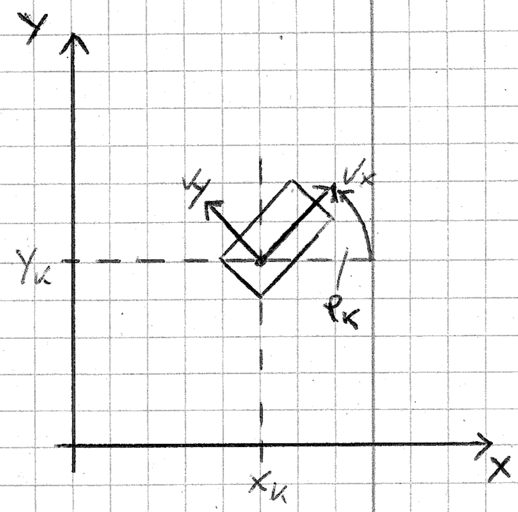
Robotic 3D Vision - Graded Online Exercise (Example)

a) state vector consists of

- robot position (x_k, y_k)
- robot orientation/heading φ_k
- robot's linear velocity (v_{x_k}, v_{y_k})

↳ can be defined either w.r.t. robot coordinate frame or global frame (here it is defined in robot coordinate frame)



$$\vec{x}_k = (x_k, y_k, \varphi_k, v_{x_k}, v_{y_k})^T$$

b) state transition model based on IMU measurements

$$\omega_k, \ \vec{a}_k = (a_{x_k}, a_{y_k})^T$$

$$\vec{u}_k = (\omega_k, a_{x_k}, a_{y_k})^T$$

$$\vec{x}_k = g(\vec{x}_{k-1}, \vec{u}_k)$$

$$\begin{pmatrix} x_k \\ y_k \end{pmatrix} = \begin{pmatrix} x_{k-1} \\ y_{k-1} \end{pmatrix} + R(\varphi_{k-1}) \cdot \left[\begin{pmatrix} v_{x_{k-1}} \\ v_{y_{k-1}} \end{pmatrix} \cdot \Delta t + \frac{1}{2} \begin{pmatrix} a_{x_k} \\ a_{y_k} \end{pmatrix} \cdot \Delta t^2 \right]$$

$$R(\varphi) = \begin{pmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{pmatrix}$$

$$\varphi_k = \varphi_{k-1} + \omega_k \cdot \Delta t$$

$$v_{x_k} = v_{x_{k-1}} + a_{x_k} \cdot \Delta t$$

$$v_{y_k} = v_{y_{k-1}} + a_{y_k} \cdot \Delta t$$

c) Observation model

$$\vec{y}_k = h(\vec{x}_k)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \cdot \vec{x}_k$$

d)

$$\vec{x}_{k-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \Rightarrow \quad R(\varphi = 0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$x_k^- = x_{k-1} + v_{x k-1} \cdot \Delta t + \frac{1}{2} a_{xk} \cdot \Delta t^2$$
$$= \frac{1}{2} \cdot a_{xk} \cdot \Delta t^2 = 0{,}125 \text{ m}$$

$$y_k^- = y_{k-1} + v_{y k-1} \cdot \Delta t + \frac{1}{2} a_{yk} \cdot \Delta t^2$$
$$= \frac{1}{2} \cdot a_{yk} \cdot \Delta t^2 = 0{,}00125 \text{ m}$$

$$\varphi_k^- = \varphi_{k-1} + \omega_k \cdot \Delta t = 0{,}125 \text{ rad} = 7{,}16^\circ$$

$$v_{xk}^- = v_{xk-1} + a_{xk} \cdot \Delta t = 0{,}5 \ \frac{\text{m}}{\text{s}}$$

$$v_{yk}^- = v_{yk-1} + a_{yk} \cdot \Delta t = 0{,}005 \ \frac{\text{m}}{\text{s}}$$

$$\vec{x}_k^- = \left(0{,}125 \text{ m},\ 0{,}00125 \text{ m},\ 0{,}125 \text{ rad},\ 0{,}5 \ \frac{\text{m}}{\text{s}},\ 0{,}005 \ \frac{\text{m}}{\text{s}}\right)^T$$

$$\Sigma_k^- = G_k \cdot \underbrace{\Sigma_{k-1}}^{=0} G_k^T + \Sigma_{dk} = \Sigma_{dk}$$

$$= \begin{pmatrix} \frac{1}{4} \cdot \sigma_{ax}^2 \cdot (\Delta t)^4 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} \sigma_{ay}^2 \cdot (\Delta t)^4 & 0 & 0 & 0 \\ 0 & 0 & \sigma_\omega^2 \cdot (\Delta t)^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{ax}^2 (\Delta t)^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{ay} (\Delta t)^2 \end{pmatrix}$$

e) No. Due to the non-linearity introduced by the rotation matrix $R(\varphi)$ in the state transition model, the model has to be linearized in every prediktion step.
Due to this approximation an optimal state estimation can't be guaranteed anymore.

f) Since the IMU measurements are received with a frequency 4 times higher than the position measurements, this means the EKF will perform 4 prediction steps before performing 1 correction step.