Variational Methods for Computer Vision: Exercise Sheet 7

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Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $Q := [0,1] \times [0,1]$ be a rectangular area and let $V = (u(x,y), v(x,y))$ be a differentiable vector field defined on Q.

   (a) Prove Green's theorem for Q using the fundamental theorem of calculus, hence show that

   $$\iint_Q \operatorname{curl} V \mathrm{dxdy} = \iint_Q \big(v_x(x,y) - u_y(x,y)\big) \mathrm{dxdy} = \oint_{\partial Q} V(s) \mathrm{d}\vec{s}.$$

   Assume the boundary curve ∂Q to be oriented counterclockwise.

   (b) Let $\Omega \subset \mathbb{R}^2$ be an area that can be represented as a disjoint union of a finite number of squares $Q_1, ..., Q_n$. Prove that Green's theorem also holds for Ω.

2. In the lecture the piecewise constant Mumford-Shah functional is written as follows:

   $$E(u_i, C) = \sum_{i=1}^{n} \int_{\Omega_i} (I(x) - u_i)^2 \mathrm{dx} + \nu |C|.$$

   Prove that by merging two regions Ω_1 and Ω_2 the energy E changes by

   $$\delta E = \frac{A_1 A_2}{A_1 + A_2} (u_1 - u_2)^2 - \nu \delta C,$$

   where A_i denotes the area of the regions in pixels, u_i the respective mean values and δC the length of the interface of both regions.

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Finish the practical exercises from the last exercise sheet.