Chapter 0 Organization and Introduction

Convex Optimization for Computer Vision and Machine Learning WS 2017/2018

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Weekly: 2-hour lecture (Virginia), 2-hour tutorial (Emanuel)

Lecture: Starts at quarter past. Short break in between

Course material (password: cvxws17) https://vision.in.tum.de/teaching/ws2017/in2330 Lectures based on the course created by M. Moeller in 2016

Office hours: please write us an email

- Virginia's office 02.09.037
- Emanuel's office 02.09.039

Assessment: weekly exercise + written/oral final exam

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Exercises

Exercise sheets will be posted and collected on Monday (you have a week for each sheet). Solutions will be discussed on Friday

Theoretical and programming (Matlab) problems

Exercises in groups of 1 or 2. Copied solutions get 0 points

60% of exercise points adds 0.3 to your final exam (above 4.0)

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Interactions

Please don't be shy to ask questions

- They make the course more interesting
- They adapt the content to your background and interests

Please don't be shy to email us

- with suggestions or questions about blurry topics
- we will clarify topics that you found confusing
- we will adapt the exercises to help you understand them

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Course Requirements

Necessary

- Basic background in Analysis
- Background in linear Algebra
- Basic Numerical Programming (Matlab)

Useful

- Image processing, computer vision, machine learning Computer Vision I and II: Variational Methods, Multiple View Geometry, Machine Learning for Robotics and Computer Vision
- Numerical Optimization
 Numerisches Programmieren

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Optimization Problems

Given $E : S \subset \mathbb{R}^n \to \mathbb{R}$, we want to find \hat{u} such that

 $\hat{u} \in \arg\min_{u} E(u)$ s.t. $u \in C$ (1) Overview

where

u is the optimization variable

C is the constraint set

E is the objective or energy function

We can only solve¹ (1) for a subset of problems

- least-squares problem $\min_u \|Au b\|^2 \rightarrow u = (A^T A)^{-1} A^T b$
- · linear program: no analytic solution, simplex algorithm
- convex problem: guarantees for existence/uniqueness
 solution

¹no restriction to local minima or exhaustive search strategies

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 $\hat{u} \in \arg\min_{u} E(u)$ s.t. $u \in C$

where *C* is a *convex set* and *E* is a *convex function*



- converge to local minima of nonconvex functions raises the question whether the model or the minimum are wrong
- sequential convex optimization of nonconvex problems (linearization or majorization)

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Optimization algorithms

To construct iterates efficiently, we exploit the structure of E/C

$$\hat{u} \in \arg\min_{u} E(u)$$
 s.t. $u \in C$

In this course, the energy function and the constraints define a computer vision or machine learning model

- · robust to noise and outliers in the data
- regular: generalize well, do not overfit the training data
- · sparse: explain the data with as few variables as possible

This usually results in large nonsmooth optimization problems

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Denoising



$$\min_{u} \|u - f\|_1 + \alpha \int_{\Omega} |\nabla u(x)| \, dx$$

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Denoising



 $\min_{u} \|u - f\|_1 + \alpha \int_{\Omega} |\nabla u(x)| \, dx$

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Image deblurring

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$$\arg\min_{u} \|k * u - f\|_{2}^{2} + \alpha \int_{\Omega} |\nabla u(x)| dx$$

Surface Reconstruction



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Reconstruct an implicit surface $\partial M = \{x : \chi(x) = 0\}$ from an oriented point cloud $\{(x_i, V_i)\}_{i=0..N}$



Stereo Matching

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from 22.10.2017, slide 14/23

Stereo Matching



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Convexification of $\min_{v} \int_{\Omega} |f^{1}(x + v(x)) - f^{2}(x)| + \alpha |\nabla v(x)| dx$

Machine Learning Framework

Find the parameters (weights) *P* of the model $\mathcal{N}(\cdot, P)$ that explains the training examples $\{(x_i, y_i)\}_{i=0}^N$

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Example taken from http://cs.stanford.edu/people/karpathy/deepimagesent/.

Optimization problem: $\min_{P} \sum_{i} \mathcal{L}(\mathcal{N}(x_i, P), y_i).$

Common Machine Learning Loss Functions

Linear regression $\min_{w} ||Xw - y||^2$

Binary labels

- SVM loss: $\min_{w} \sum_{k} [1 y_k x_k^T w]_+$
- Binary logistic loss: $\min_{w} \sum_{k} \log(1 + \exp(-y_{k} x_{k}^{T} w))$



Example taken from

https://people.eecs.berkeley.edu/-jordan/courses/294-fall09/lectures/optimization/slides.pdf

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Chapter 1: Mathematical basics and convex analysis

Basics of multivariable calculus and linear algebra:

- Open, closed, bounded and compact sets
- Continuity of functions $f : \mathbb{R}^n \to \mathbb{R}^m$
- Differentiability of functions $f : \mathbb{R}^n \to \mathbb{R}^m$, chain rule
- Linear operators in matrix form, eigenvectors, semi-definiteness

Basics of convex analysis:

- Convex sets
- Convex extended real valued functions in ℝⁿ
- Existence of minimizers
- · Optimality conditions and subdifferential calculus

Goal: Everyone knows all necessary tools to follow the lecture!

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Chapter 2: Gradient based methods

Optimization algorithms based on (generalized) gradient methods

- Gradient descent
- Gradient projection
- Proximal gradient method
- Subgradient descent
- Convergence analysis

Goal: Establish basic minimization strategies based on energy descent methods most suitable for (partly) smooth energy functions.

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Chapter 3: Convex conjugation and duality

- · Primal and dual formulation of a problem
- Convex conjugate
- Saddle point problems
- Optimality conditions

Goal: Increase the number of tools to reformulate and analyze more complex convex minimization problems.

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Chapter 4: Primal-dual optimization schemes

- Concept: Averaged operators
- Primal-dual hybrid gradient method
- Proximal point algorithm
- Douglas-Rachford splitting
- Alternating directions method of multipliers
- Convergence analysis based on maximally monotone operators
- Primal and dual residuals. Choice of primal and dual stepsizes

Goal: Learn about state-of-the-art first order optimization methods and their relations.

These are the algorithms used in most publications on variational method in imaging and computer vision

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Chapter 5: To be defined by your interests

- Majorize-Minimize algorithm
- Other splitting algorithms: Peaceman-Rachford, etc.

Example of Linear Program: Profit maximization

A company wants to maximize its profit under certain contraints given by the availability of resources.

- A company has two products.
- Producing the amount x of product 1 requires
 - using machine A for 5x units of time,
 - using machine B for 2x units of time.
- Producing the amount x of product 2 requires
 - using machine A for 1x units of time,
 - using machine B for 4x units of time.
- Product 1 sells for twice as much as product 2.
- The cost of a time unit of machine A is half the cost of a time unit of machine B.
- Machine A is available for 160 units of time.
- Machine B is available for 240 units of time.

How much of product 1 and 2 should the company produce in order to maximize its profit?

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