

**Machine Learning for Computer Vision**  
**Winter term 2018**

January 25, 2019  
Topic: Variational Inference

- a) Show that the Bernoulli distribution is a member of the exponential family, i.e. it can be written as  $h(x)g(\eta) \exp(\eta u(x))$ . What are  $h(x)$ ,  $g(\eta)$ , and  $u(x)$ ? Hint: use  $\exp(\log())$ .
- b) From the result of a) for the natural parameter  $\eta$ , derive an expression for the parameter  $\mu$ . Try to draw a rough plot of  $\mu$  as a function  $f$  of  $\eta$ . What is the common name for that function?
- c) From the expression for  $g(\eta)$ , compute the negative gradient of the logarithm, i.e.  $-\frac{d \log g(\eta)}{d\eta}$ . Use the function  $f$  derived in b) to express  $\mu$  and use the fact that the derivative of that function is  $f(\eta)(1 - f(\eta))$ . Interpret the result.
- d) The KL-divergence in bits between two discrete distributions  $p$  and  $q$  is defined as

$$KL(p||q) = - \sum_x p(x) \log_2 \frac{q(x)}{p(x)}$$

Assume that both  $p(x | \mu)$  and  $q(x | \nu)$  are Bernoulli distributions where  $\mu = 1/2$  and  $\nu = 1/4$ . Compute the KL-divergence  $KL(p||q)$ .