

Weekly Exercises 8

Room: 02.09.023

Wednesday, 15.01.2020, 12:15-14:00

Submission deadline: Monday, 13.01.2020, 16:15, Room 02.09.023

Exact Line Search

(14+6 Points)

Exercise 1 (6 Points). Let $Q \in \mathbb{R}^{n \times n}$ be a positive definite symmetric matrix. Prove the following inequality for any vector $x \in \mathbb{R}^n$

$$\frac{(x^\top x)^2}{(x^\top Q x)(x^\top Q^{-1} x)} \geq \frac{4\lambda_n \lambda_1}{(\lambda_n + \lambda_1)^2},$$

where λ_n and λ_1 are, respectively, the largest and smallest eigenvalues of Q .

Exercise 2 (6 Points). Let $Q \in \mathbb{R}^{n \times n}$ be symmetric positive definite, and $b \in \mathbb{R}^n$. As in the previous exercise, denote the eigenvalues of Q as $0 < \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$. Consider the quadratic function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, $x \mapsto \frac{1}{2}x^\top Q x - b^\top x$ and show gradient descent with exact line search has the following convergence property:

$$\|x^{k+1} - x^*\|_Q^2 \leq \left(\frac{\lambda_n - \lambda_1}{\lambda_n + \lambda_1} \right)^2 \|x^k - x^*\|_Q^2,$$

where $x^* \in \mathbb{R}^n$ denotes the global minimizer of f .

Hint: use the inequality from exercise 1.

Image Denoising (Due Date: 13.01)

(10 Points)

Given a noisy input image, we want to remove the noises by solving following minimization problem:

$$\operatorname{argmin}_u \frac{1}{2} \|u - f\|^2 + \rho H_\varepsilon(Ku).$$

where $f \in \mathbb{R}^N$ is the input image with N pixels, ρ is a scalar weighting the smooth regularizer, H_ε is the Huber function defined as before and K is the gradient operator.

Your task is using gradient descent with line search to solve it.

For detailed line search, you can refer to Algorithm 3.5 and 3.6 (Page 79) in Numerical Optimization

http://www.apmath.spbu.ru/cnsa/pdf/monograf/Numerical_Optimization2006.pdf.

For zoom function, you can directly use bisection.