Lecture: Dr. Tao Wu
Exercises: Yuesong Shen, Zhenzhang Ye
Winter Semester 2018/19

Computer Vision Group
Institut für Informatik
Technische Universität München

Weekly Exercises 7

Room: 01.09.014

Wednesday, 12.12.2018, 12:15-14:00

Submission deadline: Monday, 10.12.2018, 16:15, Room 01.09.014

Prox and Gradient descent (8 + 4 Points)

Exercise 1 (6 Points)**.** Let $Q \in \mathbb{R}^{n\times n}$ be a positive definite symmetric matrix. Prove the following inequality for any vector $x \in \mathbb{R}^n$

$$\frac{(x^\top x)^2}{(x^\top Q x)(x^\top Q^{-1} x)} \geq \frac{4\lambda_n \lambda_1}{(\lambda_n + \lambda_1)^2},$$

where λ_n and λ_1 are, respectively, the largest and smallest eigenvalues of Q.

Exercise 2 (6 Points)**.** Let $Q \in \mathbb{R}^{n\times n}$ be symmetric positive definite, and $b \in \mathbb{R}^n$. As in the previous exercise, denote the eigenvalues of Q as $0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$. Consider the quadratic function $f : \mathbb{R}^n \to \mathbb{R}$, $x \mapsto \frac{1}{2} x^\top Q x - b^\top x$ and show gradient descent with exact line search has the following convergence property:

$$\left\|x^{k+1} - x^*\right\|_Q^2 \leq \left(\frac{\lambda_n - \lambda_1}{\lambda_n + \lambda_1}\right)^2 \left\|x^k - x^*\right\|_Q^2,$$

where $x^* \in \mathbb{R}^n$ denotes the global minimizer of f.
Hint: use the inequality from exercise 1.