

Convex Optimization for Machine Learning and Computer Vision

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 Winter Semester 2017/18

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Weekly Exercises 10

Room: 02.09.023

Friday, 02.02.2018, 09:15-11:00

Submission deadline: Monday, 29.01.2018, 10:15, Room 02.09.023

Theory: Recap (0+14 Points)

Exercise 1 (4 Points). Compute the convex conjugates of the following functions:

1. $f_1 : \mathbb{R} \rightarrow \mathbb{R} \cup \{\infty\}$ where $f_1(x) = \sqrt{1+x^2}$.
2. $f_2 : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{\infty\}$ where $f_2(x) = \log(\sum_{i=1}^n e^{x_i})$.

Don't forget to specify the domains $\text{dom}(f_1^*)$, $\text{dom}(f_2^*)$.

Definition. A function $g : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{\infty\}$ is called 1-homogeneous if

$$g(\alpha x) = \alpha g(x),$$

for all $\alpha \geq 0$.

Exercise 2 (4 Points). Let $g : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{\infty\}$ be convex, closed, proper and 1-homogeneous. Show that the proximity operator of the sum $\|\cdot\|_2 + g$ is the composition of the proximity operators of $\|\cdot\|_2$ and g , i.e.

$$\text{prox}_{\|\cdot\|_2 + g} = \text{prox}_{\|\cdot\|_2} \circ \text{prox}_g.$$

Exercise 3 (6 Points). Let C be a nonempty, closed, convex subset of \mathbb{R}^n . For each $i \in \{1, \dots, m\}$, let $\alpha_i \in (0, 1)$, $\omega_i \in (0, 1)$ and $\Phi_i : C \rightarrow \mathbb{R}^n$ be an α_i -averaged operator. Prove the following statements:

- Φ_i is α_i -averaged iff

$$\|\Phi_i(u) - \Phi_i(v)\|_2^2 + \frac{1 - \alpha_i}{\alpha_i} \|(I - \Phi_i)(u) - (I - \Phi_i)(v)\|_2^2 \leq \|u - v\|_2^2,$$

for all $u, v \in C$.

- If $\sum_{i=1}^m \omega_i = 1$ and $\alpha = \max_{1 \leq i \leq m} \alpha_i$, then

$$\Phi = \sum_{i=1}^m \omega_i \Phi_i$$

is α -averaged.