Convex Optimization for Machine Learning and Computer Vision

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Weekly Exercises 1

Room: 02.09.023

Wednesday, 30.10.2019, 12:15-14:00

Submission deadline: Monday, 28.10.2019, 16:15, Room 02.09.023

Theory: Convex Sets (12+8 Points)

Exercise 1 (4 Points)**.** Let $\mathcal{C}$ be a family of convex sets in $\mathbb{R}^n$, $C_1, C_2 \in \mathcal{C}$, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, $\lambda \in \mathbb{R}$. Prove convexity of the following sets:

- $\bigcap_{C\in\mathcal{C}} C$
- $P := \{x \in \mathbb{R}^n : Ax \leq b\}$
- $C_1 + C_2 := \{x + y : x \in C_1, y \in C_2\}$ (the Minkowski sum of C_1 and C_2)
- $\lambda C_1 := \{\lambda x : x \in C_1\}$ (the λ-dilatation of C_1).

Exercise 2 (4 Points)**.** Prove that if the set $C \subset \mathbb{R}^n$ is convex, then $\sum_{i=1}^N \lambda_i x_i \in C$ with $x_1, x_2, \ldots, x_N \in C$ and $0 \leq \lambda_1, \lambda_2, \ldots, \lambda_N \in \mathbb{R}$, $\sum_{i=1}^N \lambda_i = 1$.

Hint: Use induction to prove.

Exercise 3 (4 Points)**.** Let $\emptyset \neq X \subset \mathbb{R}^n$. Prove the equivalence of the following statements:

- X is closed.
- Every convergent sequence $\{x_n\}_{n\in\mathbb{N}} \subset X$ attains its limit in X.

Exercise 4 (8 Points)**.** Some basic problems on calculus and linear algebra.

- Let $u \in \mathbb{R}^n$, compute the gradient of following function on u: $J(u) = \sqrt{u^\top Au}$, where $A \in \mathbb{R}^{n\times n}$ is full rank and $u \neq 0$.
- What happens if A is not full rank?
- Let $z \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $f \in \mathbb{R}$ and ϵ is a positive real number, compute the gradient of following function on z:

$$R(z) = \frac{z}{f^2}\sqrt{f^2\left\|Az\right\|^2 + \left\|-z - \mathbb{1}\langle x, Az\rangle\right\|^2 + \epsilon}$$

  and $\mathbb{1} = (1, 1, \ldots, 1)^\top \in \mathbb{R}^n$.