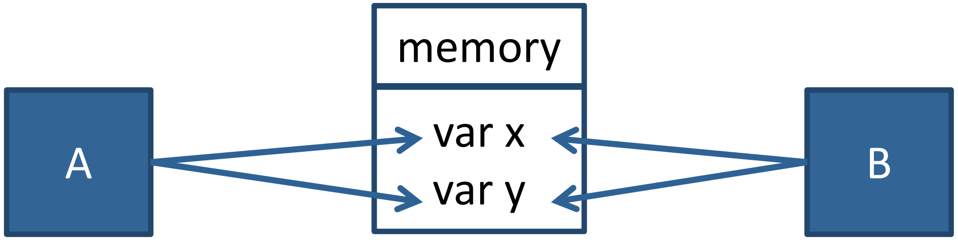


Communication Paradigms

- Message-based communication



- Direct (shared) memory access



Forms of Communication

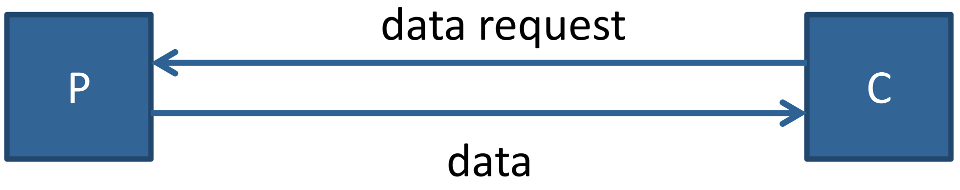
- Push
- Pull
- Publisher/subscriber
- Publish to blackboard
- Remote procedure calls / service calls
- Preemptive tasks / actions

Push

- Broadcast
- One-way communication
- Send as the information is generated by the producer P

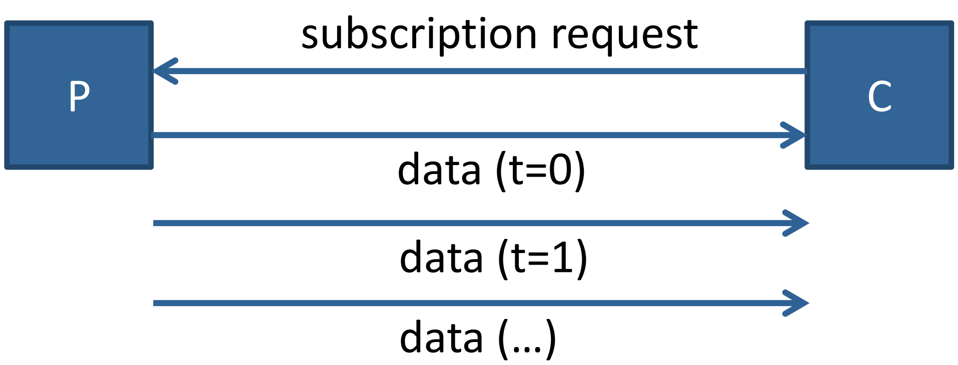
Pull

- Data is delivered upon request by the consumer C (e.g., a map of the building)
- Useful if the consumer C controls the process and the data is not required (or available) at high frequency



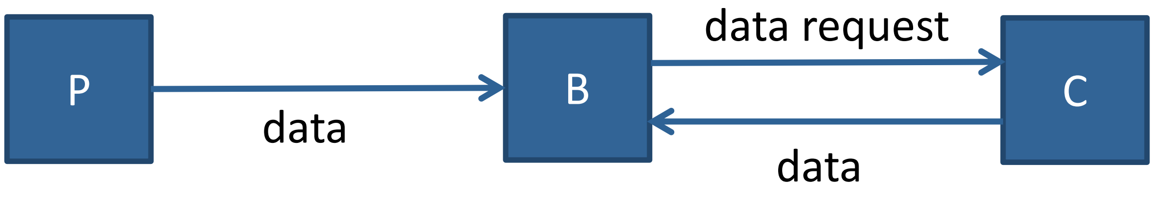
Publisher/Subscriber

- The consumer C requests a subscription for the data by the producer P (e.g., a camera or GPS)
- The producer P sends the subscribed data as it is generated to C
- Data generated according to a trigger (e.g., sensor data, computations, other messages, …)



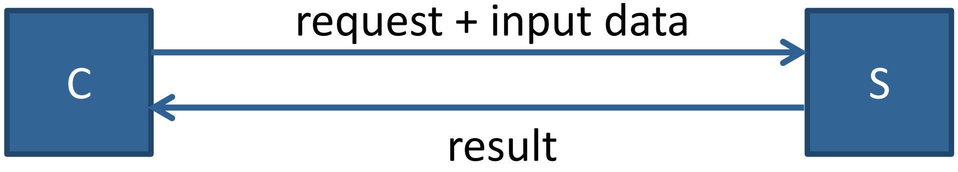
Publish to Blackboard

- The producer P sends data to the blackboard (e.g., parameter server)
- A consumer C pull data from the blackboard B
- Only the last instance of data is stored in the blackboard B



Service Calls

- The client C sends a request to the server S
- The server returns the result
- The client waits for the result (synchronous communication)
- Also called: Remote Procedure Call



Actions (Preemptive Tasks)

- The client requests the execution of an enduring action (e.g., navigate to a goal location)
- The server executes this action and sends continuously status updates
- Task execution may be canceled from both sides (e.g., timeout, new navigation goal,…)

Robot Operating System (ROS)

- We will use ROS in the lab course
- http://www.ros.org/
- Installation instructions, tutorials, docs

Concepts in ROS

- Nodes: programs that communicate with each other
- Messages: data structure (e.g., “Image”)
- Topics: typed message channels to which nodes can publish/subscribe (e.g., “/camera1/image_color”)
- Parameters: stored in a blackboard

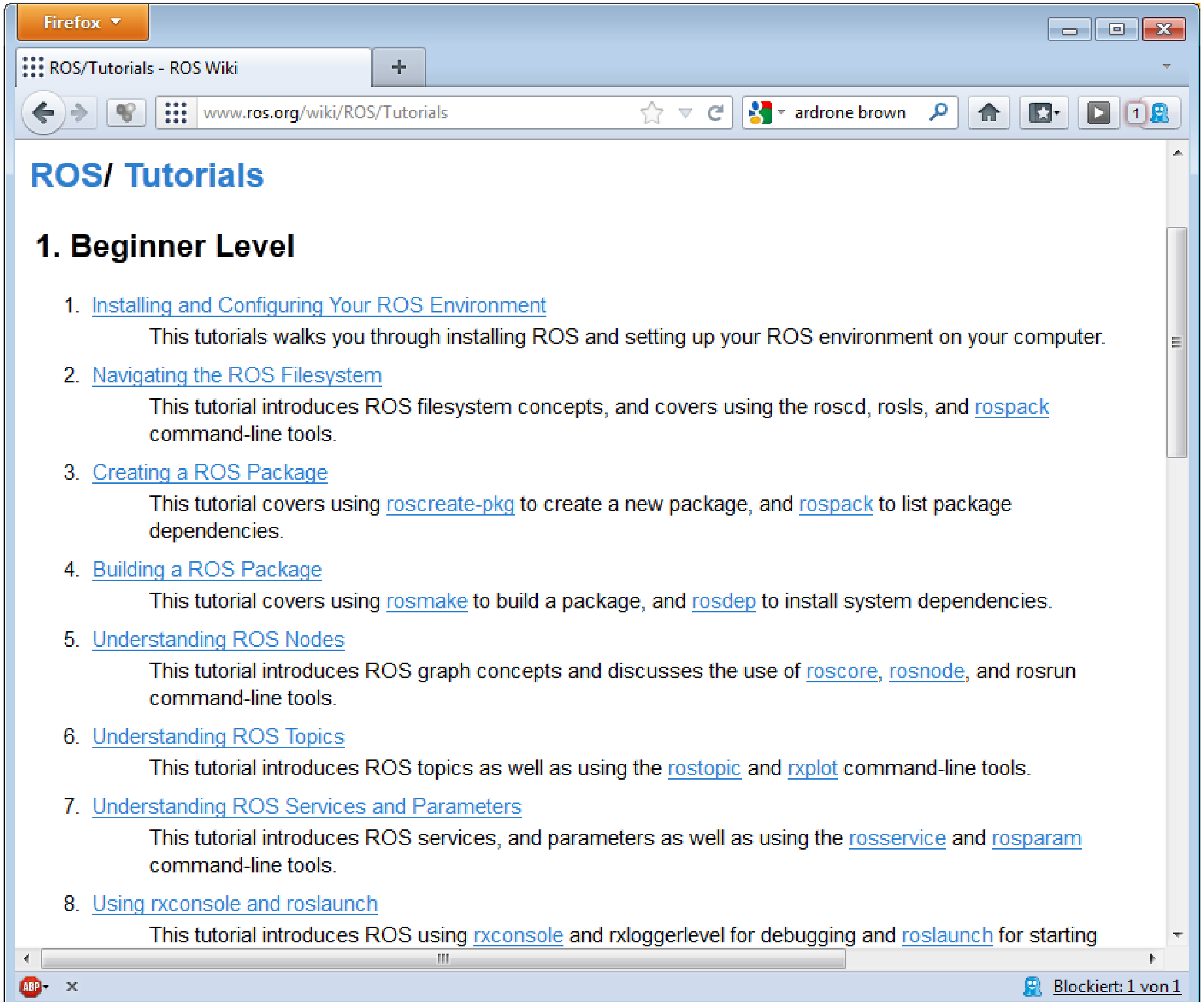
Software Management

- Package: atomic unit of building, contains one or more nodes and/or message definitions
- Build system “catkin” on top of CMake to ease building packages and ROS specific code, and to resolve dependencies between packages
- Repository: contains several packages, typically one repository per institution

Useful Tools

- catkin_init_workspace
- catkin_create_pkg
- catkin_make
- roscore
- rosnode list/info
- rostopic list/echo
- rosbag record/play
- rosrun

Tutorials in ROS



Questions ?