



Chapter 12

Photometric 3D-reconstruction

Computer Vision I: Variational Methods

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3D-scanning

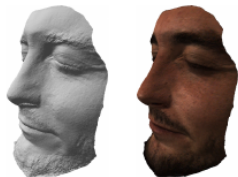
3D-scanning = estimation of shape *and* color



(a)

(b)

(c)



(d)

(e)

[Quéau et al., 2017b, (JMIV)]

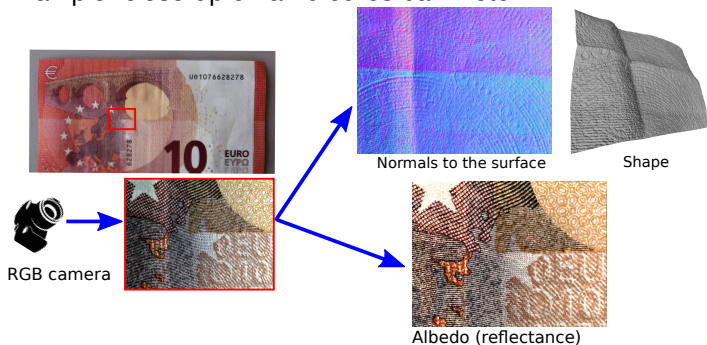
- Geometric techniques (e.g. SfM, MVS) based on multi-view consistency only recover shape (no color estimation)
- Photometric techniques (e.g., shape-from-shading and photometric stereo), which are based on **inverting the image formation model**, recover both.



Photometric techniques

Photometric techniques constitute the first choice for the recovery of very thin structures.

Example: close-up on a 10 euros banknote



[Quéau, 2015, (PhD thesis)]

Photometric 3D-reconstruction =

Shape analysis through *luminous quantities measurement* (photo-metry), by reverting the image formation process.



Photometric techniques

Photometric techniques constitute the first choice for the recovery of very thin structures.

Example: close-up on a 5 euros banknote



[Quéau et al., 2017g, (QCAV)]



Photometric techniques

Photometric techniques constitute the first choice for the recovery of very thin structures.

Example: close-up on a 50 cents euros coin



[Quéau et al., 2016a, (CVPR)]



Photometric techniques

Photometric techniques constitute the first choice for the recovery of very thin structures.

Example: close-up on human skin





1 Basics of photometric 3D-reconstruction

2 Shape-from-shading

3 Photometric stereo

Basics of photometric
3D-reconstruction

Shape-from-shading

Photometric stereo



1 Basics of photometric 3D-reconstruction

2 Shape-from-shading

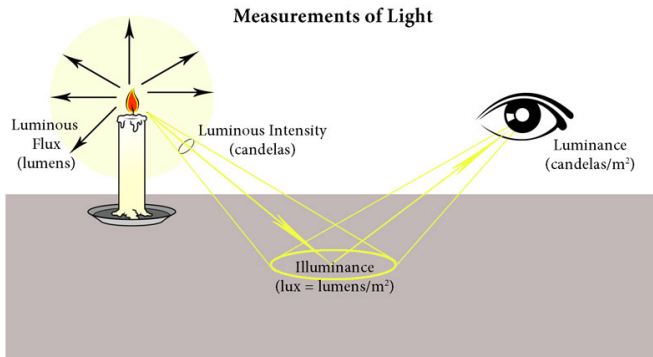
3 Photometric stereo

Basics of photometric
3D-reconstruction

Shape-from-shading

Photometric stereo

A few photometric definitions

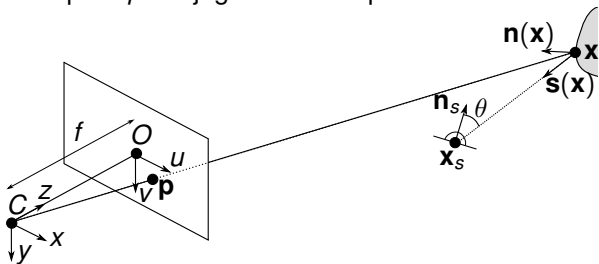


(source: <http://rsagencies.co.za>)

- luminance (radiance) = perceived luminosity of the surface - $\text{cd.m}^{-2} = \text{lm.m}^{-2}.\text{sr}^{-1}$ ($\text{W.m}^{-2}.\text{sr}^{-1}$)
- illuminance (irradiance): luminous flux received by a surface per unit area - $\text{lx} = \text{lm.m}^{-2}$ (W.m^{-2})
- reflectance: proportion of incident light which is reflected by a surface (unit-less)

Digital cameras as luminance measurement devices

Consider a pixel p conjugate to a 3D-point x on the surface.

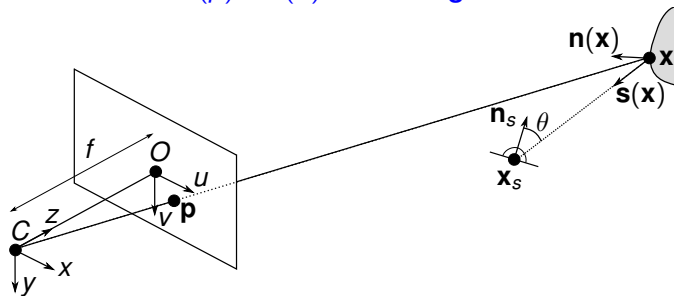


- Digital cameras essentially count the number of photons crossing the photosensitive cell i.e., the pixel p
 - $I(p) \approx \frac{t}{g} E_s(p)$ with t the exposure time, g the gain and $E_s(p)$ the sensor illuminance at pixel p (up to offset, thermal noise and saturation)
 - $E_s(p) \approx L(x)$ with $L(x)$ the luminance of the point x on the surface (up to geometric attenuation, see [Horn, 1986])
- The brightness in p is proportional to the luminance in x :

$$I(p) \approx L(x)$$



How is formula $I(p) \approx L(x)$ interesting?



Further consider that incident light in x is coming from all directions ω on the visible hemisphere $\mathcal{H}(x)$. Then:

$$I(p) \approx L(x) = \int_{\omega \in \mathcal{H}(x)} \rho(x, \dots) \phi_{\omega} \max\{0, \mathbf{s}_{\omega} \cdot \mathbf{n}(x)\} d\omega$$

with ϕ_{ω} the light intensity (in cd) in direction ω , \mathbf{s}_{ω} the unit-length lighting direction, $\rho(x)$ the reflectance in point x (**i.e., the surface material**) and $\mathbf{n}(x)$ the normal to the surface (**i.e., the surface shape**)

→ Inverting this image formation model allows the estimation of the surface shape and its material (color, roughness, etc.)



In general, we need to simplify

$$I(\rho) \approx L(x) = \int_{\omega \in \mathcal{H}(x)} \rho(x, \dots) \phi_{\omega} \max\{0, \mathbf{s}_{\omega} \cdot \mathbf{n}(x)\} d\omega$$

because of:

- the integral over all lighting directions (partial solution: spherical harmonics, cf. talks of Bjoern and Robert)
- the complexity of the reflectance function ρ , which generally depends on the angle between incident light and viewing direction (ρ is in general a BRDF, for bidirectional reflectance function, see for instance lectures on computer graphics)

(remark: considering multi-channel images adds even more complexity - be careful if you read that such extensions are straightforward ;-)





A simple, yet effective image formation model

- Assuming a **single light source**, the integrand is a Dirac:

$$\int_{\omega \in \mathcal{H}(x)} \rho(x) \phi_{\omega} \max\{0, \mathbf{s}_{\omega} \cdot \mathbf{n}(x)\} d\omega = \rho(x, \dots) \phi(x) \max\{0, \mathbf{s}(x) \cdot \mathbf{n}(x)\}$$

with $\mathbf{s}(x)$ the unit-length direction of the the luminous flux reaching x and $\phi(x)$ its density (in $\text{cd} \cdot \text{m}^{-2}$).

- Assume a **Lambertian surface: luminance is independent from the viewing angle**. Corollary:

$$\rho(x, \dots) := \frac{\rho(x)}{\pi}$$

with $\rho(x) \geq 0$ a scalar function called the **albedo** (0 = black, 1 = white).

And thus:

$$I(p) \approx \rho(x) \phi(x) \max\{0, \mathbf{s}(x) \cdot \mathbf{n}(x)\}$$

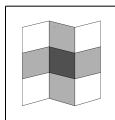
(often **abusively** referred to as Lambert's law)

Inverting the simplified image formation model

Considering the relationship:

$$I(p) \approx \rho(x)\phi(x) \max\{0, \mathbf{s}(x) \cdot \mathbf{n}(x)\}$$

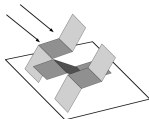
one may want to estimate shape (\mathbf{n}), reflectance (ρ) and lighting (ϕ and \mathbf{s}). **In general, this is impossible.**



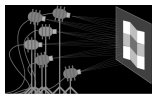
An image



Painter's explanation



Sculptor's explanation



Lighting designer's explanation

Impossible to tell **reflectance** from **shape** and **lighting**
(images from [Adelson and Pentland, 1996, (Perception as Bayesian inference)])





Basics of photometric
3D-reconstruction

Shape-from-shading

Photometric stereo

1 Basics of photometric 3D-reconstruction

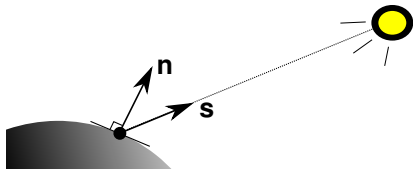
2 Shape-from-shading

3 Photometric stereo

Shape-from-shading [Horn, 1970] is the task of estimating shape, assuming known lighting and reflectance. Classically one further simplifies the previous image formation model

$$\underbrace{I(p)}_{\text{Luminance}} = \underbrace{\rho(x)}_{\text{Albedo}} \underbrace{\phi(x)}_{\text{Intensity}} \underbrace{\max\{0, \mathbf{s}(x) \cdot \mathbf{n}(x)\}}_{\text{shading}}$$

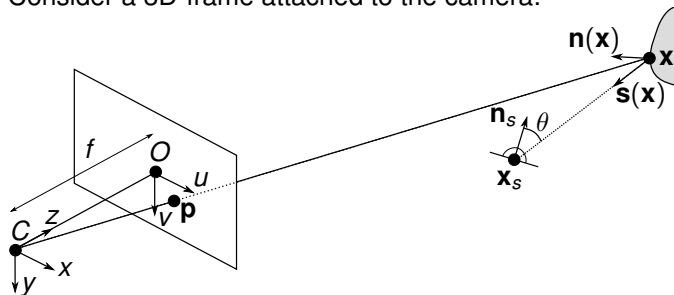
where the *shading* (i.e., the surface illuminance) is the the clamped (for self-shadows) dot product between the unit-length normal and lighting directions:



- $\mathbf{s} \in \mathbb{S}^2 \subset \mathbb{R}^3$: lighting direction
- $\mathbf{n} \in \mathbb{S}^2 \subset \mathbb{R}^3$: normal to the surface (shape)

Classical assumptions of shape-from-shading

Consider a 3D-frame attached to the camera:



and further assume:

- White surface: $\rho(x) := 1$
- Distant point light source: $\phi(x) := \phi > 0$,
 $\mathbf{s}(x) := \mathbf{s} \in \mathbb{S}^2 \subset \mathbb{R}^3$
- Calibrated source: $\phi := 1$
- Frontal lighting: $\mathbf{s} := [0, 0, -1]^\top$
- Orthographic camera: $\mathbf{n}(x) := \frac{1}{\sqrt{|\nabla z(\rho)|^2 + 1}} \begin{bmatrix} \nabla z(\rho) \\ -1 \end{bmatrix}$,
(z: **depth map**)



Plugging $\rho(x) := 1$, $\mathbf{s}(x) := [0, 0, -1]^\top$, $\phi(x) := 1$, and $\mathbf{n}(x) := \frac{1}{\sqrt{|\nabla z(\rho)|^2 + 1}} \begin{bmatrix} \nabla z(\rho) \\ -1 \end{bmatrix}$ in our model

$$\underbrace{I(\rho)}_{\text{Luminance}} = \underbrace{\rho(x)}_{\text{Albedo}} \underbrace{\phi(x)}_{\text{Intensity}} \underbrace{\max\{0, \mathbf{s}(x) \cdot \mathbf{n}(x)\}}_{\text{shading}},$$

we obtain a nonlinear PDE in z over the image domain Ω :

$$I(\rho) = \frac{1}{\sqrt{|\nabla z(\rho)|^2 + 1}}, \quad \rho \in \Omega \subset \mathbb{R}^2.$$

This is directly related to the celebrated **eikonal equation** $|\nabla z| = f$. The unique maximum viscosity solution of SfS can thus be determined by solving the eikonal equation [Lions et al., 1993].



On uniqueness in shape-from-shading

Despite the extremely restrictive assumptions listed above, SfS remains ambiguous. For instance, the two following results are equally valid explanations of the Lena image under such assumptions:

Maximum viscosity solution
[Cristiani and Falcone, 2007]

Variational solution
[Quéau et al., 2017f,
(EMMCVPR)]





Some solutions to disambiguate shape-from-shading

- 0 consider an RGB image with three colored light sources [Kontsevich et al., 1994]
- 0 relax the distant light assumption [Prados and Faugeras, 2005]
- 1 Add priors within a **variational framework** [Horn and Brooks, 1986]
- 2 Consider multiple images under varying lighting: **photometric stereo** [Woodham, 1980]

The variational approach to shape-from-shading

[Horn and Brooks, 1986] decouple local and global shape estimations, introducing an integrability prior

- 1) Estimate an **integrable** (curl-free) gradient field $(p, q) := \nabla z$:

$$\min_{p,q} \iint \left(I - \frac{1}{\sqrt{p^2 + q^2 + 1}} \right)^2 + \lambda (\partial_y p - \partial_x q)^2 dx dy$$

(nonlinear global optimization problem: hard to solve)

- 2) **Integrate** this field into a depth map:

$$\min_z \iint \|(p, q) - \nabla z\|^2 dx dy$$

(not as straightforward as it seems to be [Quéau et al., 2017c, Quéau et al., 2017d, (JMIV)])



Some recent developments [Quéau et al., 2017f, (EMMCVPR)]

(p, q) is conservative *by construction* \rightarrow joint local and global (integrated) estimation:

$$\min_{p,q,z} \iint \left(I - \frac{1}{\sqrt{p^2 + q^2 + 1}} \right)^2 dx dy$$

s.t. $(p, q) = \nabla z$

One can add **smoothness** and **shape** priors, if needed:

$$\min_{p,q,z} \iint \lambda \left(I - \frac{1}{\sqrt{p^2 + q^2 + 1}} \right)^2 + \nu \sqrt{1 + p^2 + q^2} dx dy$$
$$+ \iint \mu (Kz - z^0)^2 dx dy$$

s.t. $(p, q) = \nabla z$

Playing with parameters and integration domains, one may achieve shape-from-shading, depth denoising and inpainting, depth refinement and super-resolution, etc. Numerical solution using ADMM.



Example of result (without regularisation)

Other straightforward extensions include RGB data, spherical harmonics lighting, perspective camera, etc. Example of SfS results from [Quéau et al., 2017f] applied to a dataset from [Han et al., 2013]:



Image



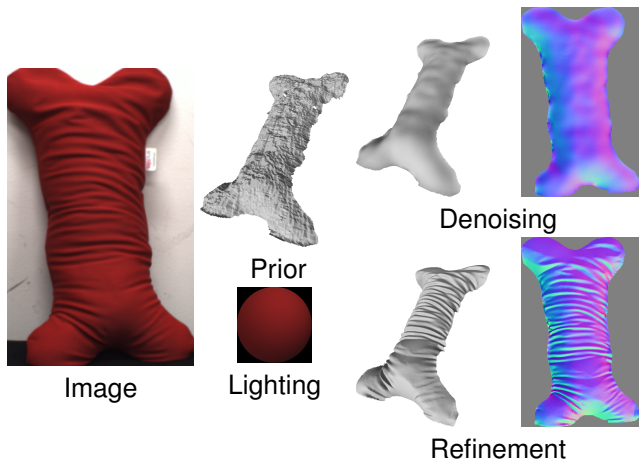
Lighting



3D-reconstruction



Example of result (with regularisation)



Matlab code available:

https://github.com/yqueau/shape_from_shading



Depth super-resolution for RGB-D sensors

By including lighting (spherical harmonics) and albedo (Mumford-Shah prior) estimations, the previous model can achieve state-of-the-art single-shot super-resolution of depth maps for RGBD sensors [Häfner et al., 2018].



Data: LR depth
+ HR image

HR depth
+ HR reflectance

(cf. seminar of Bjoern Häfner)





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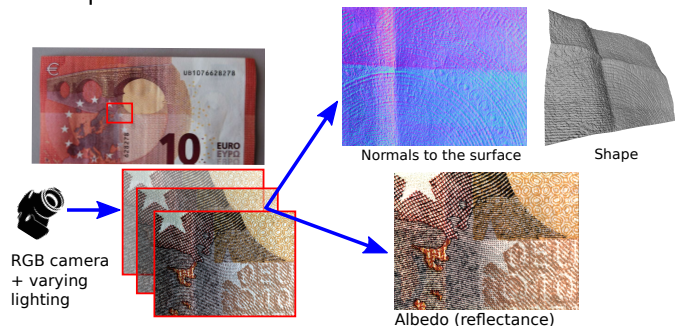


Photometric stereo

When no prior knowledge is available, it is hopeless to achieve a reasonable 3D-reconstruction based on SfS, due to its ill-posedness.

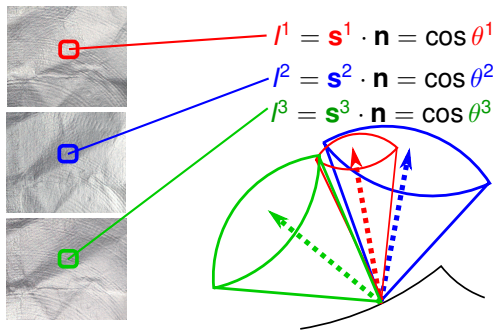
Photometric stereo [Woodham, 1980] is an extension of SfS which considers multiple images of the surface, taken from the **same viewing angle** but under **varying lighting**.

Besides its ability to unambiguously recover thin geometric structures, photometric stereo is the only computer vision technique which is able to **estimate reflectance**.



[Quéau, 2015, PhD thesis]

Why photometric stereo disambiguates SfS



- $m = 1$ image (SfS), known albedo: infinitely many possible normals in each point
- $m = 2$ images, known albedo: two possible normals [Quéau et al., 2017e, (IVC)]
- $m \geq 3$: unique approximate solution - problem is over-constrained and thus albedo can be estimated



Photometric stereo as a linear system of equations

Recall our initial model:

$$\underbrace{I(p)}_{\text{Luminance}} = \underbrace{\rho(x)}_{\text{Albedo}} \underbrace{\phi(x)}_{\text{Intensity}} \underbrace{\max\{0, \mathbf{s}(x) \cdot \mathbf{n}(x)\}}_{\text{shading}}$$

Consider $m \geq 3$ images I^i under known, varying lighting $(\phi^i(x), \mathbf{s}^i(x))$. Then one needs to solve, in each pixel p , the system of equations in $(\rho(x) > 0, \mathbf{n}(x) \in \mathbb{S}^2)$:

$$I^i(p) = \rho(x) \phi^i(x) \max\{0, \mathbf{s}^i(x) \cdot \mathbf{n}(x)\}, \quad i \in \{1, \dots, m\}$$

Assuming distant light sources ($s^i(x) = \phi^i(x) \mathbf{s}^i(x) := \mathbf{s}^i \in \mathbb{R}^3$), neglecting self-shadows ($\max\{a, 0\} := a$), and plugging together the unknowns into

$$m(x) := \rho(x) \mathbf{n}(x),$$

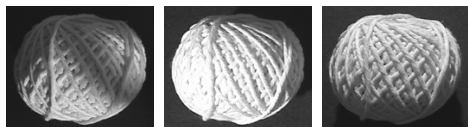
then we obtain a linear system of equations in $m(x) \in \mathbb{R}^3$:

$$I^i(p) = \mathbf{s}^i \cdot m(x), \quad i \in \{1, \dots, m\}$$

which we can solve and then deduce $\rho(x) := |m(x)| > 0$ and $\mathbf{n}(x) := \frac{m(x)}{|m(x)|}$.



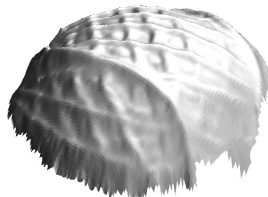
Two-stages resolution of photometric stereo



→
(1)



↓ (2)



(1) Estimation of surface normals
(and albedo)

(2) Integration of normals
[Quéau et al., 2017c, (JMIV)]

This approach can even be extended to unknown lighting, for instance by using TV regularisation [Quéau et al., 2015].



Basic photometric stereo strategy

Data

- lighting vectors
 $s^1, s^2, s^3, \dots \in \mathbb{R}^3$
- brightness
 $I^1, I^2, I^3, \dots > 0$



I^1



I^2



I^3



Albedo ρ



Normals \mathbf{n}



Depth z

1

$$\min_{m \in \mathbb{R}^3} \sum_{i=1}^m \Phi (|s^i \cdot m - I^i|)$$

with Φ some robust estimator

2

$$\rho = \|m\|$$

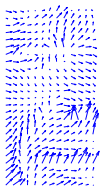
3

$$\mathbf{n} = \frac{m}{\|m\|}$$

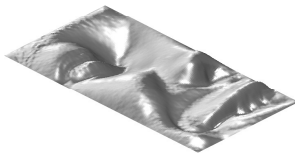
4

$\mathbf{n} \rightarrow z$ (integration)

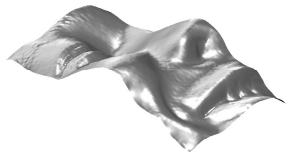
Integrating the normals (1)



Normals



Integration (Dirichlet)



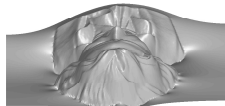
Integration (periodic)



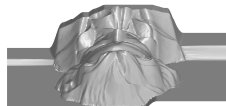
Integration (natural Neumann)



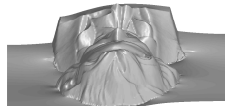
$$\min_z \iint_{x \in \Omega} \Phi \left(\left\| \nabla z - [-m_1/m_3, -m_2/m_3]^T \right\| \right) dx$$



$\Phi = \text{least-squares}$



$\Phi = \text{norm 1}$



$\Phi = \text{non-convex}$

- Least squares: easy / over-smoothing
- Norm 1: convex / staircasing
- Non-convex: harder / better (not faster, but stronger ;)

[Quéau et al., 2017d, (JMIV)]

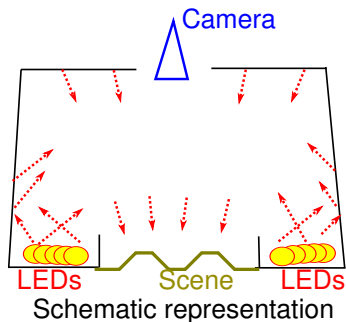


Experimental setup

(joint work with the Pixience company and Toulouse Tech Transfer)



Dermoscope with LEDs



[Quéau et al., 2017g, QCAV]



3D-reconstruction of metallic coins



1 euro (Italy)



50 cents (Spain)



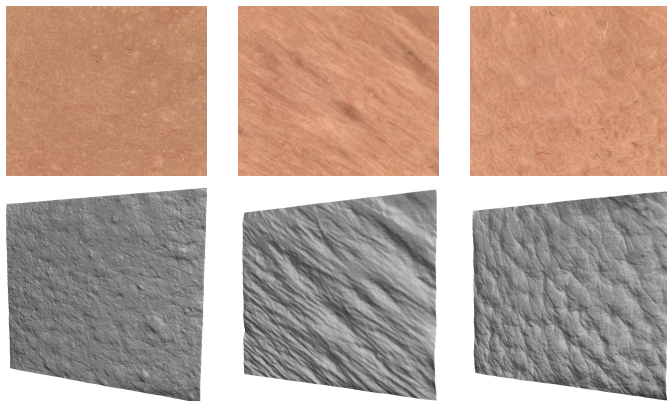
1 yuan (China)



3D-reconstructions [Quéau et al., 2016a, (CVPR)]



3D-reconstructions of synthetic skin samples



3D-reconstructions [Quéau et al., 2016a, (CVPR)]

3D-reconstructions of the human skin

Image 3D de ride

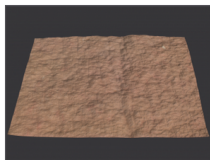


Image 3D de cicatrice

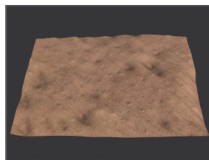
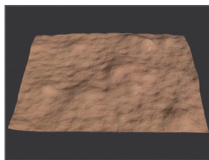
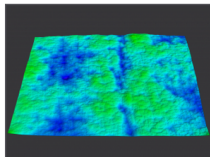


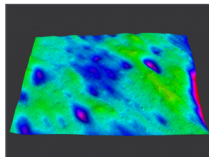
Image 3D d'acné



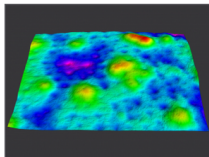
Carte d'élévation de ride



Carte d'élévation de cicatrice



Carte d'élévation d'acné



Source : <http://www.pixience.com/produits-2/c-cube-recherche-clinique/module-3d/>



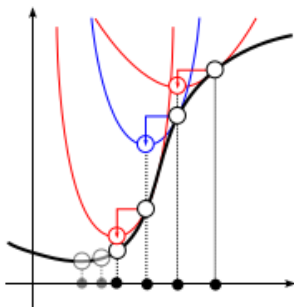
Integrated variational approach

Direct estimation of depth $z : \Omega \rightarrow \mathbb{R}$ and albedo $\rho : \Omega \rightarrow \mathbb{R}$
solutions of $l^i = \rho \mathbf{s}^i \cdot \frac{[\nabla z^\top, -1]^\top}{\sqrt{|\nabla z|^2 + 1}}$ [Quéau et al., 2017i, (CVPR)]

Linearisation $\rho := \frac{\rho}{\sqrt{|\nabla z|^2 + 1}}$, then

$$\min_{z, \rho} \sum_{i=1}^m \iint_{x \in \Omega} \Phi \left(\left| \rho(x) \mathbf{s}^i \cdot [\nabla z(x)^\top, -1]^\top - l^i(x) \right| \right) dx$$

Non-convex problem:
optimisation by alternating reweighted least-squares (alternating MM) [Quéau et al., 2017b, Quéau et al., 2017i, Quéau et al., 2017h, Mélou et al., 2017]



Straightforward extensions of the integrated variational approach



$$\min_{z, \rho} \sum_{i=1}^m \iint_{x \in \Omega} \Phi \left(\left| \rho(x) \mathbf{s}^i \cdot [\nabla z(x)^\top, -1]^\top - I^i(x) \right| \right) dx$$

- Perspective projection [Quéau et al., 2017i, (CVPR)]
- Uncalibrated PS [Quéau et al., 2017i, (CVPR)]: just include lighting vectors \mathbf{s}^i in the unknowns – there is still a unique solution under perspective projection
- Color images and reflectance [Quéau et al., 2017b, (JMIV)]
- Nearby, anisotropic light sources [Quéau et al., 2017b, Quéau et al., 2017h, (SSVM + JMIV)] (\mathbf{s}^i becomes a nonlinear function of z)
- Depth prior from RGBD sensor [Peng et al., 2017, Quéau et al., 2017a, ICCV + TS]

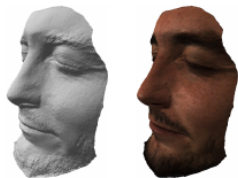
Example: LED-based photometric stereo [Quéau et al., 2017b, (JMIV)]



(a)

(b)

(c)



(d)

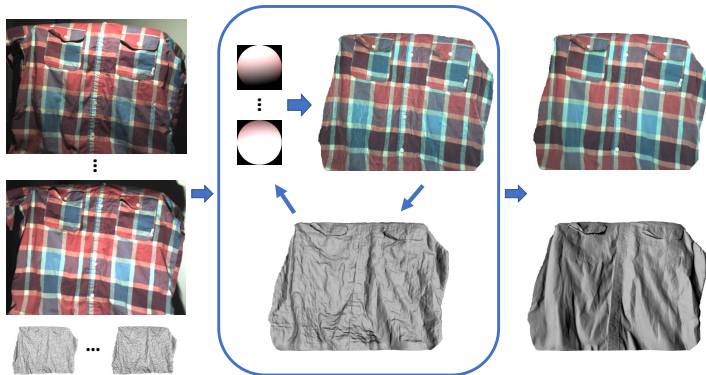
(e)

Codes: https://github.com/yqueau/near_ps



Example: uncalibrated photometric stereo-based depth super-resolution [Peng et al., 2017, (ICCV)]

(cf. seminar of Bjoern Haefner)



Codes: <https://github.com/pengsongyou/SRmeetsPS>



Just for finishing this series of lectures on a cool (who said useless ?) application

We can use PS to scan people while they are watching their holidays pictures [Quéau et al., 2016b]



Bibliography I



Adelson, E. H. and Pentland, A. P. (1996).
The perception of shading and reflectance.
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