

Robotic 3D Vision

Lecture 8: Visual Odometry 3 – Direct Methods

WS 2020/21

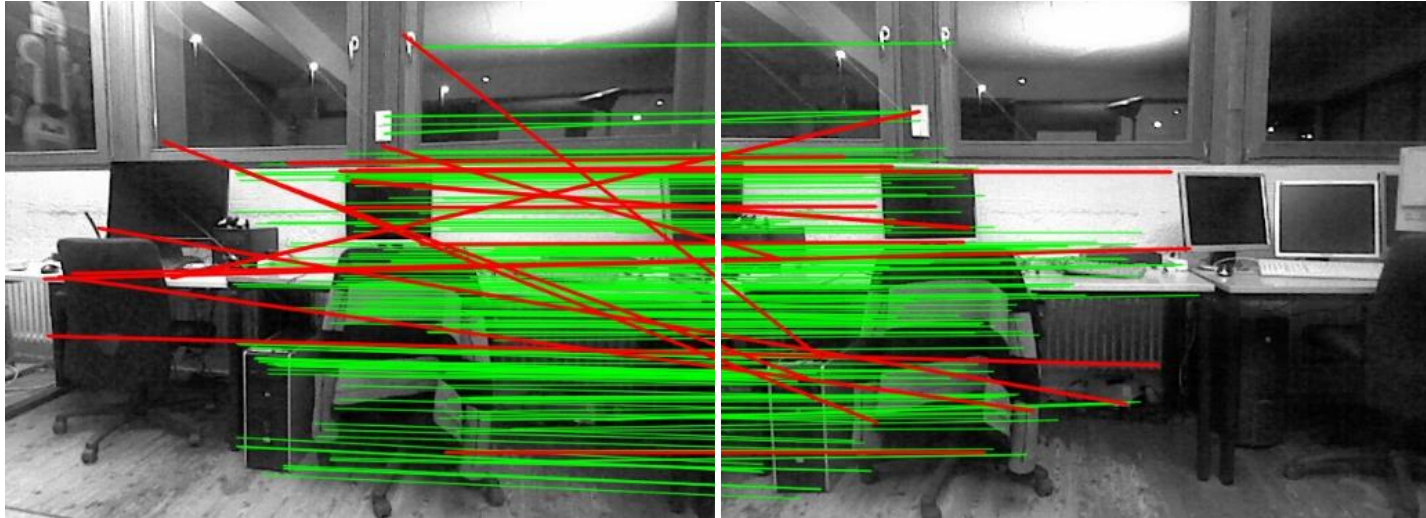
Dr. Niclas Zeller

Artisense GmbH

What We Will Cover Today

- RANSAC (leftover from last lecture)
- Direct visual odometry methods
 - Principles of direct image alignment
 - Photometric alignment
 - Geometric alignment
- Direct visual odometry for RGB-D cameras
- Direct visual odometry for monocular cameras
 - Semi-dense monocular odometry
- Photometric calibration
-
- Stereo extensions

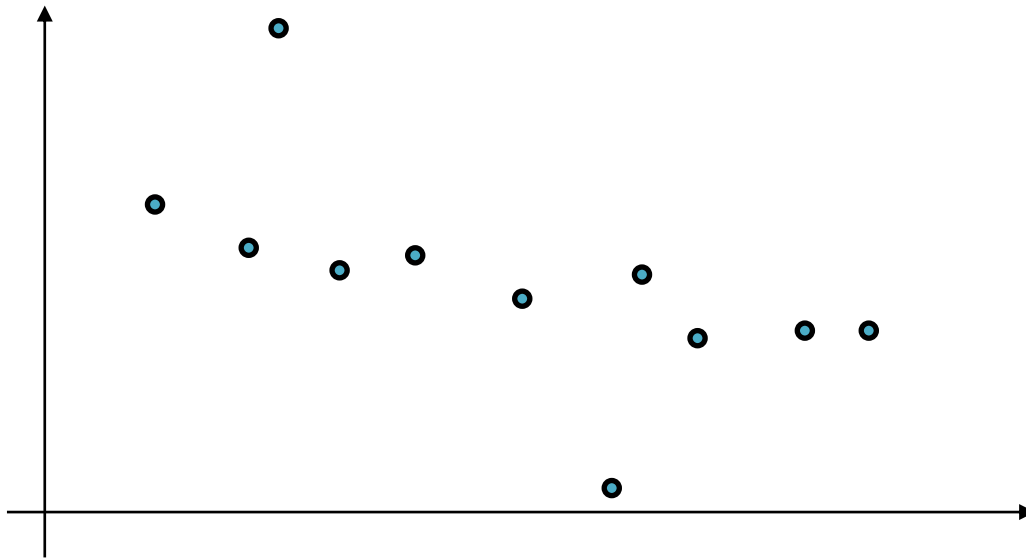
Recap: Keypoint Matching



- Only accept matches with distance smaller a threshold
- What else can we do?

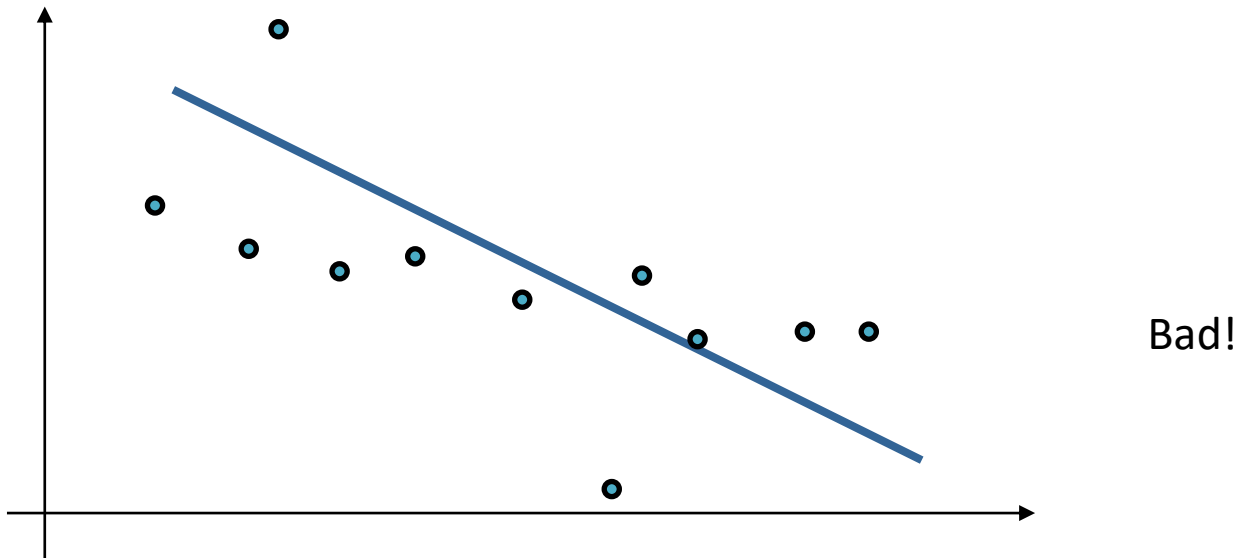
Random Sample Consensus (RANSAC)

- Model fitting in presence of noise and outliers
- Example: fitting a line through 2D points



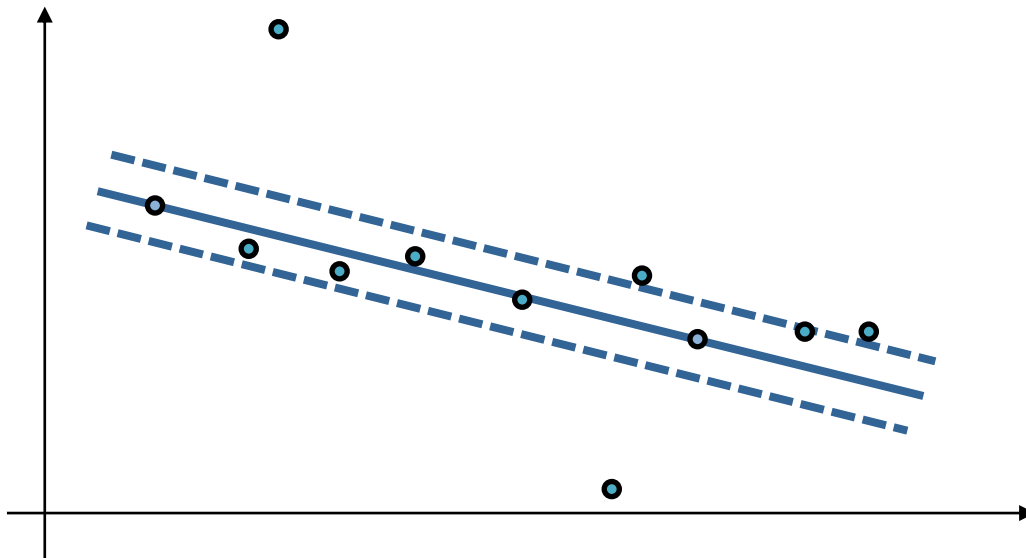
RANSAC

- Least-squares solution, assuming constant noise for all points



RANSAC

- We only need 2 points to fit a line. Let's try 2 random points



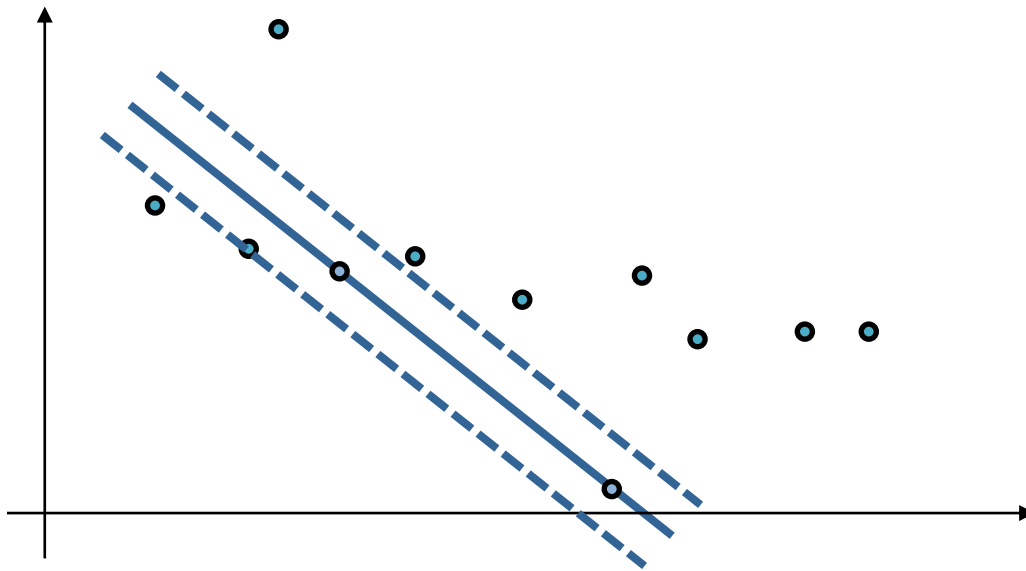
Quite ok

7 inliers

4 outliers

RANSAC

- Let's try 2 other random points



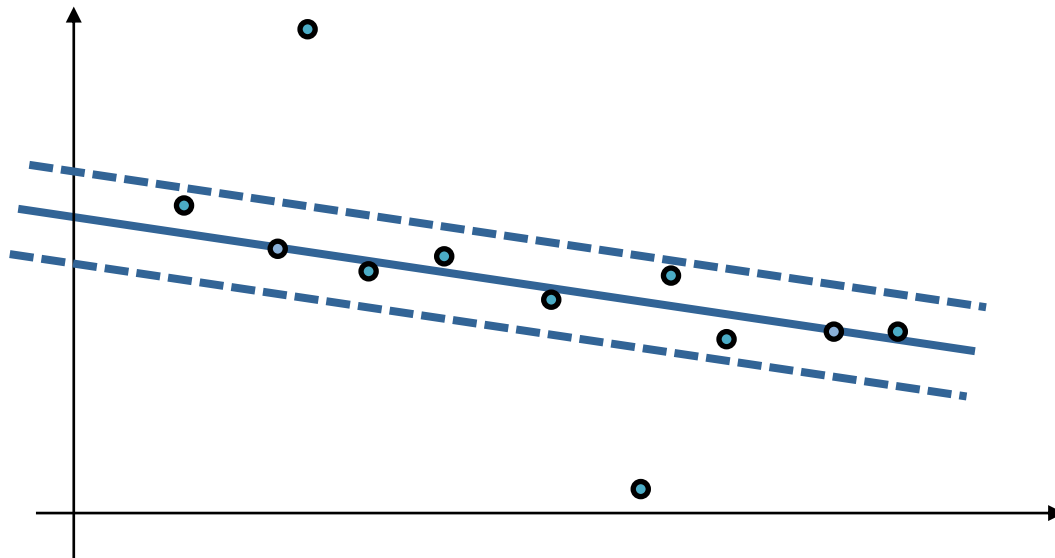
Quite bad

3 inliers

8 outliers

RANSAC

- Let's try yet another 2 random points

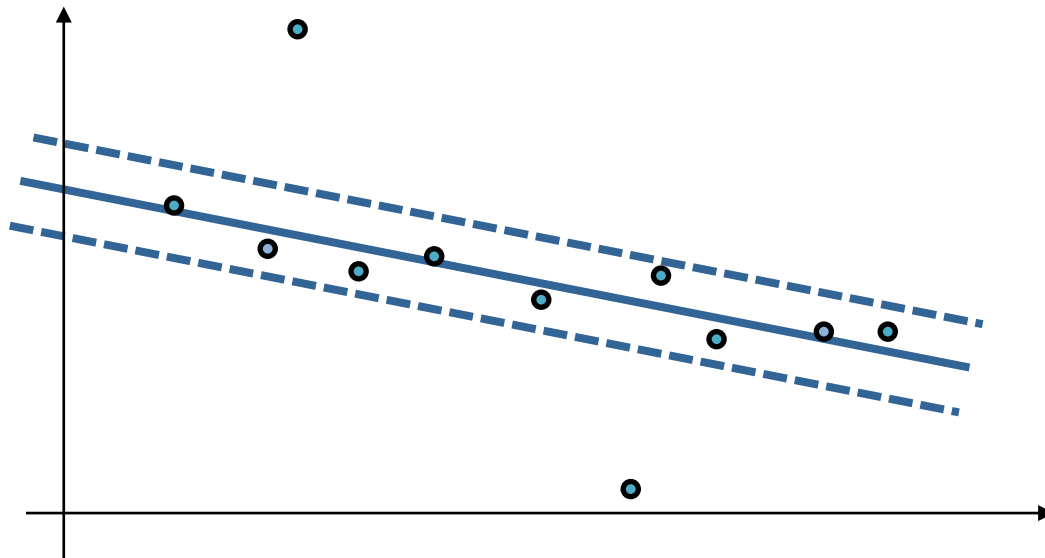


Quite good!

9 inliers
2 outliers

RANSAC

- Let's use the inliers of the best trial so far to perform least squares fitting



Even better!

RANSAC

- How many iteration do we need to find the optimal solution
 - p - probability of finding the correct solution
 - ϵ - outlier ration $\rightarrow w = 1 - \epsilon$ (inlier ratio)
 - s - number of data points required to calculate solution
 - N - number of iterations

Probability of picking at least one outlier

$$1 - p = (1 - w^s)^N = (1 - (1 - \epsilon)^s)^N$$

Probability of not a single correct solution

Probability of picking s inliers

$$N \geq \frac{\log(1 - p)}{\log(1 - (1 - \epsilon)^s)}$$

RANSAC Algorithm

- RANdOm SAmpLe Consensus algorithm formalizes this idea
- Algorithm:

Input: data D , s required #data points for fitting, success probability p , outlier ratio ϵ

Output: inlier set

1. Compute required number of iterations $N \geq \frac{\log(1-p)}{\log(1-(1-\epsilon)^s)}$
2. For N iterations do:
 1. Randomly select a subset of s data points
 2. Fit model on the subset
 3. Count inliers and keep model/subset with largest number of inliers
3. Refit model using found inlier set

RANSAC

N for $p = 0.99$

	Required points s	Outlier ratio ϵ						
		10%	20%	30%	40%	50%	60%	70%
Line	2	3	5	7	11	17	27	49
Plane	3	4	7	11	19	35	70	169
Essential matrix	8	9	26	78	272	1177	7025	70188

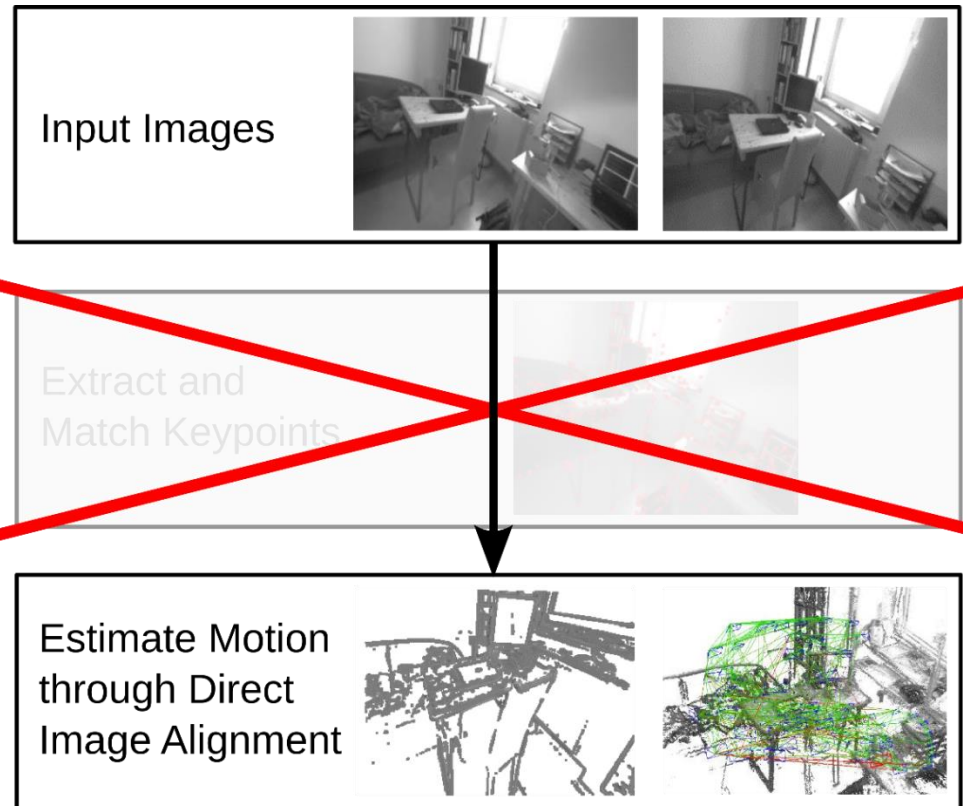
Direct Visual Odometry Pipeline

- Avoid manually designed keypoint detection and matching
- Instead: direct image alignment

$$E(\xi) = \int_{\mathbf{y} \in \Omega} |I_1(\mathbf{y}) - I_2(\omega(\mathbf{y}, \xi))| d\mathbf{y}$$

$$E(\xi) = \sum_i |I_1(\mathbf{y}_i) - I_2(\omega(\mathbf{y}_i, \xi))|$$

- Warping requires depth
 - RGB-D
 - Fixed-baseline stereo
 - Temporal stereo, tracking and (local) mapping



Direct Visual Odometry Example (RGB-D)

Robust Odometry Estimation for RGB-D Cameras

Christian Kerl, Jürgen Sturm, Daniel Cremers



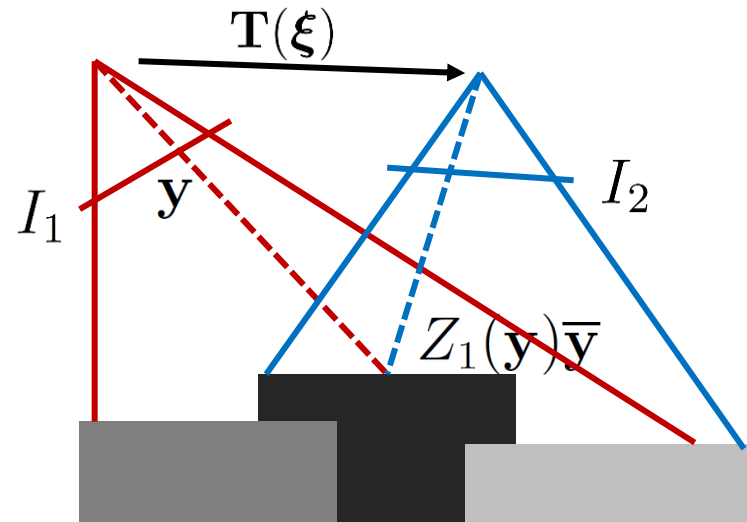
Computer Vision and Pattern Recognition Group
Department of Computer Science
Technical University of Munich



(Kerl, Sturm, Cremers, ICRA 2013)

<https://www.youtube.com/watch?v=TMqPwoCCmto>

Direct Image Alignment Principle



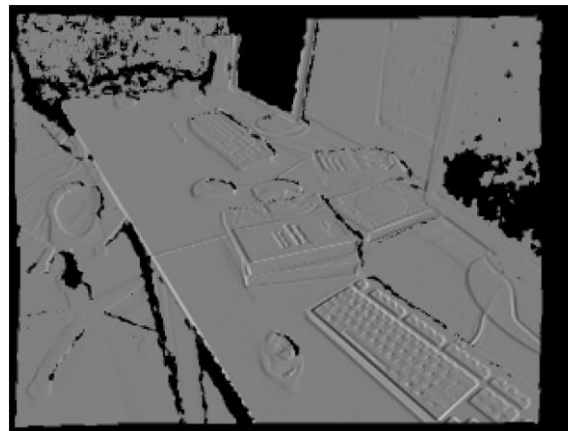
- If we know pixel depth, we can synthesize an image from a different view point
- Ideally, the intensities of the synthesized warped image are the same as from the real one

$$I_1(\mathbf{y}) = I_2(\pi(\mathbf{T}(\boldsymbol{\xi})Z_1(\mathbf{y})\bar{\mathbf{y}}))$$

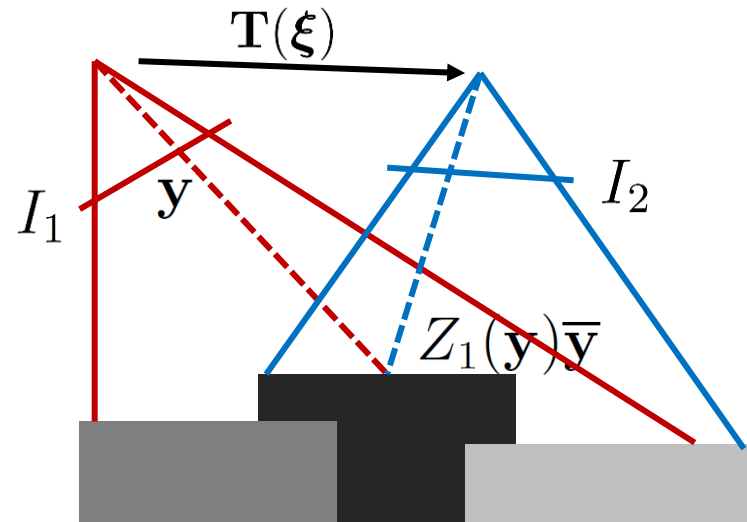
Derivative of Image Warp



Images from Kerl et al., ICRA 2013



Direct RGB-D Image Alignment



- RGB-D cameras measure depth, we only need to estimate camera motion!
- In addition to the **photometric error**

$$I_1(\mathbf{y}) = I_2(\pi(\mathbf{T}(\xi)Z_1(\mathbf{y})\bar{\mathbf{y}}))$$

we can measure **geometric error** directly

$$[\mathbf{T}(\xi)Z_1(\mathbf{y})\bar{\mathbf{y}}]_z = Z_2(\pi(\mathbf{T}(\xi)Z_1(\mathbf{y})\bar{\mathbf{y}}))$$

Probabilistic Direct Image Alignment

- Measurements are affected by noise

$$I_1(\mathbf{y}) = I_2(\pi(\mathbf{T}(\boldsymbol{\xi})Z_1(\mathbf{y})\bar{\mathbf{y}})) + \epsilon$$

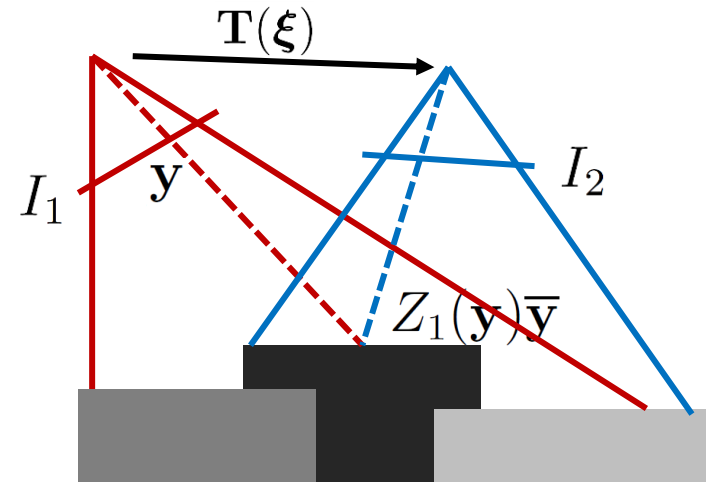
- A convenient assumption is Gaussian noise

$$\epsilon \sim \mathcal{N}(0, \sigma_I^2)$$

- If we further assume that noise of pixel intensities is stochastically independent across the image, we can formulate the a-posteriori probability

$$p(\boldsymbol{\xi} \mid I_1, I_2) \propto p(I_1 \mid \boldsymbol{\xi}, I_2)p(\boldsymbol{\xi})$$

$$\propto p(\boldsymbol{\xi}) \prod_{\mathbf{y} \in \Omega} \mathcal{N}(I_1(\mathbf{y}) - I_2(\pi(\mathbf{T}(\boldsymbol{\xi})Z_1(\mathbf{y})\bar{\mathbf{y}})); 0, \sigma_I^2)$$



Optimization Approach

- Optimize negative log-likelihood
 - Product of exponentials becomes a summation over quadratic terms
 - Normalizers are independent of the pose
 - We ignore the pose prior $p(\xi)$

$$E(\xi) = \sum_{\mathbf{y} \in \Omega} \frac{r(\mathbf{y}, \xi)^2}{\sigma_I^2} \quad , \text{ stacked residuals:} \quad E(\xi) = \mathbf{r}(\xi)^\top \mathbf{W} \mathbf{r}(\xi)$$

$$r(\mathbf{y}, \xi) = I_1(\mathbf{y}) - I_2(\pi(\mathbf{T}(\xi) Z_1(\mathbf{y}) \bar{\mathbf{y}}))$$

- Non-linear least squares problem can be efficiently optimized using standard optimization tools (Gauss-Newton, Levenberg-Marquardt)

Recap: Gauss-Newton Method

- Approximate Newton's method to minimize $E(\mathbf{x})$
 - Approximate $E(\mathbf{x})$ through linearization of residuals

$$\begin{aligned}\tilde{E}(\mathbf{x}) &= \frac{1}{2} \tilde{\mathbf{r}}(\mathbf{x})^\top \mathbf{W} \tilde{\mathbf{r}}(\mathbf{x}) \\ &= \frac{1}{2} (\mathbf{r}(\mathbf{x}_k) + \mathbf{J}_k (\mathbf{x} - \mathbf{x}_k))^\top \mathbf{W} (\mathbf{r}(\mathbf{x}_k) + \mathbf{J}_k (\mathbf{x} - \mathbf{x}_k)) \quad \mathbf{J}_k := \nabla_{\mathbf{x}} \mathbf{r}(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_k} \\ &= \frac{1}{2} \mathbf{r}(\mathbf{x}_k)^\top \mathbf{W} \mathbf{r}(\mathbf{x}_k) + \underbrace{\mathbf{r}(\mathbf{x}_k)^\top \mathbf{W} \mathbf{J}_k}_{=: \mathbf{b}_k^\top} (\mathbf{x} - \mathbf{x}_k) + \frac{1}{2} (\mathbf{x} - \mathbf{x}_k)^\top \underbrace{\mathbf{J}_k^\top \mathbf{W} \mathbf{J}_k}_{=: \mathbf{H}_k} (\mathbf{x} - \mathbf{x}_k)\end{aligned}$$

- Find root of $\nabla_{\mathbf{x}} \tilde{E}(\mathbf{x}) = \mathbf{b}_k^\top + (\mathbf{x} - \mathbf{x}_k)^\top \mathbf{H}_k$ using Newton's method, i.e.

$$\nabla_{\mathbf{x}} \tilde{E}(\mathbf{x}) = \mathbf{0} \text{ iff } \mathbf{x} = \mathbf{x}_k - \mathbf{H}_k^{-1} \mathbf{b}_k$$

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \mathbf{H}_k^{-1} \mathbf{b}_k$$

Recap: Levenberg-Marquardt Method

- Gradually transition between gradient descent and Gauss-Newton
 - Augment Hessian approximation of Gauss-Newton (damping)

$$\mathbf{x}_{k+1} = \mathbf{x}_k - (\mathbf{H}_k + \lambda \mathbf{I})^{-1} \mathbf{b}_k$$

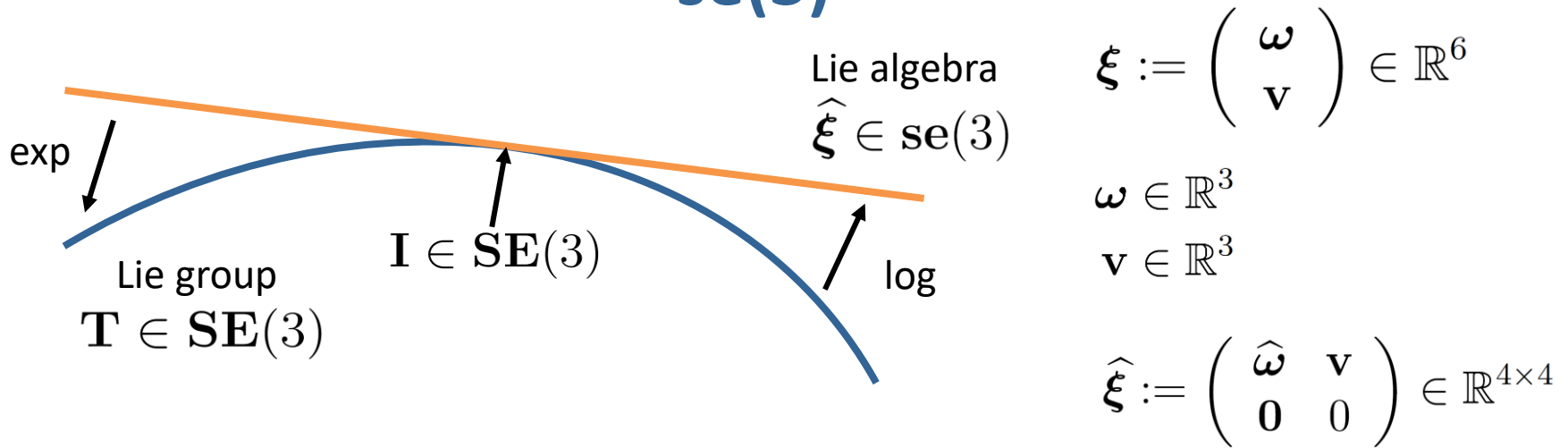
- Adaptive weighting: $\mathbf{x}_{k+1} = \mathbf{x}_k - (\mathbf{H}_k + \lambda \text{diag}(\mathbf{H}_k))^{-1} \mathbf{b}_k$
- Start with $\lambda = 0.1$
- Accept step and decrease lambda $\lambda \leftarrow \lambda/2$ if error function decreases, otherwise discard step and increase lambda $\lambda \leftarrow 2\lambda$ (akin line search)

Pose Parametrization for Optimization

- Requirements on pose parametrization
 - No singularities
 - Minimal to avoid constraints
- Various pose parametrizations available
 - Direct matrix representation => not minimal
 - Quaternion / translation => not minimal
 - Euler angles / translation => singularities (gimbal lock)
 - **Twist coordinates** of elements in Lie Algebra $se(3)$ of $SE(3)$ (axis-angle / translation)

Recap: Representing Motion using Lie Algebra

$se(3)$



- $\mathbf{SE}(3)$ is a smooth manifold, i.e. a Lie group
- Its Lie algebra $se(3)$ provides an elegant way to parametrize poses for optimization
- Its elements $\hat{\xi} \in se(3)$ form the **tangent space** of $\mathbf{SE}(3)$ at identity
- The $se(3)$ elements can be interpreted as rotational and translational velocities (**twists**)

Optimization with Twist Coordinates

- Twists provide a minimal local representation without singularities
- We can decompose transformations in each optimization step into the transformation itself and an infinitesimal increment

$$\mathbf{T}(\xi) = \exp(\widehat{\delta\xi}) \mathbf{T}(\xi) = \mathbf{T}(\delta\xi \oplus \xi) \quad \text{But!} \quad \mathbf{T}(\delta\xi + \xi) \neq \mathbf{T}(\delta\xi)\mathbf{T}(\xi)$$

- We perform optimization with respect to auxiliary variable $\delta\xi$
 - Example: Gradient descent on the auxiliary variable

$$\delta\xi^* = -\eta \nabla_{\delta\xi} E(\xi_k, \delta\xi)$$

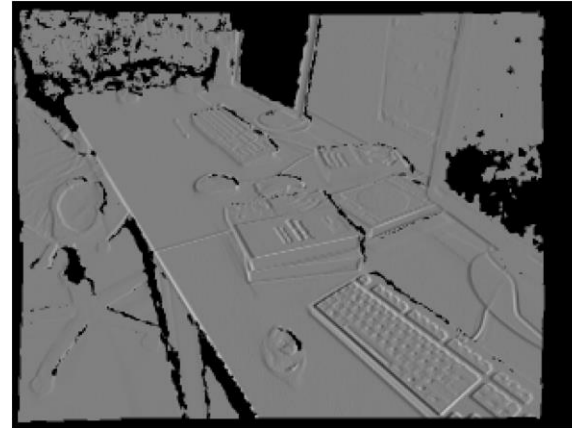
$$\mathbf{T}(\xi_{k+1}) = \exp(\widehat{\delta\xi^*}) \mathbf{T}(\xi_k)$$

- Similar for Gauss-Newton: calculate Jacobian of $\mathbf{r}(\xi_k, \delta\xi)$ with respect to $\delta\xi$
- Make sure the increment is applied from the correct side

Properties of Residual Linearization



$$|I_1 - I_2|$$



$$\frac{\partial I_2}{\partial v_x}$$

$$r(\mathbf{y}, \xi) = I_1(\mathbf{y}) - I_2(\omega(\mathbf{y}, \xi)) \quad \text{with} \quad \omega(\mathbf{y}, \xi) := \pi(\mathbf{T}(\xi)Z_1(\mathbf{y})\bar{\mathbf{y}})$$

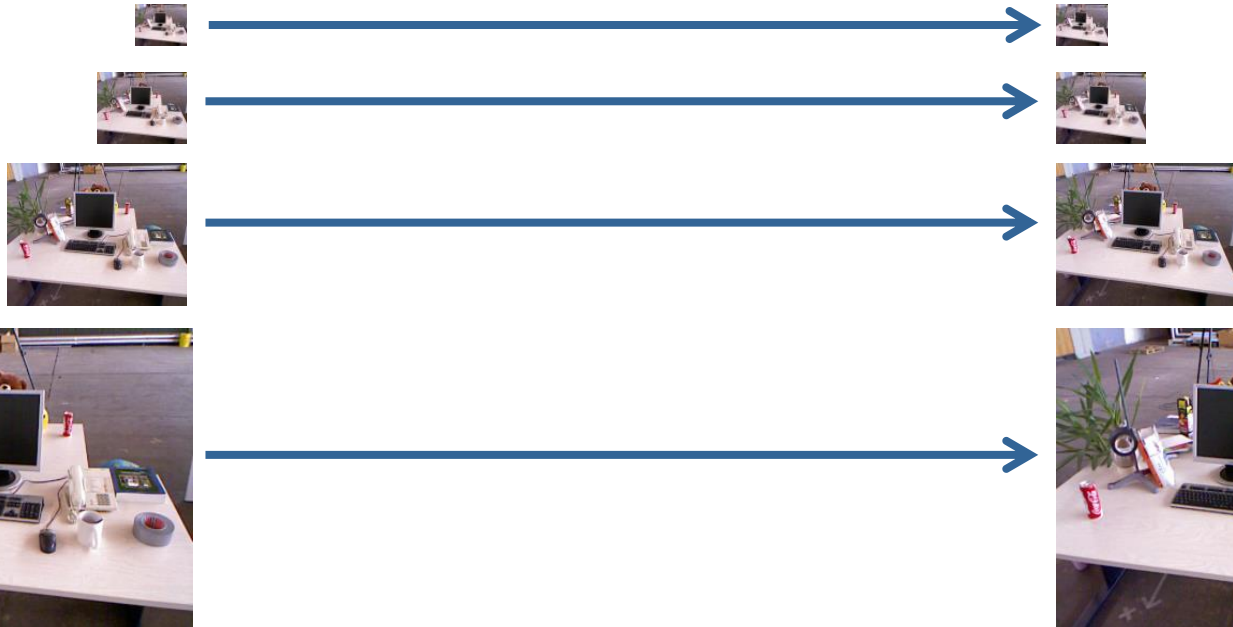
- Gradient of residuals w.r.t. pose

$$\nabla_{\xi} r(\mathbf{y}, \xi) = -\nabla_{\omega} I_2(\omega(\mathbf{y}, \xi)) \nabla_{\xi} \omega(\mathbf{y}, \xi)$$

- Linearization is only valid for motions that change the projection in a small image neighborhood that is captured by the local gradient
- $E(\xi)$ is far from being a convex function (many local minima)

Coarse-To-Fine Optimization

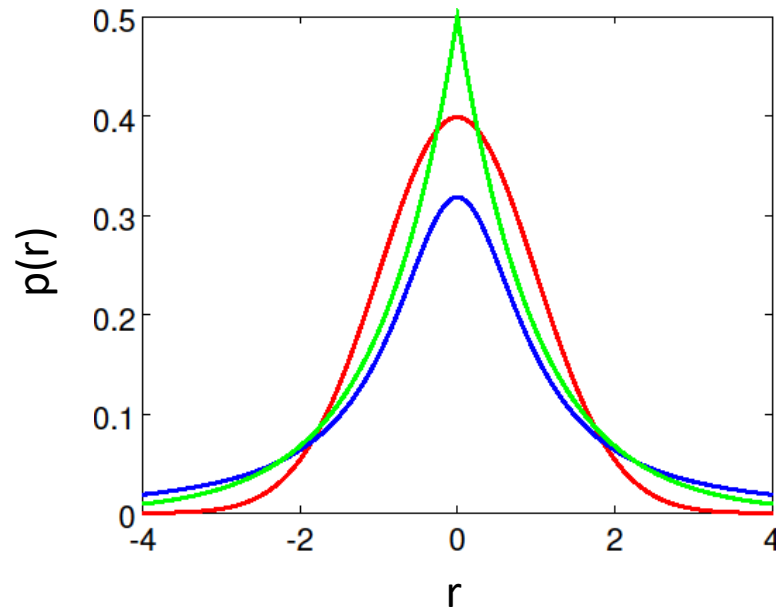
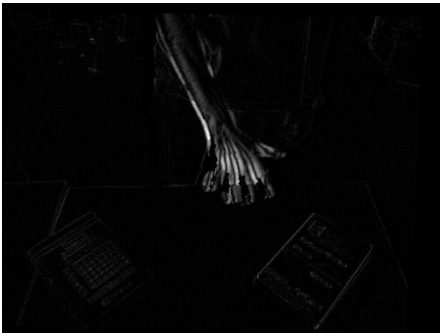
coarse motion



fine motion

- Important: smooth image during downscaling
 - E.g. average over four neighboring pixels

Residual Distributions

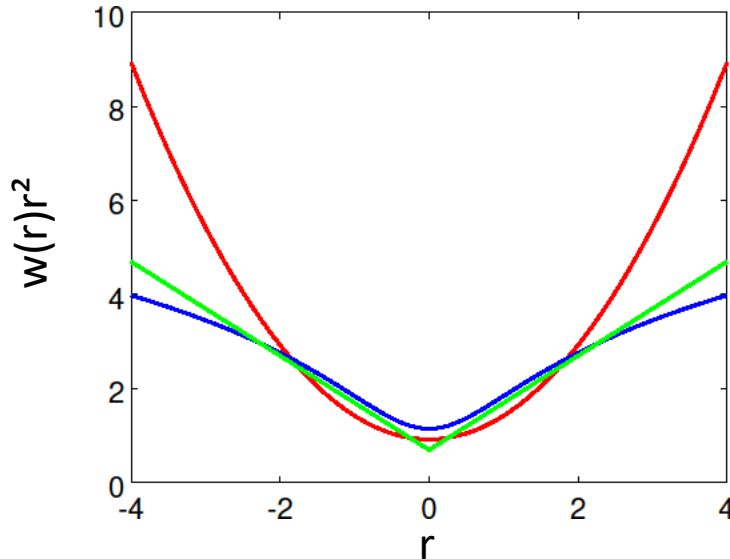


- Normal distribution
- Laplace distribution
- Student-t distribution

- Gaussian noise assumption on photometric residuals oversimplifies
- Outliers (occlusions, motion, etc.):
 - Residuals are distributed with more mass on the larger values

Images from Kerl et al., ICRA 2013

Optimizing Non-Gaussian Measurement Noise



- Normal distribution
- Laplace distribution
- Student-t distribution

- Can we change the residual distribution in least squares optimization?
- For specific types of distributions: yes!
- Iteratively reweighted least squares: Reweight residuals in each iteration

$$E(\xi) = \sum_{\mathbf{y} \in \Omega} w(r(\mathbf{y}, \xi)) \frac{r(\mathbf{y}, \xi)^2}{\sigma_I^2}$$

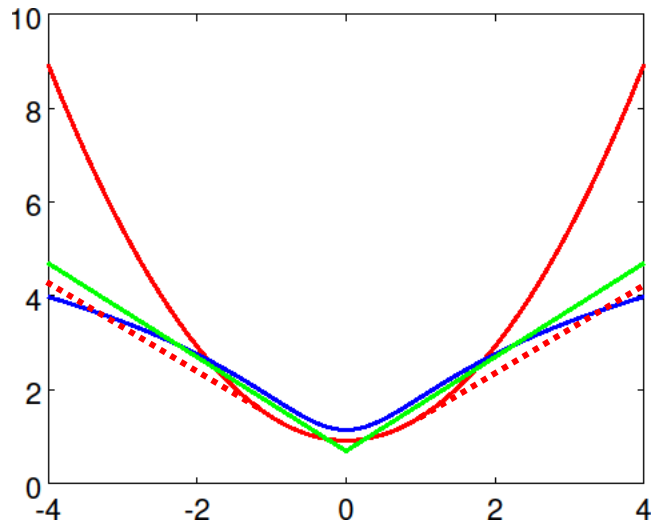
Laplace distribution:

$$w(r(\mathbf{y}, \xi)) = |r(\mathbf{y}, \xi)|^{-1}$$

Huber Loss

- Huber-loss „switches“ between Gaussian (locally at mean) and Laplace distribution

$$\|r\|_{\delta} = \begin{cases} 0.5r^2 & \text{for } |r| \leq \delta \\ \delta(|r| - 0.5\delta) & \text{otherwise} \end{cases}$$



- Normal distribution
- Laplace distribution
- Student-t distribution
- Huber-loss for $\delta = 1$

Efficient Non-Linear Least Squares

- Gauss-Newton / Levenberg-Marquardt can be applied very efficiently to direct image alignment:
 - \mathbf{H}_k is only a 6x6 matrix
 - $\mathbf{b}_k = \mathbf{J}_k^T \mathbf{W} \mathbf{r}(\xi_k)$ is a 6x1 vector
 - Since we treat each pixel stochastically independent from neighboring pixels, \mathbf{H}_k and \mathbf{b}_k are summed over individual pixels

$$\mathbf{H}_k = \sum_{\mathbf{y} \in \Omega} \frac{w(\mathbf{y}, \xi_k)}{\sigma_I^2} \mathbf{J}_{k,\mathbf{y}}^T \mathbf{J}_{k,\mathbf{y}} \quad \mathbf{b}_k = \sum_{\mathbf{y} \in \Omega} \frac{w(\mathbf{y}, \xi_k)}{\sigma_I^2} \mathbf{J}_{k,\mathbf{y}}^T r(\mathbf{y}, \xi_k)$$

$$\mathbf{J}_{k,\mathbf{y}} := \nabla_{\delta \xi} r(\mathbf{y}, \delta \xi \oplus \xi_k)$$

Algorithm: Direct RGB-D Visual Odometry

Input: RGB-D image sequence $I_{0:t}, Z_{0:t}$

Output: aggregated camera poses $\mathbf{T}_{0:t}$

Algorithm:

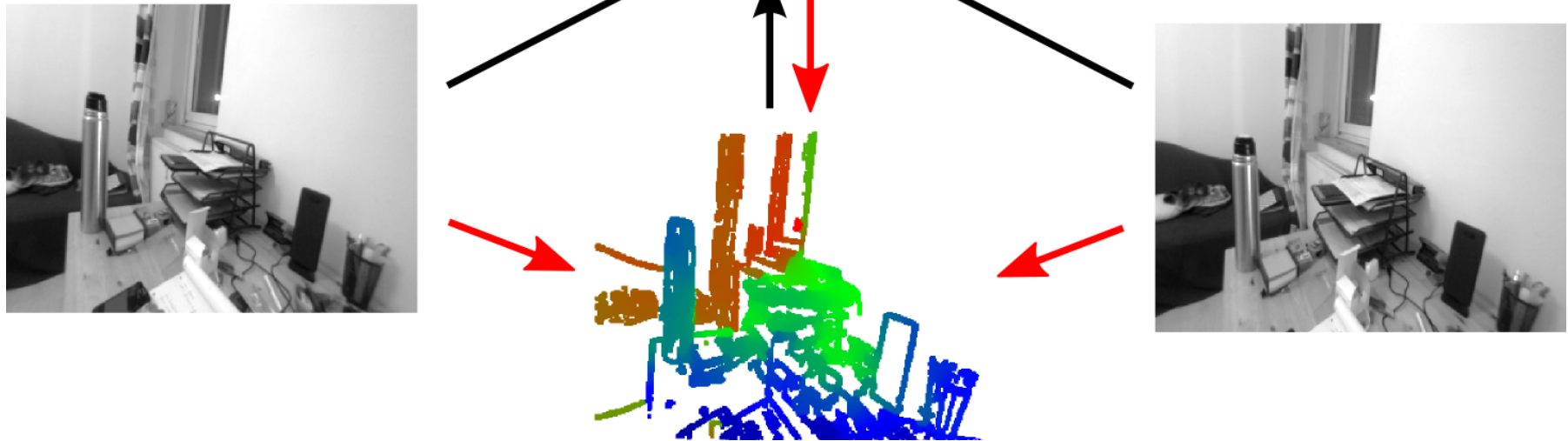
For each current RGB-D image I_k, Z_k :

1. Estimate relative camera motion \mathbf{T}_k^{k-1} towards the previous RGB-D frame using direct image alignment
2. Concatenate estimated camera motion with previous frame camera pose to obtain current camera pose estimate

$$\mathbf{T}_k = \mathbf{T}_{k-1} \mathbf{T}_k^{k-1}$$

Monocular Direct Visual Odometry

- Estimate motion and depth concurrently

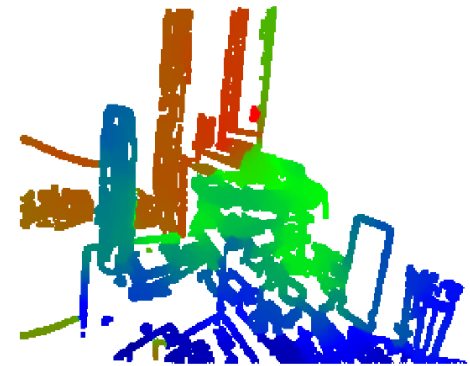


- Alternating optimization: **Tracking** and **Mapping**

Images from: Engel et al., ICCV 2013

Semi-Dense Mapping

- Estimate inverse depth and variance at high gradient pixels
- Correspondence search along epipolar line (5-pixel intensity SSD)

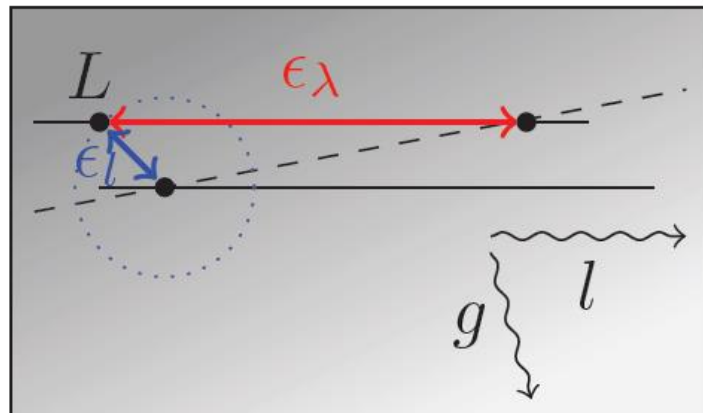
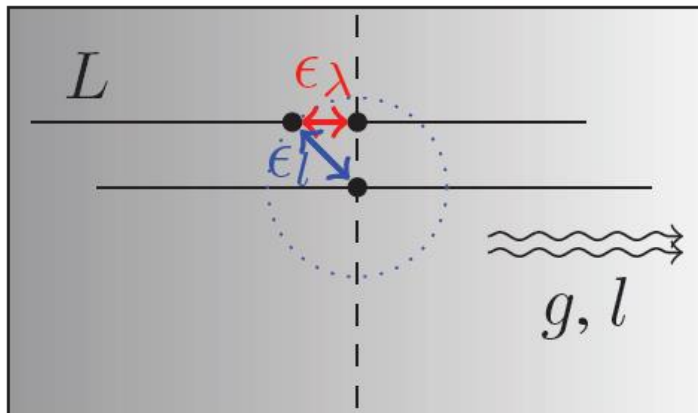


- Kalman-filtering of depth map:
 - Propagate depth map & variance from previous frame
 - Update depth map & variance with new depth observations

Images from: Engel et al., ICCV 2013

Semi-Dense Mapping

- Estimate for inverse depth uncertainty from geometric and intensity noise
 - Very simplified model, but works quite well in reality



Geometric noise

$$\sigma_{\lambda(\xi, \pi)}^2 = \frac{\sigma_l^2}{\langle g, l \rangle^2}$$

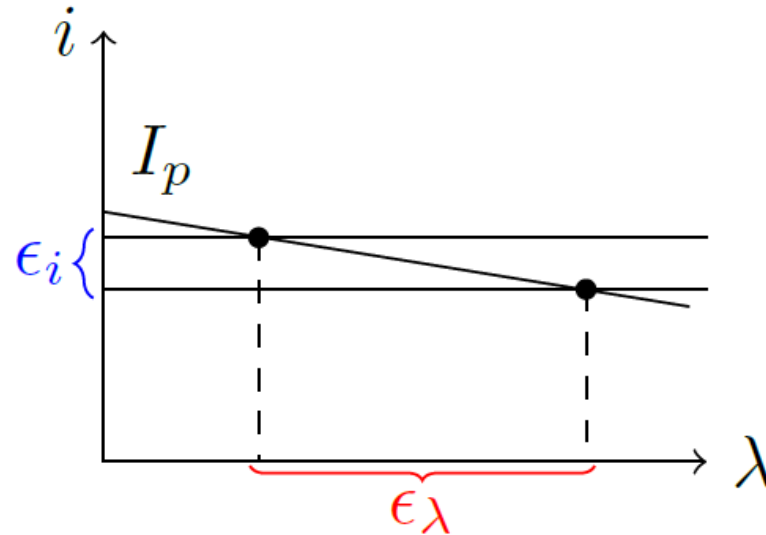
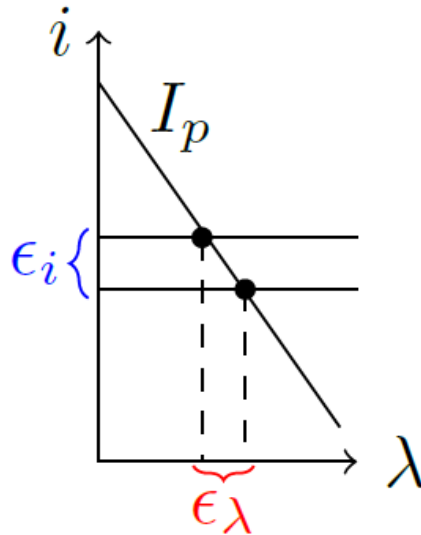
pos. variance of epipolar line
 gradient direction
 epipolar line direction

λ is the estimated disparity
 approx. proportional to
 inverse depth

Images from: Engel et al., ICCV 2013

Semi-Dense Mapping

- Estimate for inverse depth uncertainty from geometric and intensity noise



Intensity noise

$$\sigma_{\lambda(I)}^2 = \frac{2\sigma_i^2}{g_p^2}$$

intensity noise variance

image gradient magnitude at epipolar line

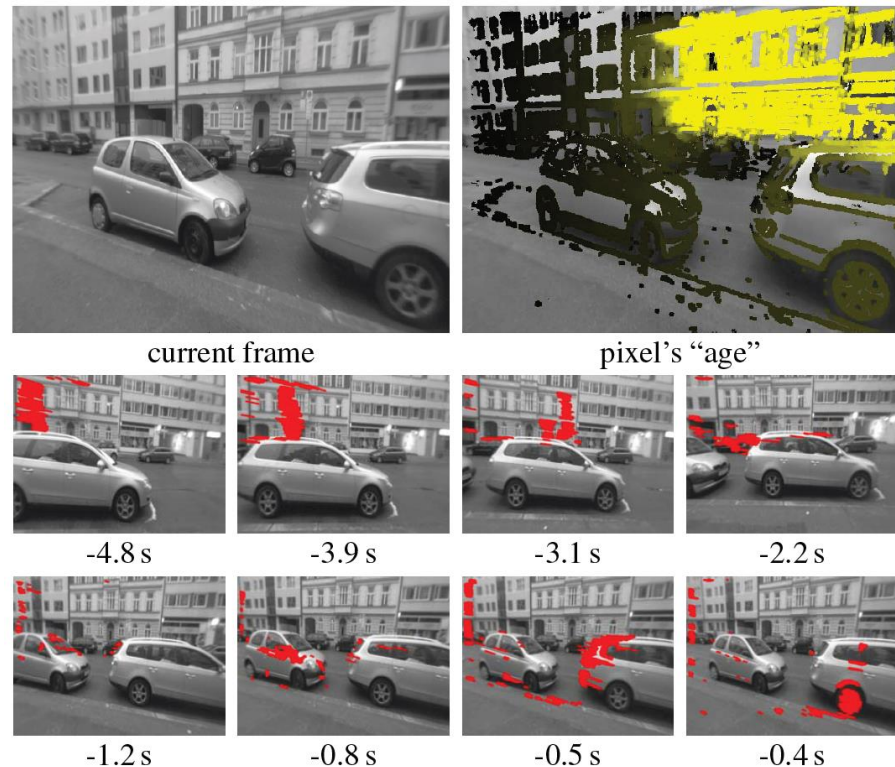
Paper:

https://openaccess.thecvf.com/content_iccv_2013/papers/Engel_Semi-dense_Visual_Odometry_2013_ICCV_paper.pdf

Images from: Engel et al., ICCV 2013

Choosing the Stereo Reference Frame

- Naive: use one specific reference frame (f.e. the previous frame or a keyframe)
- We can also select the reference frame for stereo comparisons for each pixel individually in order to achieve a trade-off between accuracy and computation time



Heuristics from Engel et al., ICCV 2013:
Use oldest frame in which pixel still visible but disparity search range and observation angle below threshold

Images from: Engel et al., ICCV 2013

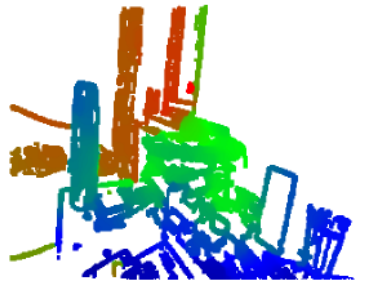
Semi-Dense Direct Image Alignment



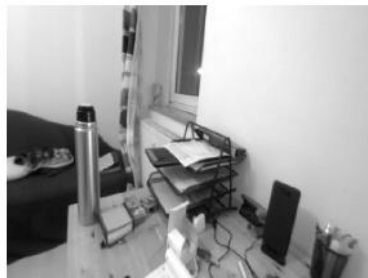
I_1

$$E(\xi) = \sum_{\mathbf{y} \in \Omega^Z} w(r(\mathbf{y}, \xi)) \frac{r(\mathbf{y}, \xi)^2}{\sigma_{Z(\mathbf{y})}^2}$$

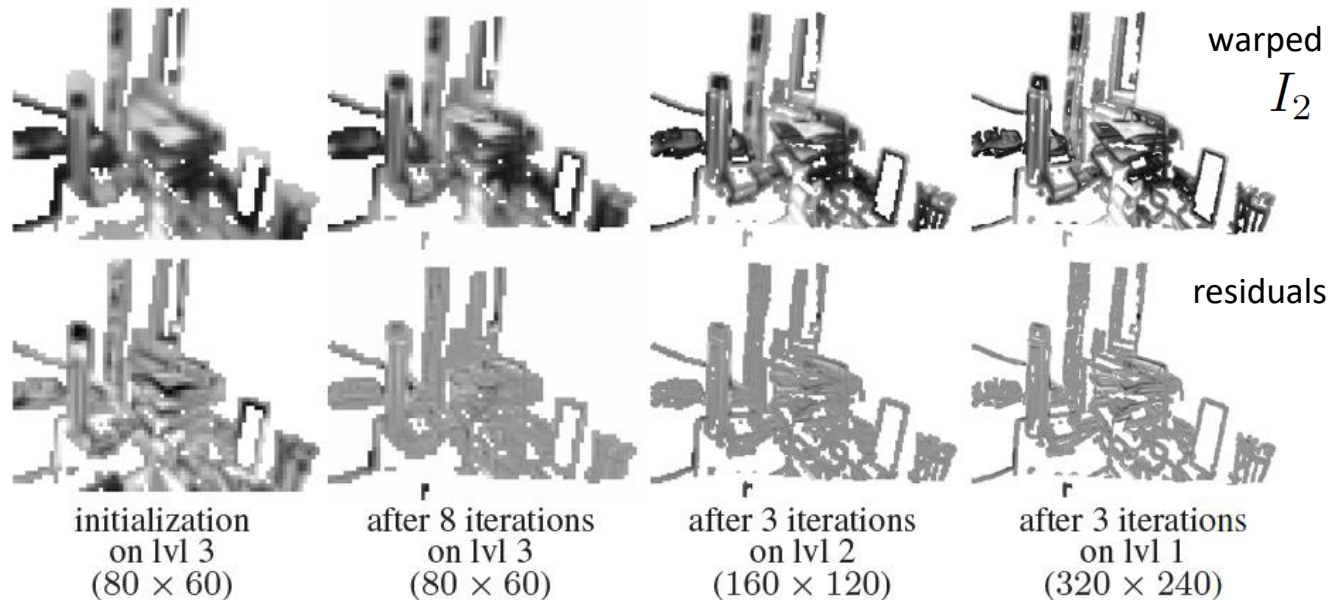
$$r(\mathbf{y}, \xi) = I_1(\mathbf{y}) - I_2(\pi(\mathbf{T}(\xi)Z_1(\mathbf{y})\bar{\mathbf{y}}))$$



Z_1



I_2



Images from: Engel et al., ICCV 2013

Algorithm: Direct Monocular Visual Odometry

Input: Monocular image sequence $I_{0:t}$

Output: aggregated camera poses $\mathbf{T}_{0:t}$

Algorithm:

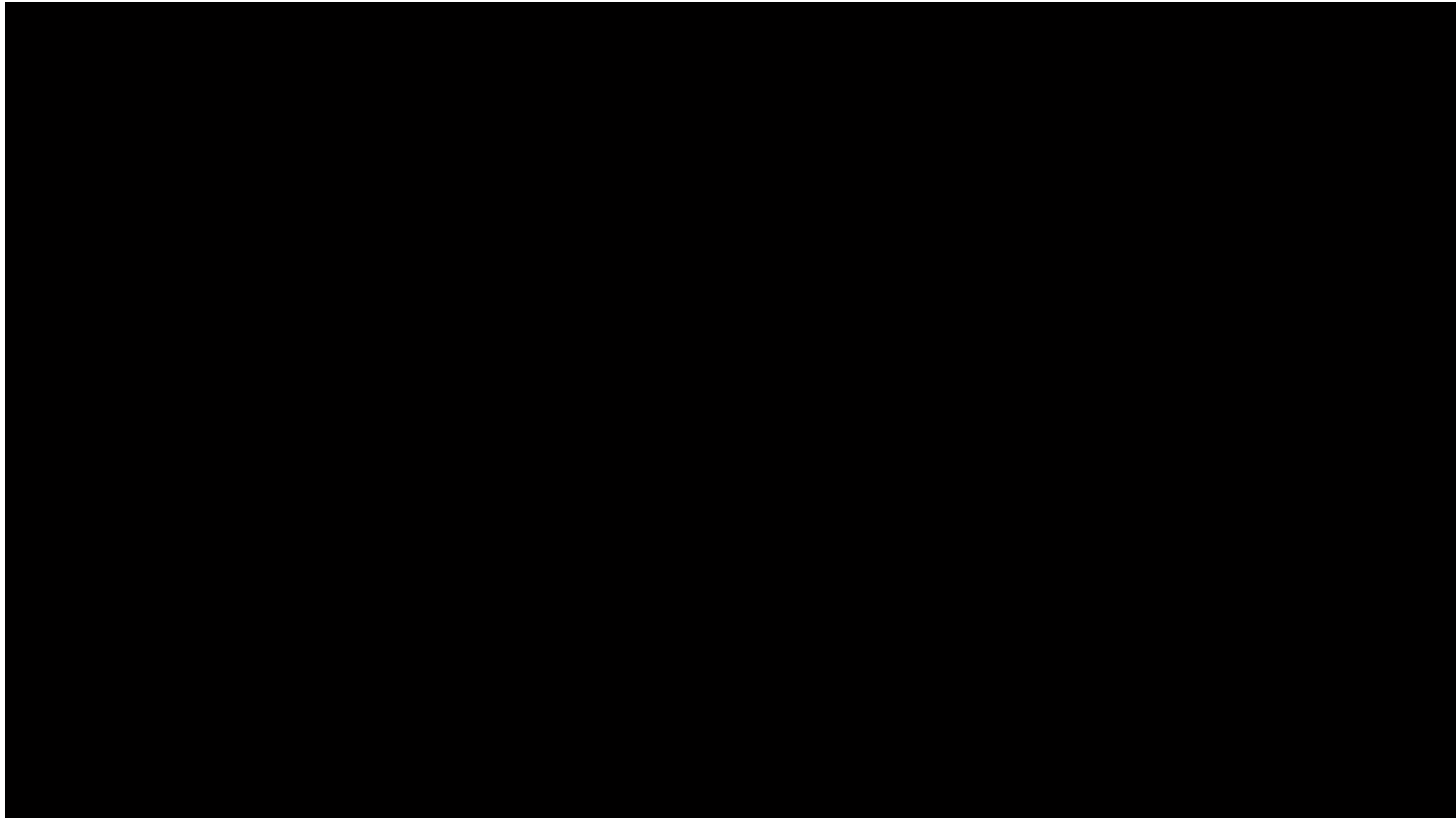
Initialize depth map Z_0

- E.g. from first two frames with a point-based method

For each current image I_k :

1. Estimate relative camera motion \mathbf{T}_k^{k-1} towards the previous image with estimated semi-dense depth map Z_{k-1} using direct image alignment
2. Concatenate estimated camera motion with previous frame camera pose to obtain current camera pose estimate $\mathbf{T}_k = \mathbf{T}_{k-1} \mathbf{T}_k^{k-1}$
3. Propagate semi-dense depth map Z_{k-1} from previous frame to current frame to obtain \tilde{Z}_k
4. Update propagated semi-dense depth map \tilde{Z}_k with temporal stereo depth measurements to obtain Z_k

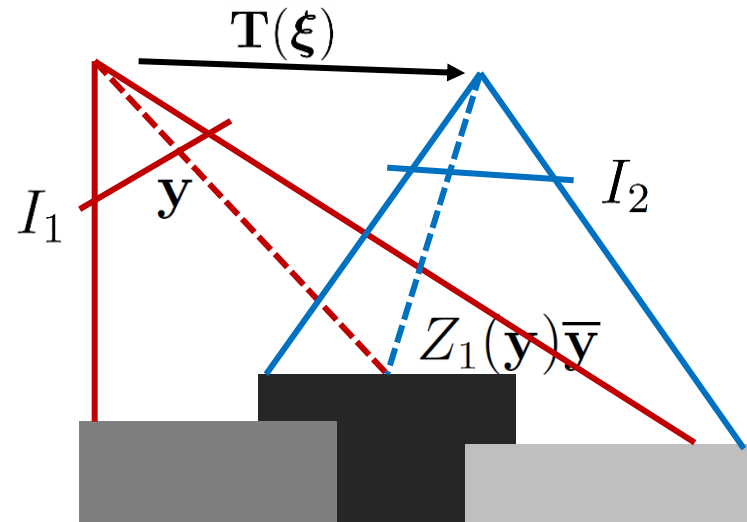
Direct Visual Odometry Example (Mono)



(Engel, Sturm, Cremers, ICCV 2013)

<https://www.youtube.com/watch?v=LZChzEcLNzI>

Direct Image Alignment Revisited



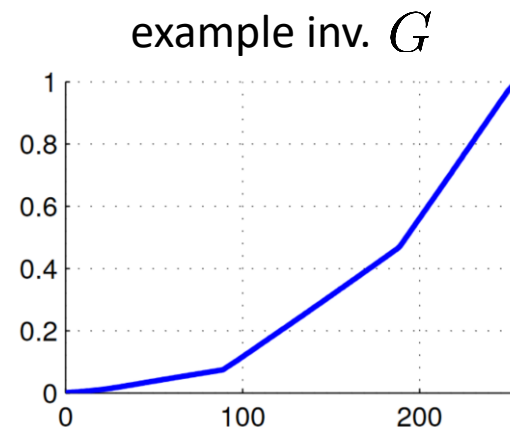
- If we know pixel depth, we can „simulate“ an image from a different view point
- Ideally, the warped image is the same as the image taken from that pose:

$$I_1(\mathbf{y}) = I_2(\pi(\mathbf{T}(\boldsymbol{\xi})Z_1(\mathbf{y})\bar{\mathbf{y}}))$$

- What do we mean with „ideally“ ?

Recap: Camera Response Function

- The objects in the scene radiate light which is focused by the lens onto the image sensor
- The pixels of the sensor observe an irradiance $B : \Omega \rightarrow \mathbb{R}$ for an exposure time t
- The camera electronics translates the accumulated irradiance into intensity values according to a non-linear camera response function $G : \mathbb{R} \rightarrow [0, 255]$
- The measured intensity is $I(\mathbf{x}) = G(tB(\mathbf{x}))$



Recap: Vignetting

uncorrected



corrected

- Lenses gradually focus more light at the center of the image than at the image borders
- The image appears darker towards the borders
- Also called “lens attenuation”
- Lens vignetting can be modelled as a map $V : \Omega \rightarrow [0, 1]$

- Intensity measurement model

$$I(\mathbf{x}) = G(tV(\mathbf{x})B(\mathbf{x}))$$

$V(\mathbf{x})$

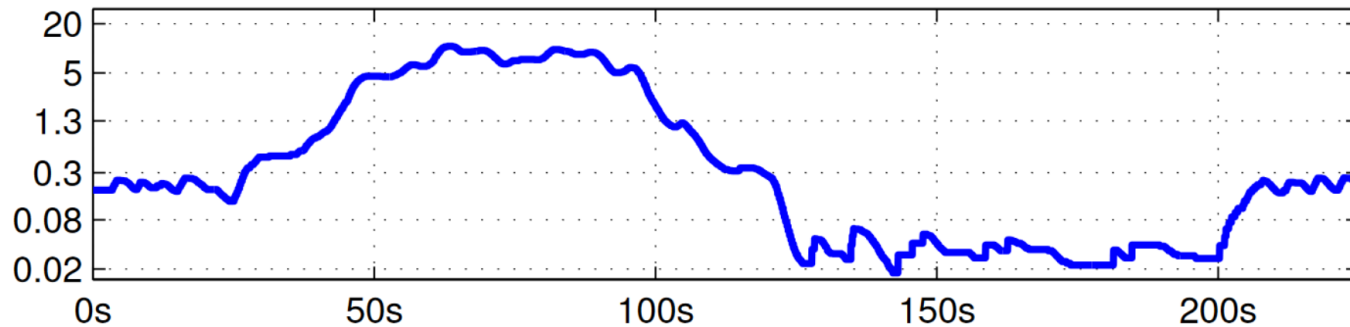


Brightness Constancy Assumption Revisited

- Camera images include vignetting effects and non-linear camera response function
- Idea: invert vignetting and camera response function using a known calibration
- Perform direct image alignment on irradiance images:

$$I'(\mathbf{y}) = tB(\mathbf{y}) = \frac{G^{-1}(I(\mathbf{y}))}{V(\mathbf{y})}$$

Brightness Constancy Assumption Revisited

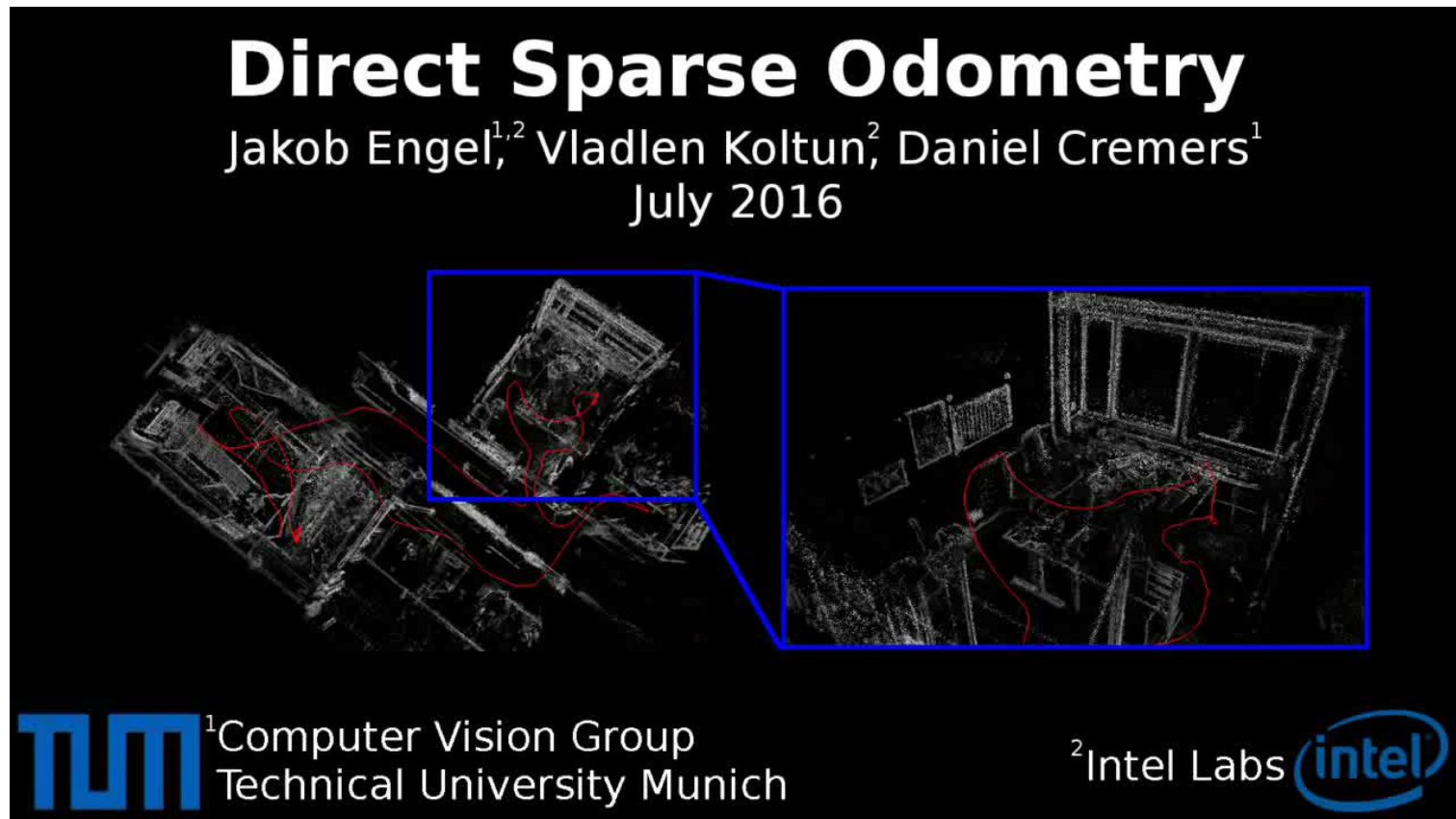


- Automatic exposure adjustment needed in realistic environments
- Add exposure parameters explicitly to objective function:

$$(I_2(\omega(\mathbf{y}, \boldsymbol{\xi}, Z_1(\mathbf{y}))) - b_2) - \frac{t_2 \exp(a_2)}{t_1 \exp(a_1)} (I_1(\mathbf{y}) - b_1)$$

Image: Engel et al. PAMI 2018

Direct Sparse Visual Odometry (Mono)



(Engel, Koltun, Cremers, T-PAMI 2018)

How does the robot move?

<https://www.youtube.com/watch?v=C6-xwSOOdqQ>

Direct Mapping with Stereo Cameras

- For stereo cameras, we can exploit the known camera extrinsics to estimate depth from static stereo (left-right images) in addition to temporal stereo (successive left or right images)



no information from static
stereo

no information from temporal stereo

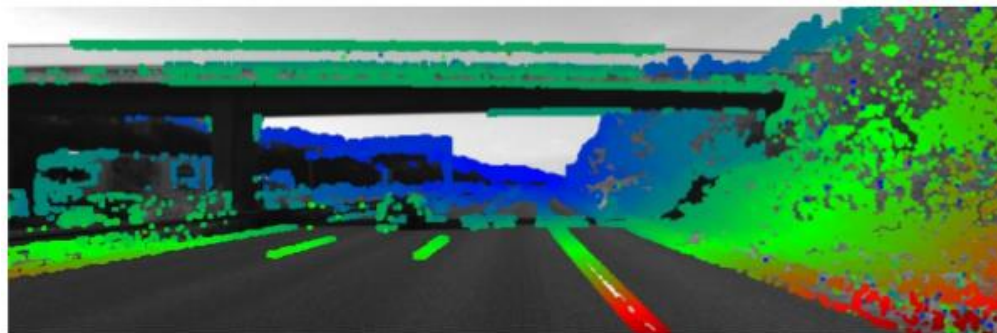



Image from: Engel et al. IROS 2015

Direct Sparse Visual Odometry (Stereo)


Large-Scale Direct Sparse Visual Odometry
with Stereo Cameras

Rui Wang*, Martin Schwörer*, Daniel Cremers
ICCV 2017, Venice



*Equally contributed

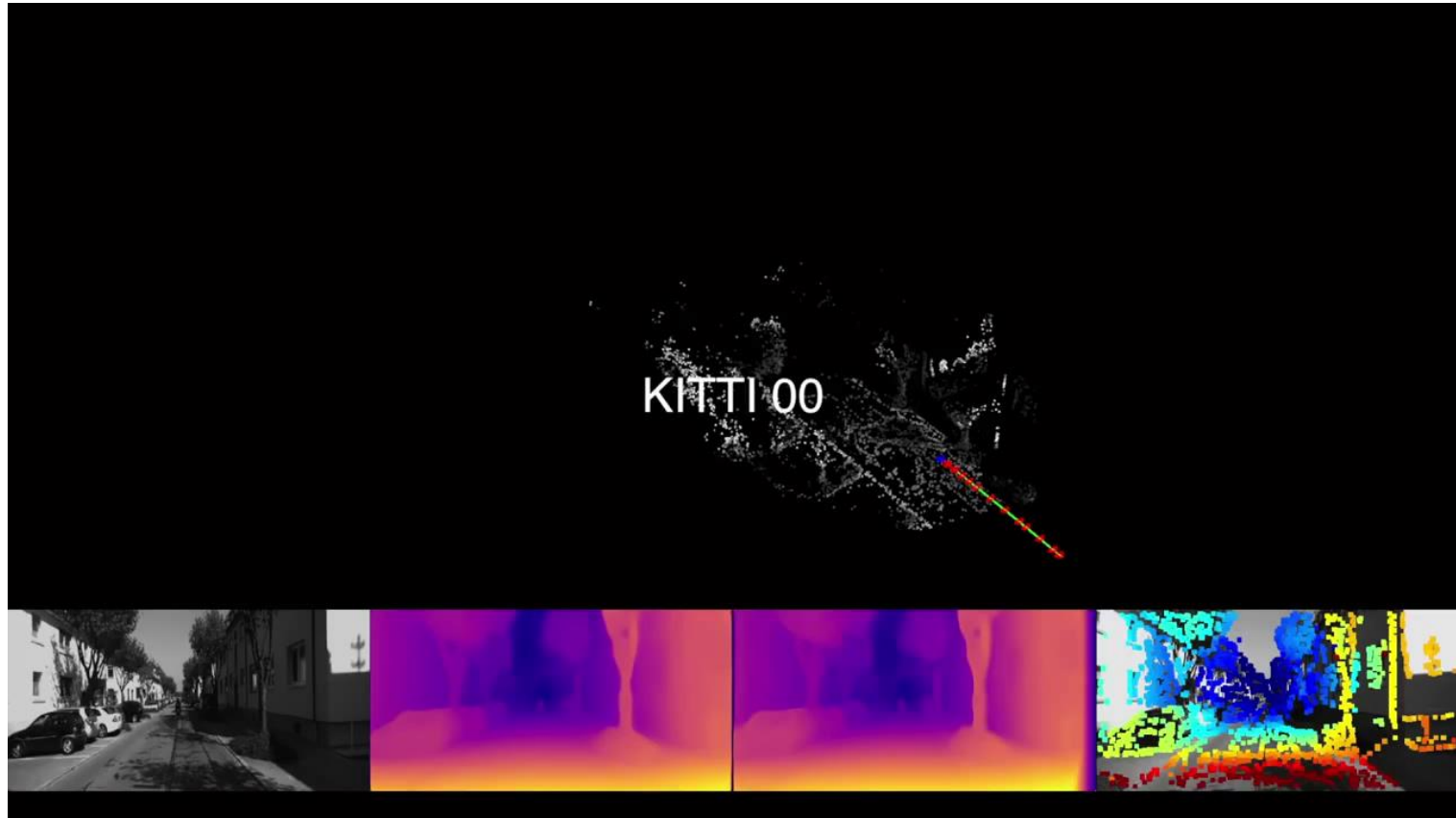
Computer Vision Group
Technical University of Munich



(Wang, Schwörer, Cremers, ICCV 2017)

<https://www.youtube.com/watch?v=A53vJO8eygw>

Deep Direct Sparse VO (Mono)



(Yang, Wang, Stückler, Cremers, ECCV 2018)

https://www.youtube.com/watch?v=sLZOeC9z_tw&t=7s

Lessons Learned Today

- Direct image alignment avoids manually designed keypoints and can use all available image information
- Direct visual odometry
 - Dense RGB-D odometry by direct image alignment with measured depth
 - Direct image alignment for monocular cameras requires depth estimation from temporal stereo
 - Stereo cameras: Direct depth estimation using static and temporal stereo
- Direct image alignment as non-linear least squares problem
 - Linearization of the residuals requires a coarse-to-fine optimization scheme
 - SE(3) Lie algebra provides an elegant way of motion representation for gradient-based optimization
 - Iteratively reweighted least squares allows for wider set of residual distributions than Gaussians
- Photometric calibration and exposure parameter estimation

Thanks for your attention!

Slides Information

- These slides have been initially created by Jörg Stückler as part of the lecture “Robotic 3D Vision” in winter term 2017/18 at Technical University of Munich.
- The slides have been revised by myself (Niclas Zeller) for the same lecture held in winter term 2020/21
- Acknowledgement of all people that contributed images or video material has been tried (please kindly inform me if such an acknowledgement is missing so it can be added).