Robotic 3D Vision – Graded Online Exercise (Example)

Duration: 1.5 h Winter Term 2020/2021 February 24, 2021

This example does not have the extent of a full regular exam.

1. **EKF based localization**

You are supposed to implement a visual-inertial localization algorithm based on an Extended Kalman Filter (EKF) to localize an autonomous wheeled robot. Since the robot is only able to move on the ground plane, it is sufficient estimate its pose (position and orientation) only with respect to a 2D plane.

To perform localization you obtain on one side the measurements from an IMU mounted on the robot-platform (angular velocity $\omega_k \in \mathbb{R}$ and linear acceleration $\boldsymbol{a}_k = (a_{xk}, a_{yk})^\top \in \mathbb{R}^2$) and on the other side the measured absolute 2D position of the robot $\boldsymbol{y}_k = (x_k, y_k)^\top \in \mathbb{R}^2$ (for simplicity we assume that the position can be calculated directly based on observations of multiple landmarks with known locations).

Both the IMU measurements ω_k and $\boldsymbol{a}_k$, as well as the $\boldsymbol{y}_k$ are Gaussian distributed with covariances

$$\Sigma_\omega = \sigma_\omega^2, \qquad \boldsymbol{\Sigma_a} = \begin{pmatrix} \sigma_{ax}^2 & 0 \\ 0 & \sigma_{ay}^2 \end{pmatrix}, \qquad \boldsymbol{\Sigma_y} = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix},$$

respectively.

Hints:

- Consider the fact that IMU measurements are obtained with respect to a robot-centric coordinate frame.
- Use of the IMU measurements ω_k and $\boldsymbol{a}_k$ during the prediction step and the measured position $\boldsymbol{y}_k$ during the correction step.

a) Define the state vector $\boldsymbol{x}_k$ of the Extended Kalman Filter. Consider the fact that you also want to integrate the IMU measurements.

b) Define the state transition model:

$$\boldsymbol{x}_k = g(\boldsymbol{x}_{k-1}, \boldsymbol{u}_k).$$

For simplicity one can consider the motion to be constant between two time steps.

c) Define the observation model:

$$\boldsymbol{y}_k = h(\boldsymbol{x}_k).$$

d) Assume that the robot was initially standing still and we have perfect knowledge about the initial state. Hence, the state vector is given as follows:

$$\boldsymbol{x}_{k-1} \sim \mathcal{N}(\boldsymbol{\mu}_{k-1}, \boldsymbol{\Sigma}_{k-1}) \qquad \text{with} \qquad \boldsymbol{\mu}_{k-1} = \mathbf{0}, \qquad \boldsymbol{\Sigma}_{k-1} = \mathbf{0}.$$

Calculate the predicted state vector $\boldsymbol{x}_k^-$ and the corresponding covariance matrix $\boldsymbol{\Sigma}_k^-$ based on the following IMU measurements:

$$\begin{aligned} \omega_k &= 0.25 \text{ rad/s}, & \sigma_\omega &= 0.01 \text{ rad/s} \\ a_{xk} &= 1.0 \text{ m/s}^2, & \sigma_{ax} &= 0.02 \text{ m/s}^2 \\ a_{yk} &= 0.01 \text{ m/s}^2, & \sigma_{ay} &= 0.02 \text{ m/s}^2 \end{aligned}$$

Assume a time interval of $\Delta t = 0.5$ s between $(k-1)$ and k .

e) In certain cases the Kalman Filter provides the optimal solution to the state estimation problem. Does this also apply to the model defined above? (assume that we made perfect assumption on the state transition and observation model) Briefly explain your answer.

f) Assume that the IMU measurements ω_k and $\boldsymbol{a}_k$ are obtained with a frequency of 100 Hz, while the position measurements $\boldsymbol{y}_k$ are obtained at 25 Hz. What does this mean for the predict-correct cycle of the Kalman Filter?