Variational Methods for Computer Vision: Exercise Sheet 9

Exercise: January 8, 2020

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $I(x) : \Omega \to \mathbb{R}$ with $\Omega \subset \mathbb{R}^2$ be an image. Consider this generalized version of the Chan-Vese functional:

$$E(\phi) = \int_\Omega f_1(x) H(\phi(x)) dx + \int_\Omega f_2(x)(1 - H(\phi(x))) dx + \nu \int_\Omega |\nabla H(\phi(x))| dx, \qquad (1)$$

   where f_i is a data term which arises from a general gray value distribution p_i, hence

$$f_i(x) = -\log p_i(I(x)).$$

   $H(\phi(x))$ denotes the Heaviside step function:

$$H(\phi(x)) = \begin{cases} 1 & \text{if} \quad \phi(x) > 0, \\ 0 & \text{else.} \end{cases}$$

   Prove that the Euler-Lagrange equation of (1) can be written as follows:

$$\frac{dE}{d\phi} = \delta(\phi) \left[f_1 - f_2 - \nu \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) \right] = 0.$$

   The delta distribution $\delta(\phi)$ can be considered as the derivative of H.

2. The level set formulation of geodesic active contours can be formulated as the following functional:

$$E(\phi) = \int_\Omega g(x) |\nabla H(\phi(x))| dx, \qquad (2)$$

   where $g : \Omega \to \mathbb{R}$ denotes some edge indicator function and $H(\phi(x))$ is defined as in exercise 1. Compute the Euler-Lagrange equation of $E(\phi)$. Note that there is no constraint on the gradient $\nabla \phi$ to have norm 1.

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

In this practical exercise we are going to consider the Chan-Vese functional:

$$E(\phi) = \int_\Omega (I(x) - \mu_1)^2 H(\phi(x)) dx + \int_\Omega (I(x) - \mu_2)^2 (1 - H(\phi(x))) dx + \nu \int_\Omega |\nabla H(\phi(x))| dx. \quad (3)$$

1. Download the archive file `vmcv_ex09.zip` from the homepage and unzip it in your home folder and complete the code in `chan_vese.m` as follows.
   (a) In order to minimize the above functional, optimal μ_1 and μ_2 have to be obtained. For a given curve the optimal values for μ_1 and μ_2 are the mean values of the inner and outer region. Implement the function `[mu1,mu2]=approxRegions(Phi,I)` that returns for a given image I and a level set function Phi the mean values inside and outside the contour.
   (b) Further implement the function `dPhi=update(Phi,I)` which computes a gradient descent direction using the result of theory exercise 1.
   (c) Implement an energy minimization of (3). Initialize the level set function with a circle of radius R in the center of the image.
2. Test your implementation on the image `image.pgm` with various radii R.