Photometric Stereo with General, Unknown Lighting

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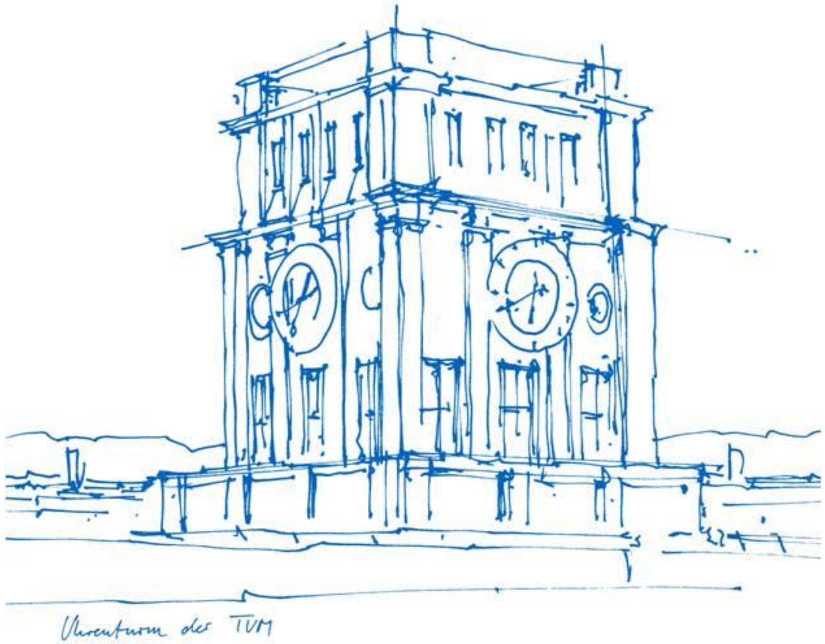
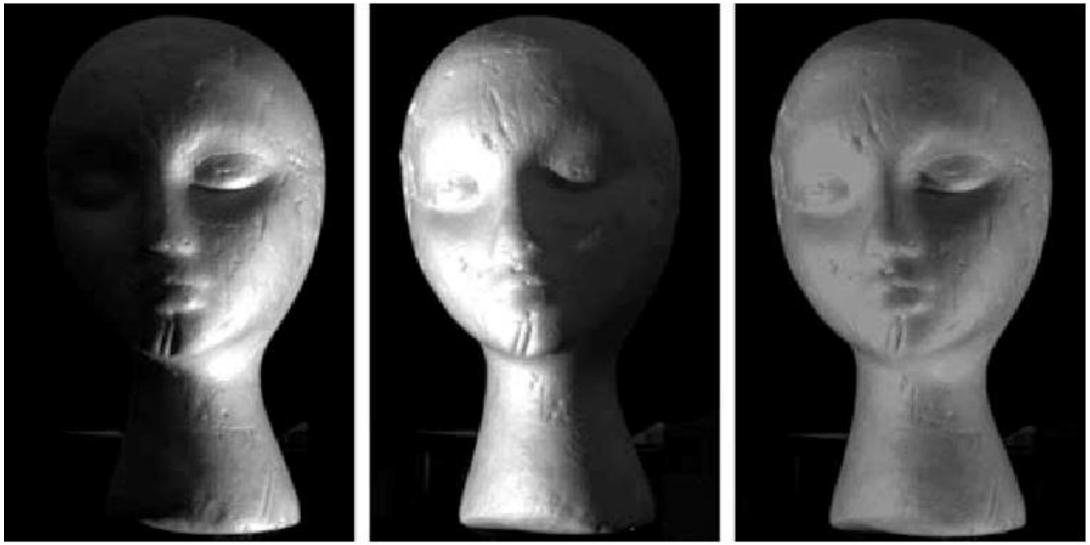
13. November 2019



Outline

- Basics
- Method
- Experiments and Results
- Conclusions
- Discussion

Basics: Photometric Stereo

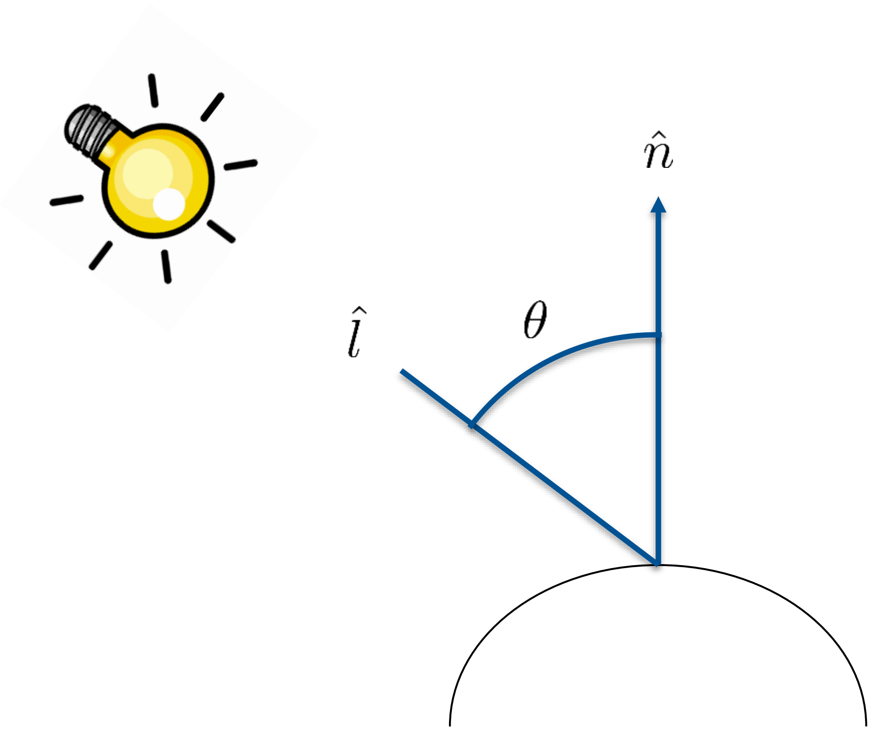


Classic approach:

Given are several images of a lambertian object under **varying lighting conditions**

➡ Extract **surface normals**

Basics: Lambertian Reflection



with
I reflected Light intensity
E incoming Light intensity
ρ albedo
θ incidance angle

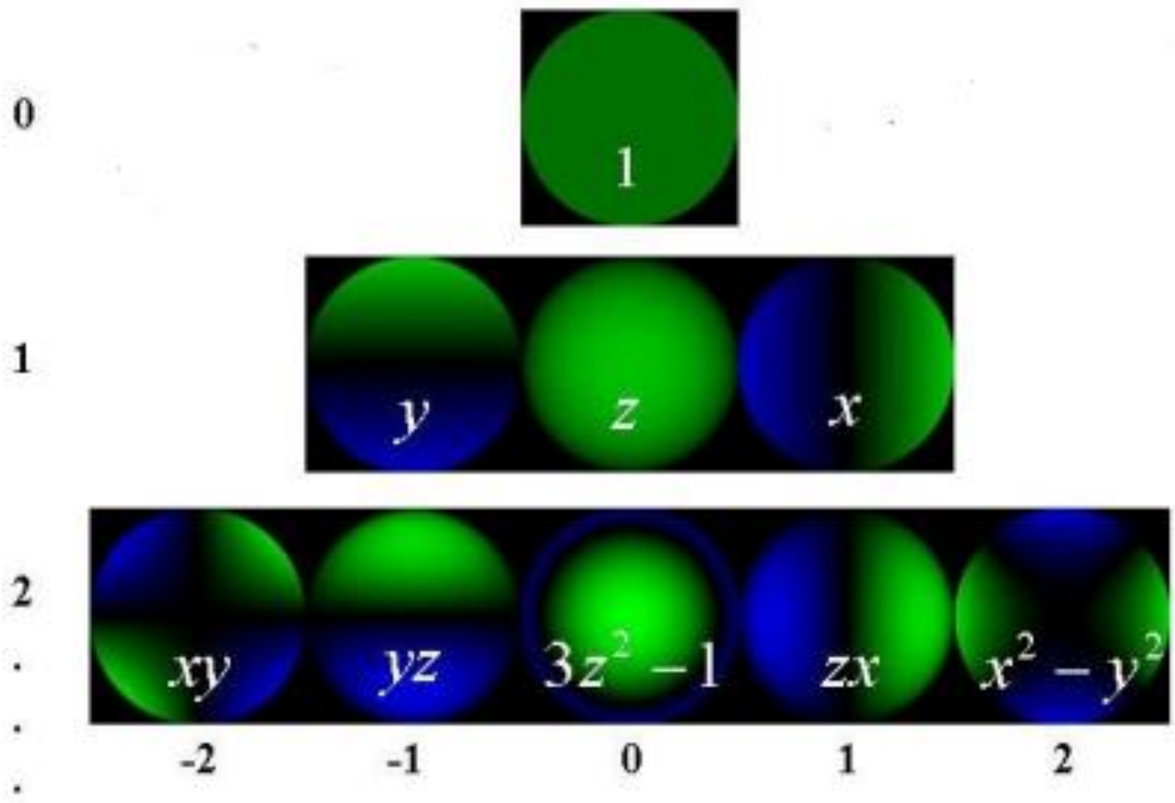
$$I = E\rho cos\theta \quad (1)$$

$$I = E\hat{l}\rho\hat{n} = l\vec{^T}\vec{n} \quad (2)$$

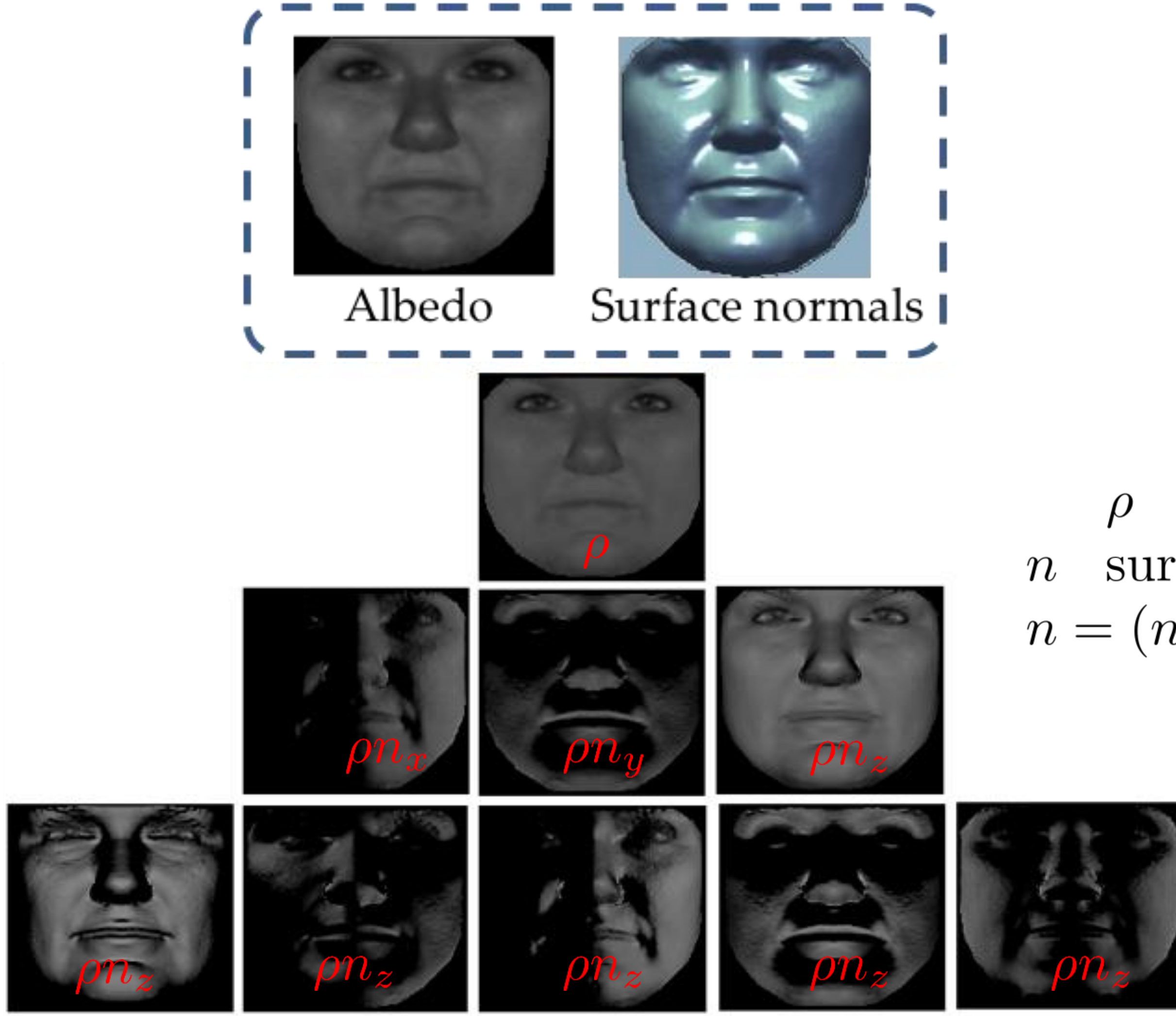
Basics: Photometric Stereo and Lambert's Law

$$\begin{bmatrix} \text{Image } 1 \\ \vdots \\ \text{Image } 2 \end{bmatrix} = \underbrace{\begin{bmatrix} M_{11} & \dots & M_{1p} \\ \vdots & & \vdots \\ M_{f1} & \dots & M_{fp} \end{bmatrix}}_{\mathbf{M}} = \underbrace{\begin{bmatrix} l_{1x} & l_{1y} & l_{1z} \\ \vdots & \vdots & \vdots \\ l_{fx} & l_{fy} & l_{fz} \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} n_{x1} & \cdots & n_{xp} \\ n_{y1} & \cdots & n_{yp} \\ n_{z1} & \cdots & n_{zp} \end{bmatrix}}_{\mathbf{S}}$$

Basics: Spherical Harmonics



Basics: Harmonic Images



ρ albedo

n surface normal

$n = (n_x, n_y, n_z)$

Method: Shape Recovery

Strategy:

Extract basis for „image space“ from M and therefore albedo and surface normals

$$\underbrace{\begin{bmatrix} \text{Image 1} \\ \vdots \\ \text{Image f} \end{bmatrix}}_{\mathbf{M}} \approx \underbrace{\begin{bmatrix} \text{Light 1} \\ \vdots \\ \text{Light f} \end{bmatrix}}_{\hat{\mathbf{L}}} \underbrace{\begin{bmatrix} \rho \\ \rho n_x \\ \rho n_y \\ \rho n_z \\ \rho(3n_z^2 - 1) \\ \rho(n_x^2 - n_y^2) \\ \rho n_x n_y \\ \rho n_x n_z \\ \rho n_y n_z \end{bmatrix}}_{\hat{\mathbf{S}}}$$

Method: Shape Recovery

First Step: Get major components

- Apply SVD :

$$M = U\Delta V^T \tag{1}$$

- Choose

$$\tilde{L} = U\sqrt{\Delta^{(fr)}} \tag{2}$$

$$\tilde{S} = \sqrt{\Delta^{(rp)}}V^T \tag{3}$$

- Problem:

$$M = \tilde{L}A^{-1}A\tilde{S} \tag{4}$$

- Aim: Find Matrix A such that

$$\hat{S} = A\tilde{S} \tag{5}$$

Shape Recovery: Case of Four Harmonics

$$M = \begin{bmatrix} M_{11} & \cdots & M_{1p} \\ M_{21} & \cdots & M_{2p} \\ M_{31} & \cdots & M_{3p} \\ M_{41} & \cdots & M_{4p} \end{bmatrix} = \hat{L} \begin{bmatrix} \rho_1 & \cdots & \rho_p \\ \rho_1 n_{x1} & \cdots & \rho_p n_{xp} \\ \rho_1 n_{y1} & \cdots & \rho_p n_{yp} \\ \rho_1 n_{z1} & \cdots & \rho_p n_{zp} \end{bmatrix} = \hat{L} \begin{bmatrix} | & \cdots & | \\ p_1 & \cdots & p_p \\ | & \cdots & | \end{bmatrix} \quad (0)$$

$$\rho_1 = \sqrt{\rho_1^2 n_{x1}^2 + \rho_1^2 n_{y1}^2 + \rho_1^2 n_{z1}^2} \Leftrightarrow \vec{p^T} J \vec{p} = 0 \quad (1)$$

- Not necessarily the case for estimated S, so we require

$$\vec{q^T} A^{-1} J A \vec{q} = \vec{q^T} B \vec{q} = 0 \quad (2)$$

- Find A by solving a system of linear equations and SVD, but

$$B = \vec{q^T} A^{-1} J A \vec{q} = \vec{q^T} A^{-1} C^T J C A \vec{q} \quad (3)$$

Shape Recovery: Case of Nine Harmonics

$$M = \begin{bmatrix} M_{11} & \cdots & M_{1p} \\ M_{21} & \cdots & M_{2p} \\ M_{31} & \cdots & M_{3p} \\ M_{41} & \cdots & M_{4p} \end{bmatrix} = \hat{L} \begin{bmatrix} \rho_1 & \cdots & \rho_p \\ \rho_1 n_{x1} & \cdots & \rho_p n_{xp} \\ \rho_1 n_{y1} & \cdots & \rho_p n_{yp} \\ \rho_1 n_{z1} & \cdots & \rho_p n_{zp} \\ \rho_1 (3n_{z1}^2 - 1) & \cdots & \rho_p (3n_{zp}^2 - 1) \\ \rho_1 (n_{x1}^2 - n_{y1}) & \cdots & \rho_p (n_{xp}^2 - n_{yp} \\ \rho_1 n_{x1} n_{y1} & \cdots & \rho_p n_{xp} n_{yp} \\ \rho_1 n_{x1} n_{z1} & \cdots & \rho_p n_{xp} n_{zp} \\ \rho_1 n_{y1} n_{z1} & \cdots & \rho_p n_{yp} n_{zp} \end{bmatrix} \tag{0}$$

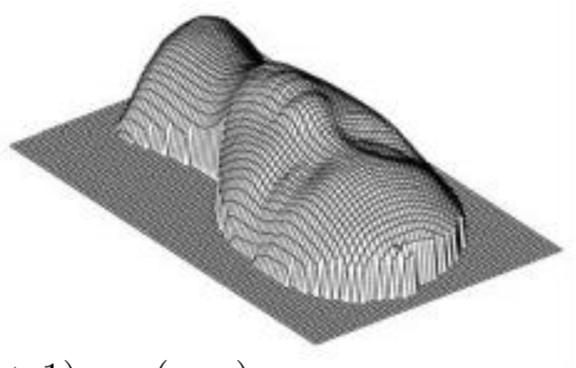
Look for A such that

$$A\tilde{S} = (\vec{h_2}, \vec{h_3}, \vec{h_4})^T \tag{1}$$

By iteratively minimizing error

$$E(A) = \min_L \|M - LS_A\| \tag{2}$$

Surface Reconstruction and Integrability



Given: a normal field $n(x, y)$

How to recover the height $z(x, y)$?

Estimate the partial derivatives $p = -\frac{n_x}{n_z}$ and $q = -\frac{n_y}{n_z}$

Approximate partial derivatives $z(x+1, y) - z(x, y)$ and $z(x, y+1) - z(x, y)$

Solve

$$\min \left((z(x+1, y) - z(x, y) - p)^2 + (z(x, y+1) - z(x, y) - q)^2 \right)$$

➡ Solves ambiguity up to generalized bas-relief ambiguity

Experiments and Results

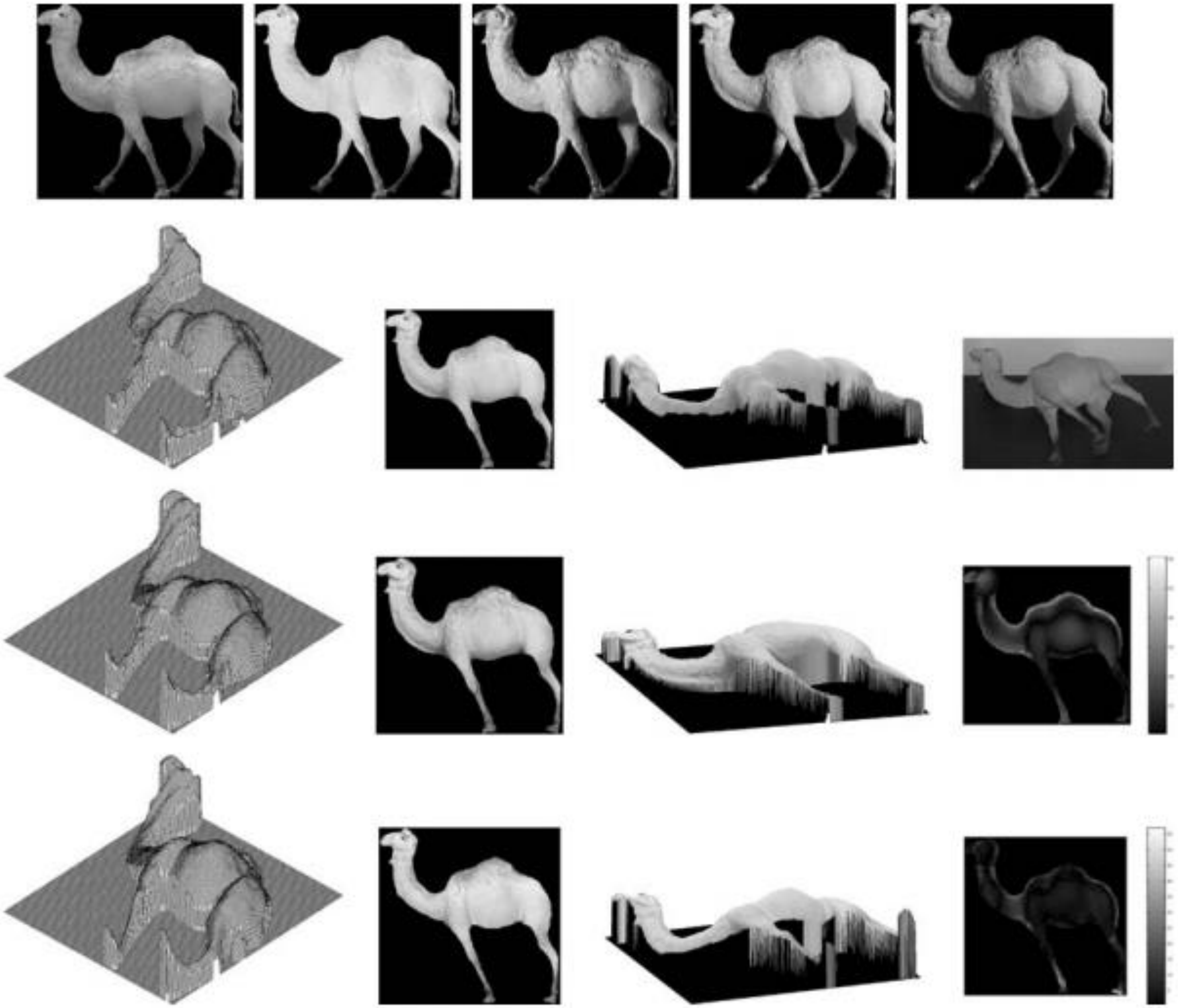
- Experiments on synthetic data:

  4D: mean error of 3,6°

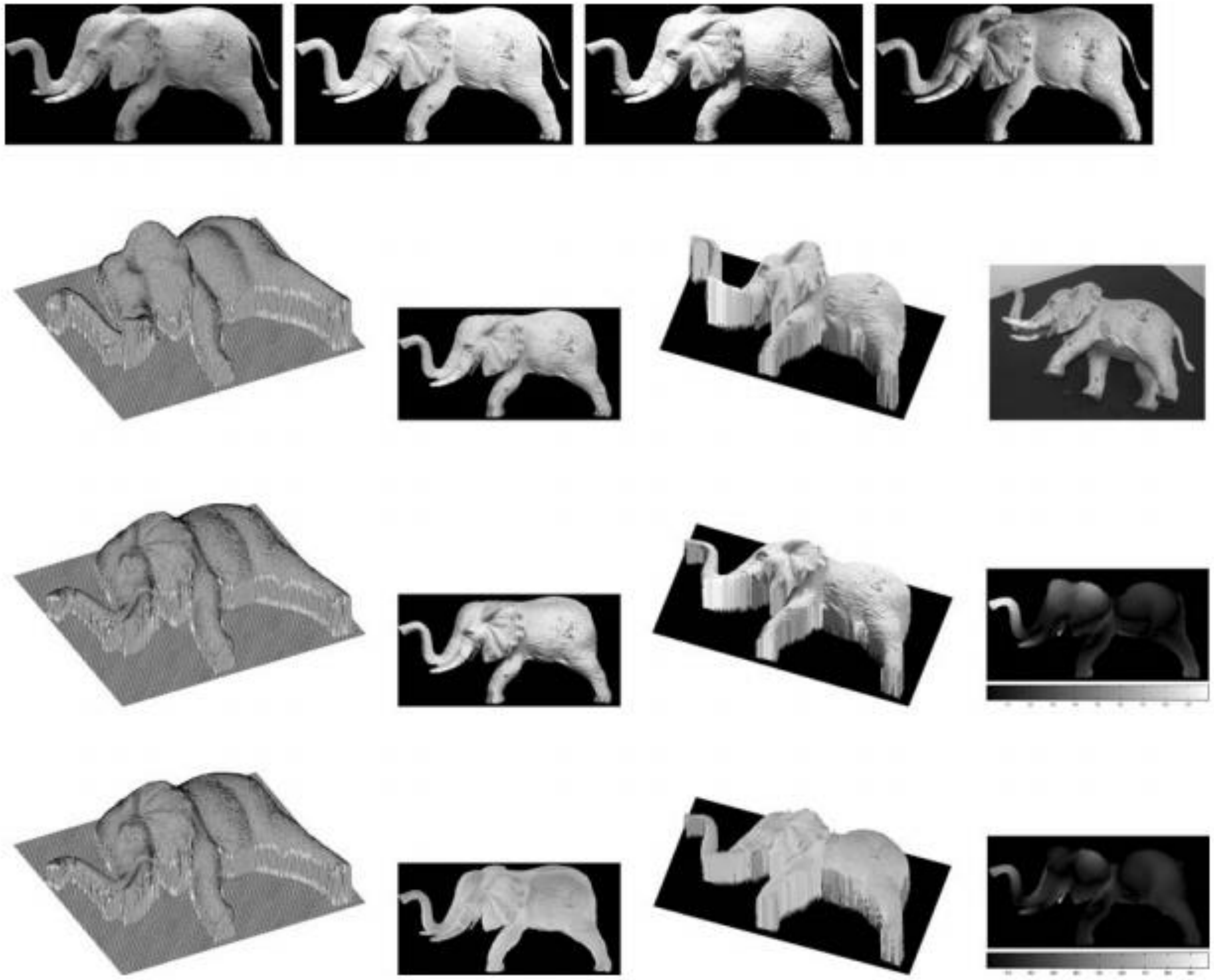
  9D: mean error of 2,8°

  in 97% of cases 9D finds a solution with error <1%

Experiments and Results



Experiments and Results



Experiments and Results

- **Experiments on synthetic data:**
  4D: mean error of 3,6°
  9D: mean error of 2,8°
  in 97% of cases 9D finds a solution with error <1%

- **Experiments on unrestricted lighting situations**

Table 1. Relative accuracy of reconstruction: $1 - \|\hat{z} - z\|^2 / \|z\|^2$, where $\hat{z}$ denotes the reconstructed depth and z denotes the laser scanned depth.

	4D	9D
Hippo	0.96	0.97
Elephant	0.95	0.98
Camel	0.97	0.99
Dino	0.98	0.99

4D: ~ 95 - 98%

9D: ~ 97 - 99%

relative accuracy compared to laser scanned depth.

Conclusions

Pros

- Relatively accurate geometry reconstruction method
- Allows arbitrary lighting conditions

Cons

- Remaining ambiguity
- Not exact: Low order spherical harmonics approximation, SVD, Lambertian model, camera noise

Questions and Discussion