Variational Methods for Computer Vision: Exercise Sheet 10

Exercise: January 15, 2020

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. The Chan-Vese functional $E(\phi)$ from last exercise sheet has been reformulated by Chan, Esodoglu and Nikolova by associating $u \equiv H(\phi)$ where $u : \Omega \to [0, 1]$. The resulting functional can be written as follows:

$$E(u) = \int_\Omega f_1(x)u(x) + f_2(x)(1 - u(x)) + \nu|\nabla u(x)| \, \mathrm{dx}. \tag{1}$$

   (a) Prove that (1) is a convex functional.

   (b) Prove that $U = \{u : \Omega \to [0, 1]\}$ is a convex set.

   (c) The projection $f_U \in U$ of a given function $f : \Omega \to \mathbb{R}$ onto the set U can be written as the minimizer of the following functional

$$f_U := \arg\min_{u \in U} \left(\int_\Omega (f(x) - u(x))^2 \, \mathrm{dx} \right).$$

   Show that :

$$f_U(x) = \begin{cases} 1, & \text{if} \quad f(x) > 1, \\ 0, & \text{if} \quad f(x) < 0, \\ f(x), & \text{otherwise.} \end{cases}$$

   (d) Prove that the Euler-Lagrange equation of (1) can be written as follows:

$$\left[f_1 - f_2 - \nu \, \mathrm{div} \left(\frac{\nabla u}{|\nabla u|} \right) \right] = 0.$$

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Implement the minimization of the Chan-Esodoglu-Nikolova functional and make sure the optimization stays in the constrained space of functions U from the theoretical exercise by doing a re-projection by clipping (as in exercise 1c).
2. Test your implementation on the image `image.png` from last exercise sheet by initializing the the algorithm with a circle of radius R in the center of the image.
3. After obtaining the global minimizer visualize the segmentation result by thresholding the resulting function i.e by using the command `imagesc(u<0.5)`.