

Weekly Exercises 12

Room: 01.09.014

Wednesday, 30.01.2019, 12:15-14:00

Submission deadline: Monday, 28.01.2019, 16:15, Room 01.09.014

Consensus optimization

(4+4 Points)

Exercise 1 (8 Points). Consider the *consensus optimization* problem:

$$\begin{aligned} \min_{\{x_i\}_{i=1}^l \subset \mathbb{R}^n, x_0 \in \mathbb{R}^n} \quad & \sum_{i=1}^l f_i(x_i) \\ \text{subject to} \quad & x_i = x_0 \quad \forall i \in \{1, 2, \dots, l\}. \end{aligned} \tag{1}$$

Here each function $f_i : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{\infty\}$ is proper, convex, and lower-semicontinuous.

- Write down the augmented Lagrangian functional for (1) (which will involve multipliers $\{y_i\}_{i=1}^l \subset \mathbb{R}^n$).
- Formulate an alternating direction of multipliers (ADMM) method for (1). Update the variables in the order of $\{x_i\}_{i=1}^l, \{y_i\}_{i=1}^l, x_0$.
- One can interpret the ADMM scheme in (b) as a generalized proximal iteration on $(x_0, \{y_i\}_{i=1}^l)$:

$$0 \in M \begin{bmatrix} x_0^{k+1} - x_0^k \\ y_1^{k+1} - y_1^k \\ \vdots \\ y_l^{k+1} - y_l^k \end{bmatrix} + R \left(\begin{bmatrix} x_0^{k+1} \\ y_1^{k+1} \\ \vdots \\ y_l^{k+1} \end{bmatrix} \right).$$

Identify the positive semidefinite matrix M and the monotone operator R in the above equation.