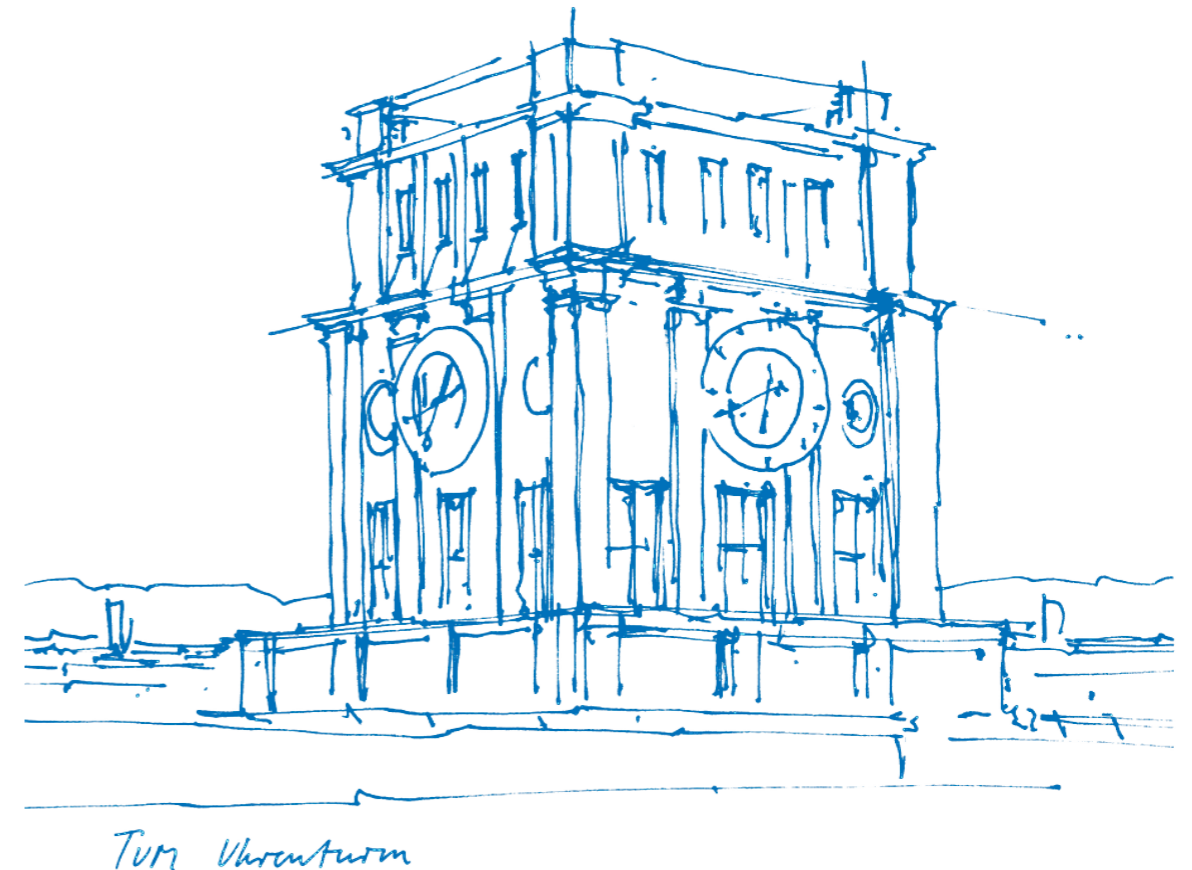


Practical Course: Vision Based Navigation

Lecture 3: Keypoint Detection, Matching and Motion Estimation

Dr. Vladyslav Usenko, Nikolaus Demmel, David Schubert
Prof. Dr. Daniel Cremers



Topics Covered



- Keypoint detection
 - Corner detection
 - Blob detection
 - Rotation estimation
 - Scale selection
- Keypoint description
 - Scale-Invariant Feature Transform (SIFT)
 - Binary Features: BRIEF, ORB
- Keypoint matching
- Robust model fitting with RANSAC
- Epipolar constraint
- Keypoint-based motion estimation
- Place recognition with bag of words

Local Features

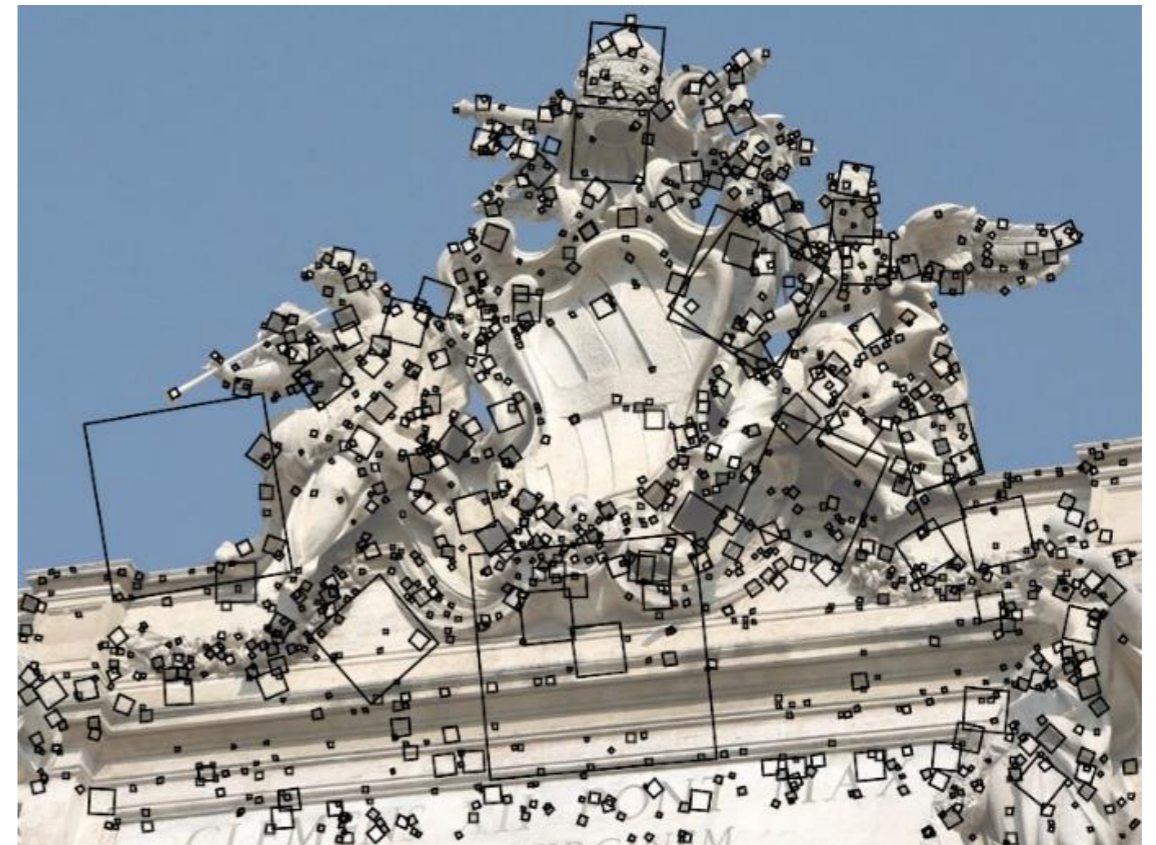
Keypoint Detection

Desirable properties of keypoint detectors for visual SLAM / SfM:

- High repeatability
- Localization accuracy
- Robustness
- Invariance
- Computational efficiency



Harris Corners

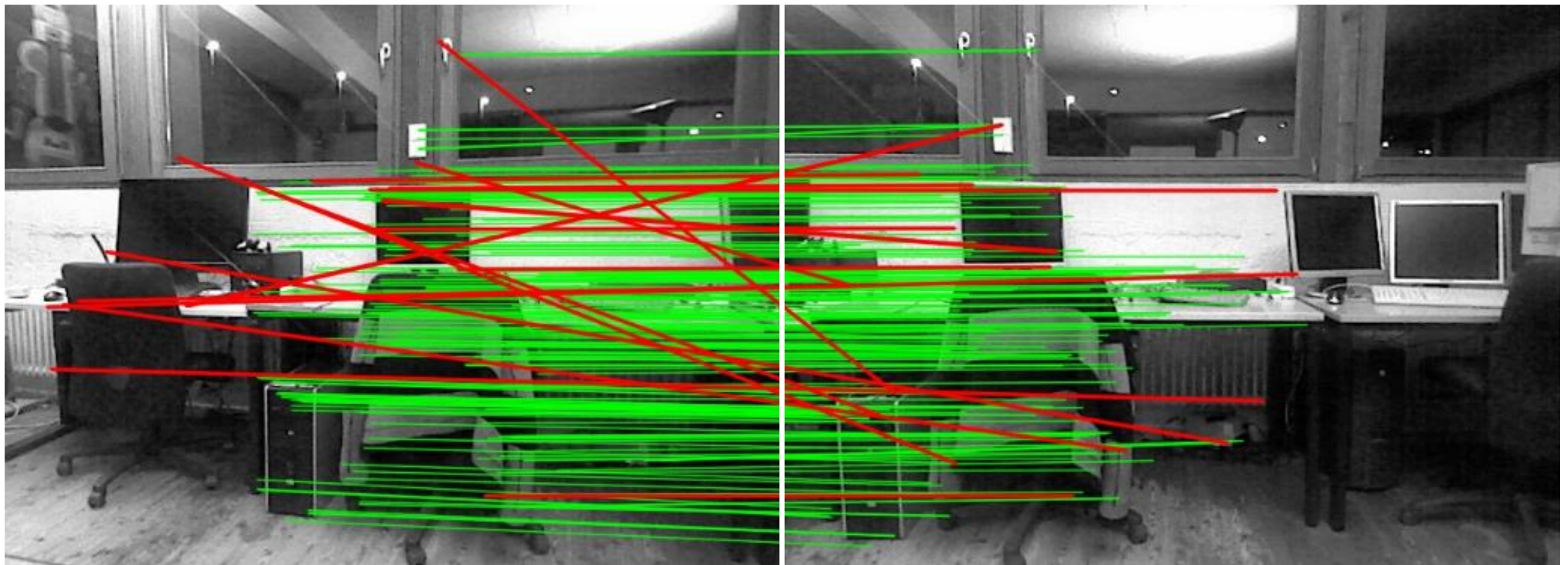


DoG (SIFT) blobs

Keypoint Matching

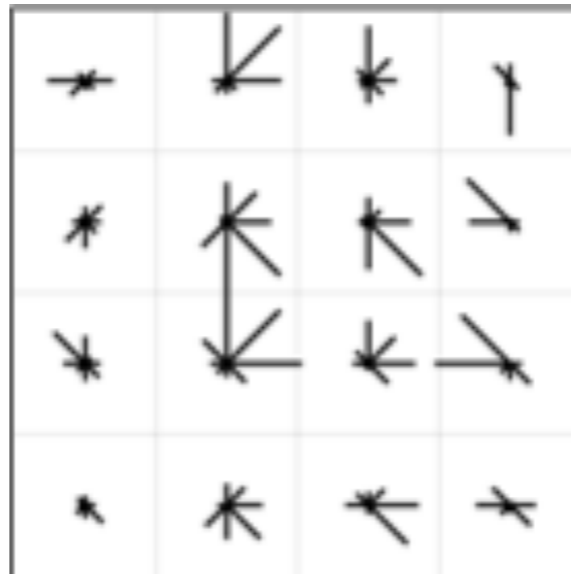
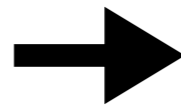
Desirable properties of keypoint matching for visual SLAM / SfM:

- High recall
- Precision
- Robustness
- Computational efficiency
- One possible approach to keypoint matching: by descriptor

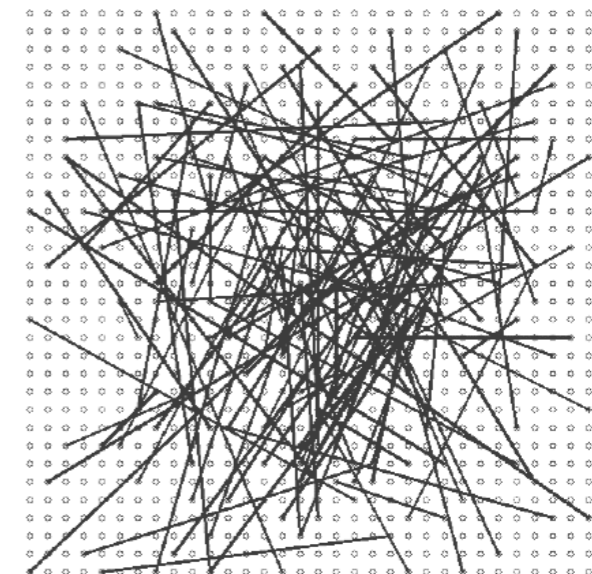


Local Feature Descriptors

- Desirable properties for SLAM / SfM: distinctiveness, robustness, invariance
- Extract signatures that describe local image regions, examples:
 - Histograms over image gradients (SIFT)
 - Histograms over Haar-wavelet responses (SURF)
 - Binary patterns (BRIEF, BRISK, FREAK, etc.)
 - Learning-based descriptors (f.e. Calonder et al., ECCV 2008)
- Rotation-invariance: Align with dominant orientation in local region
- Scale-invariance: Adapt described region extent to keypoint scale

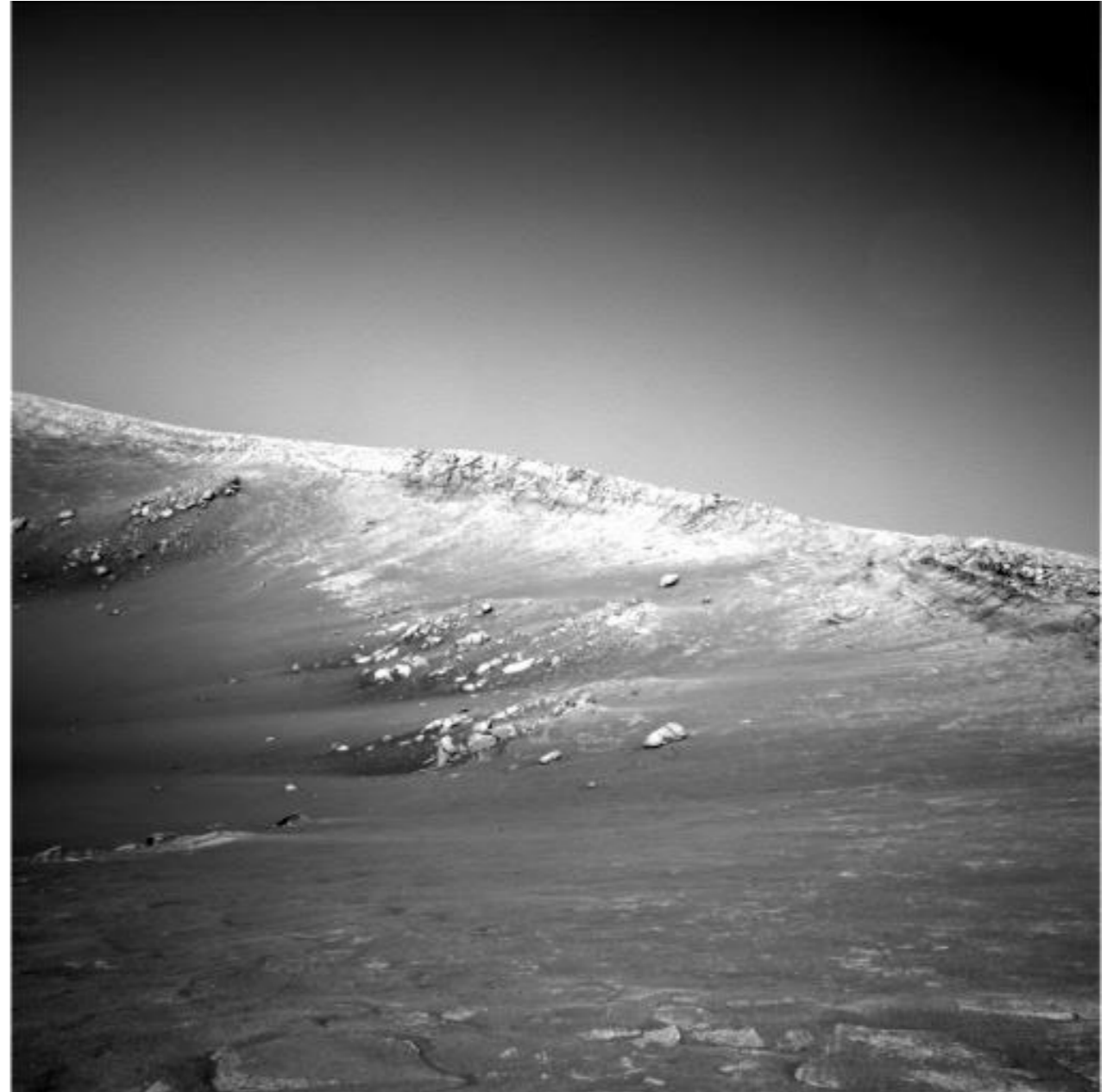


SIFT gradient pooling



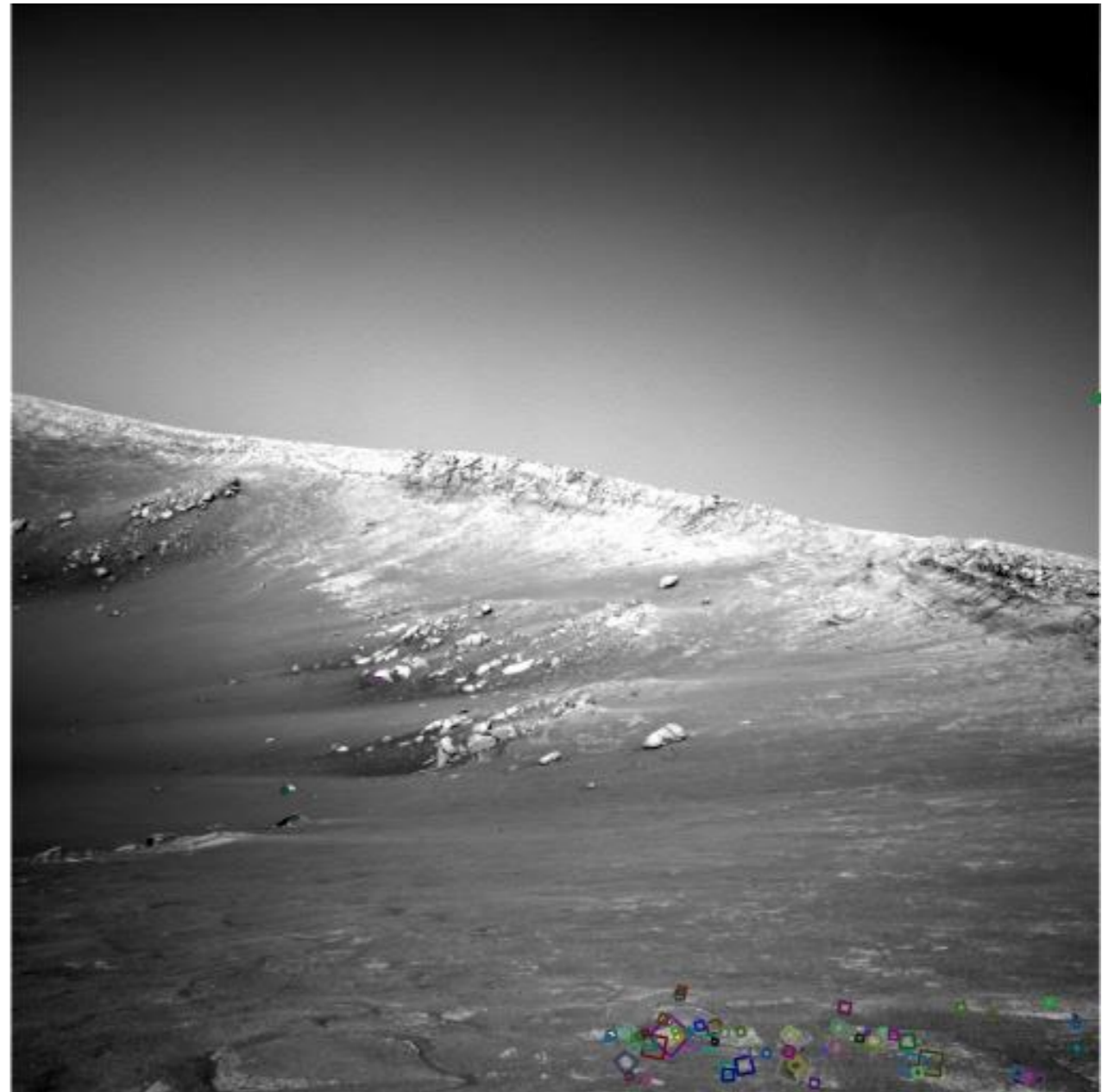
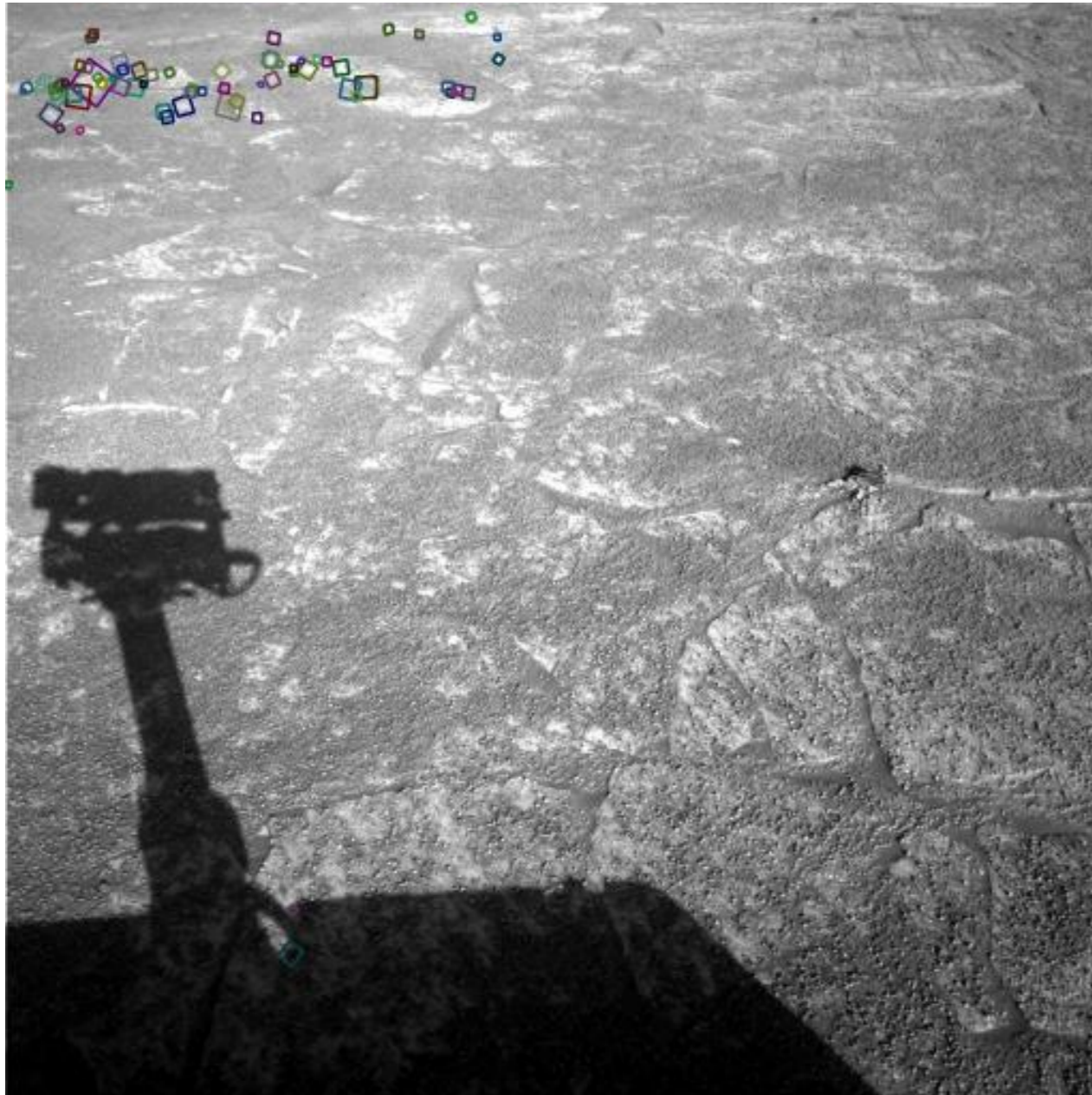
BRIEF binary tests

Image Matching



NASA Mars Rover images

Image Matching

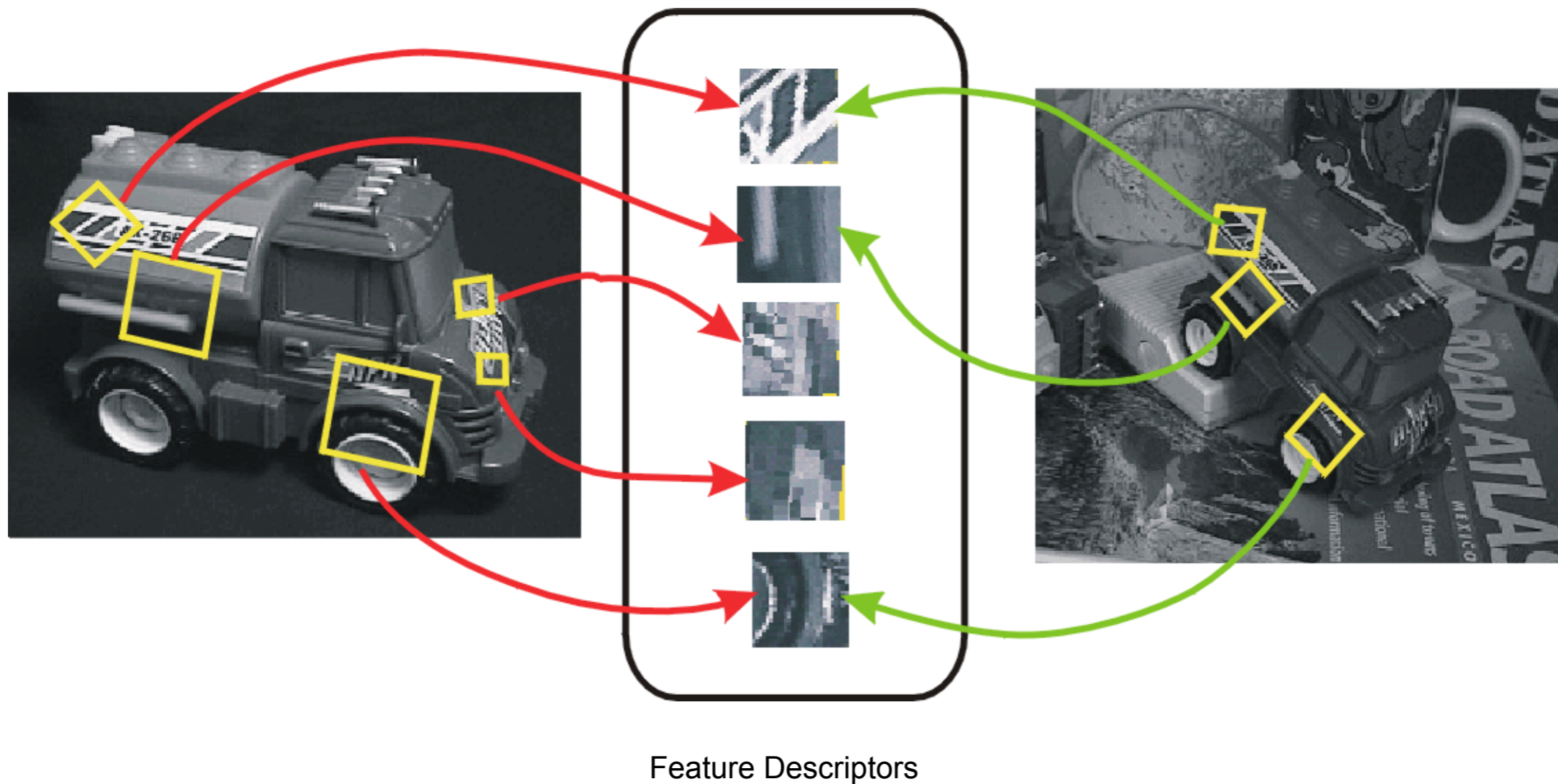


NASA Mars Rover images
with SIFT feature matches

Invariant Local Features

Find features that are invariant to transformations

- geometric invariance: translation, rotation, scale
- photometric invariance: brightness, exposure, ...



Advantages of Local Features

Locality

- features are local, so robust to occlusion and clutter

Distinctiveness

- can differentiate a large database of objects

Quantity

- hundreds or thousands in a single image

Efficiency

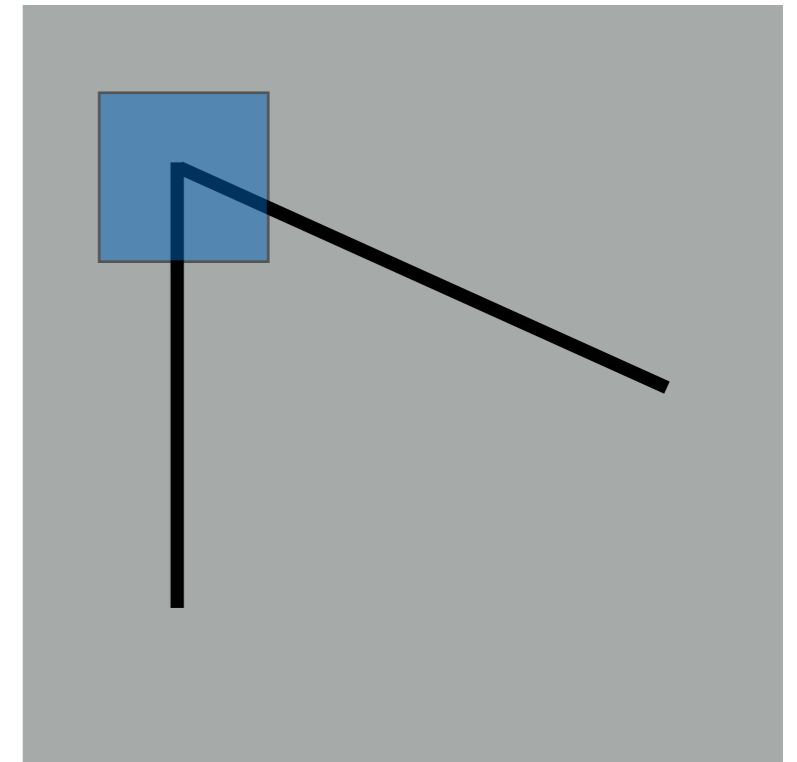
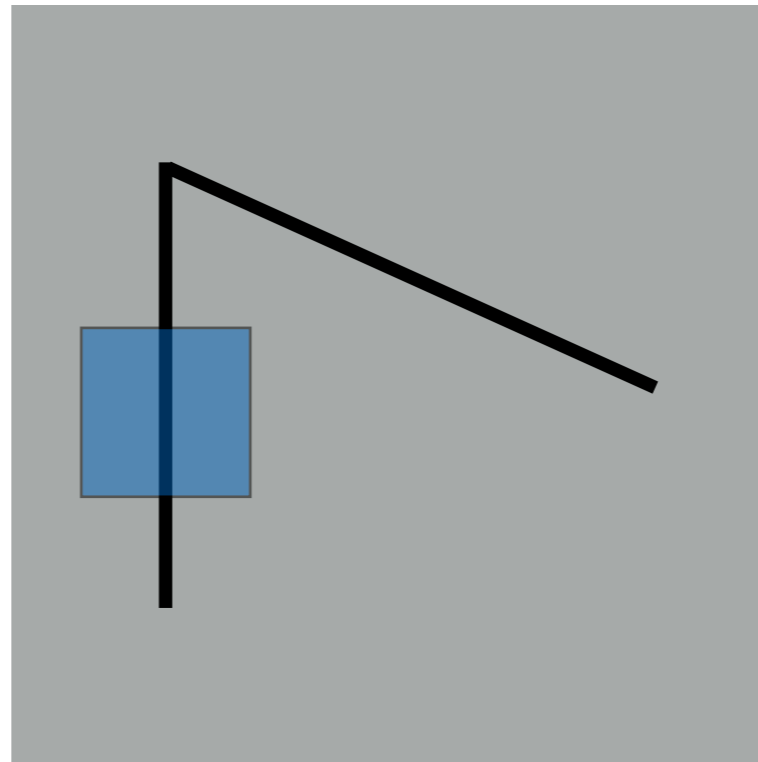
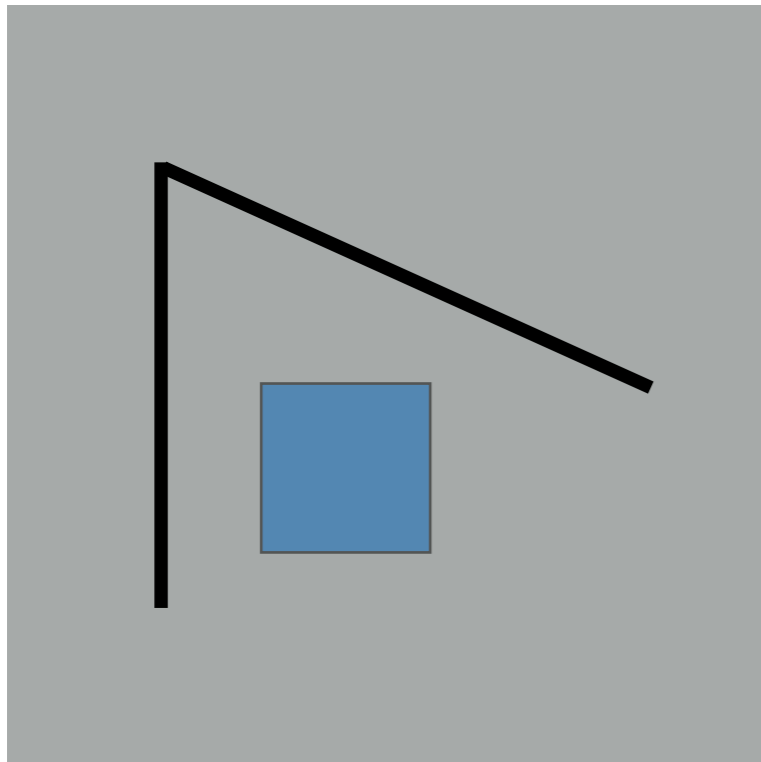
- real-time performance achievable

Keypoint Detection

Local Measure of Uniqueness

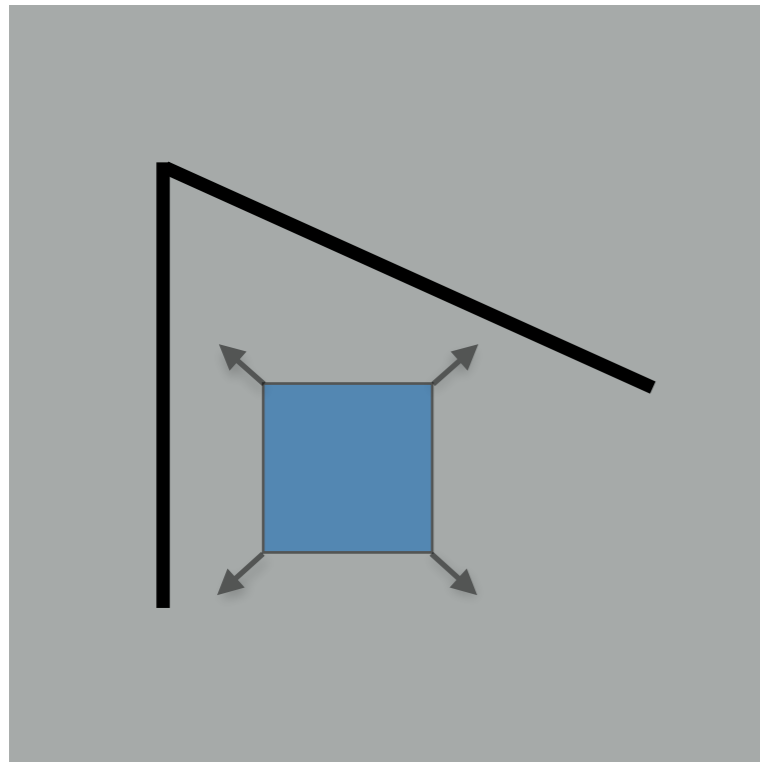
Suppose we only consider a small window of pixels

- What defines whether a feature is well localized and unique?

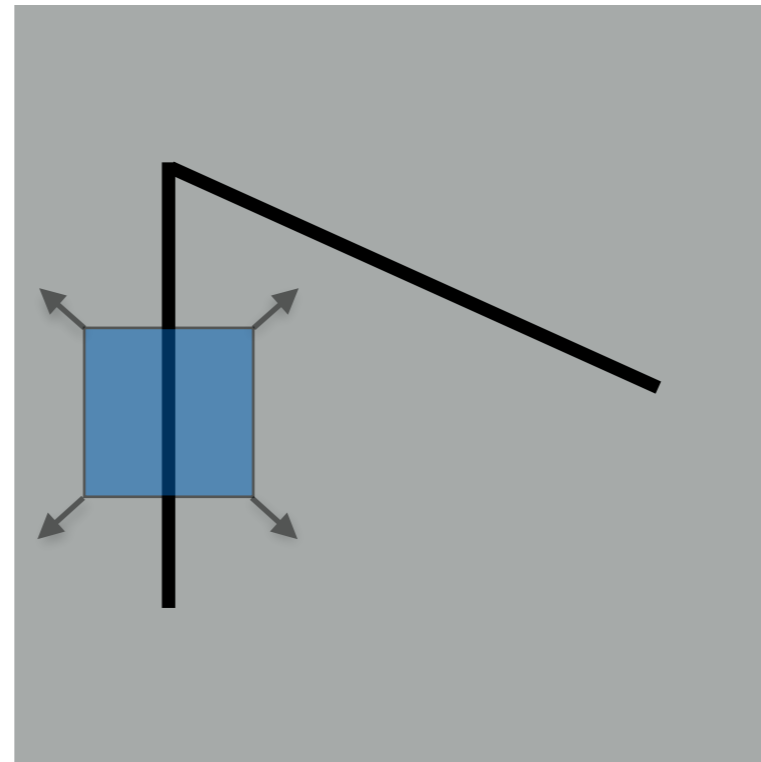


Local Measure of Uniqueness

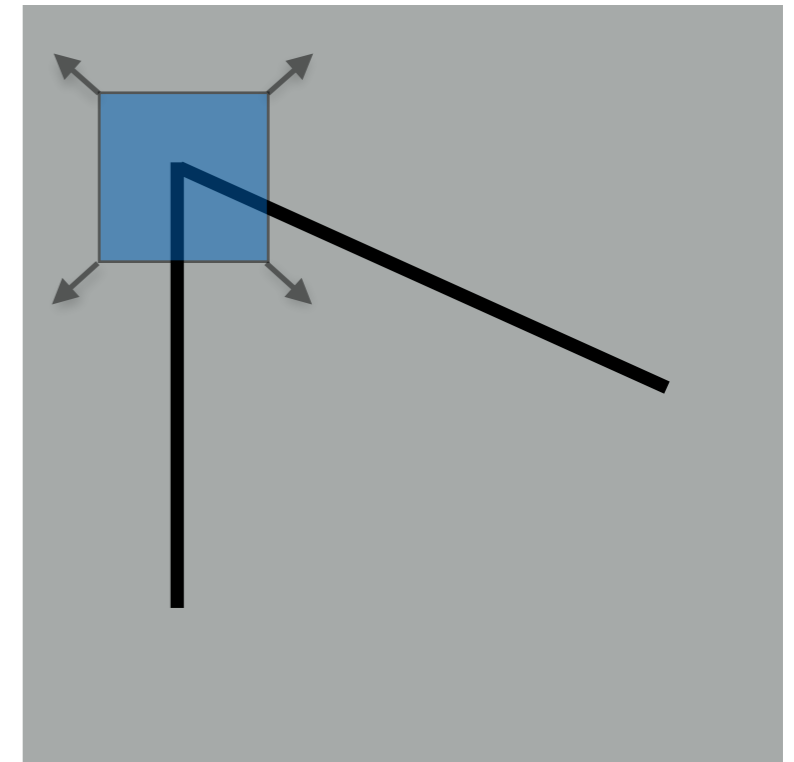
How does the window change when you shift by a small amount?



“flat” region:
no change in any
directions



“edge”:
no change along
the edge direction

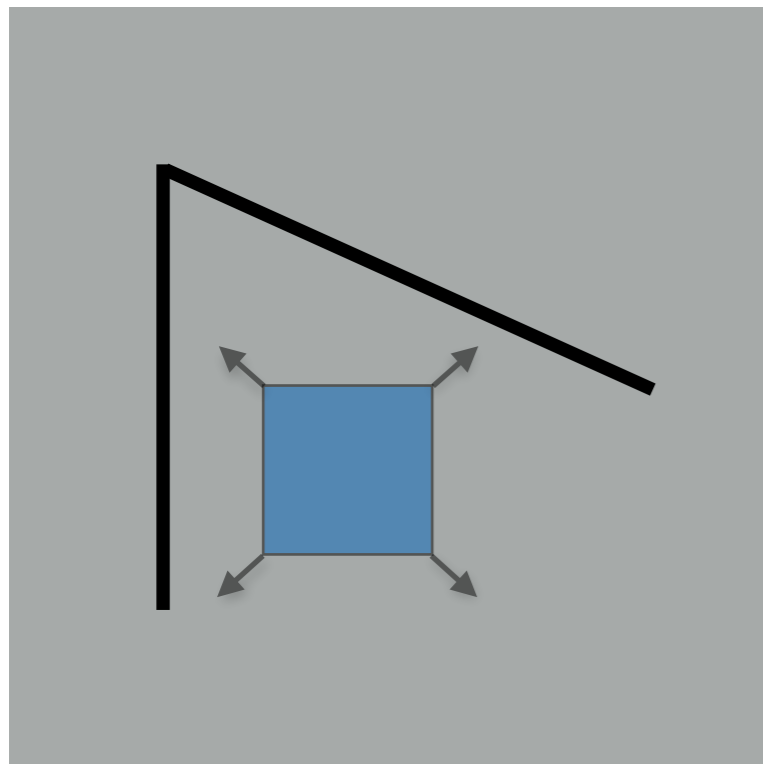


“corner”:
significant change
in all directions

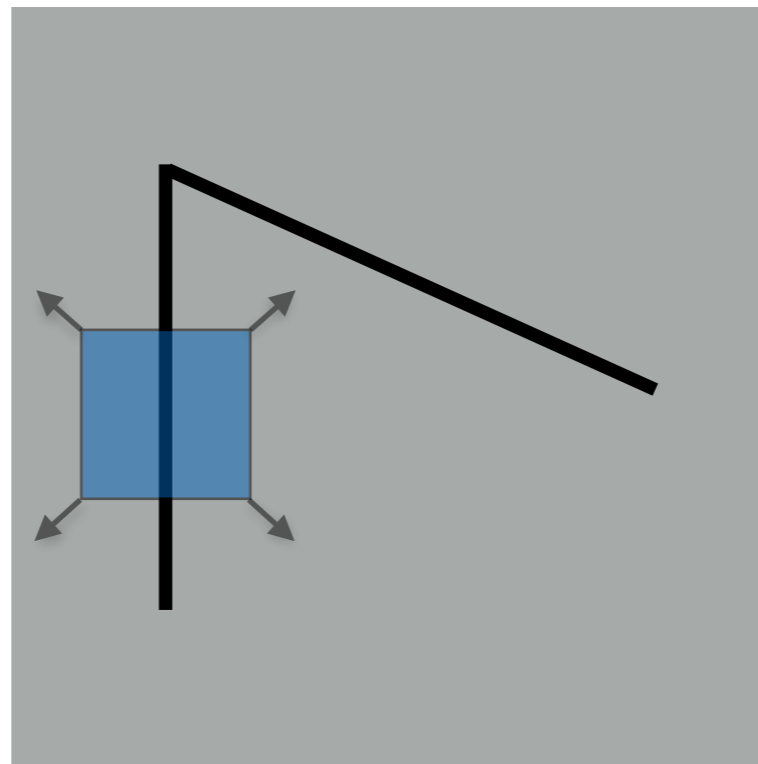
Local Measure of Uniqueness

Define:

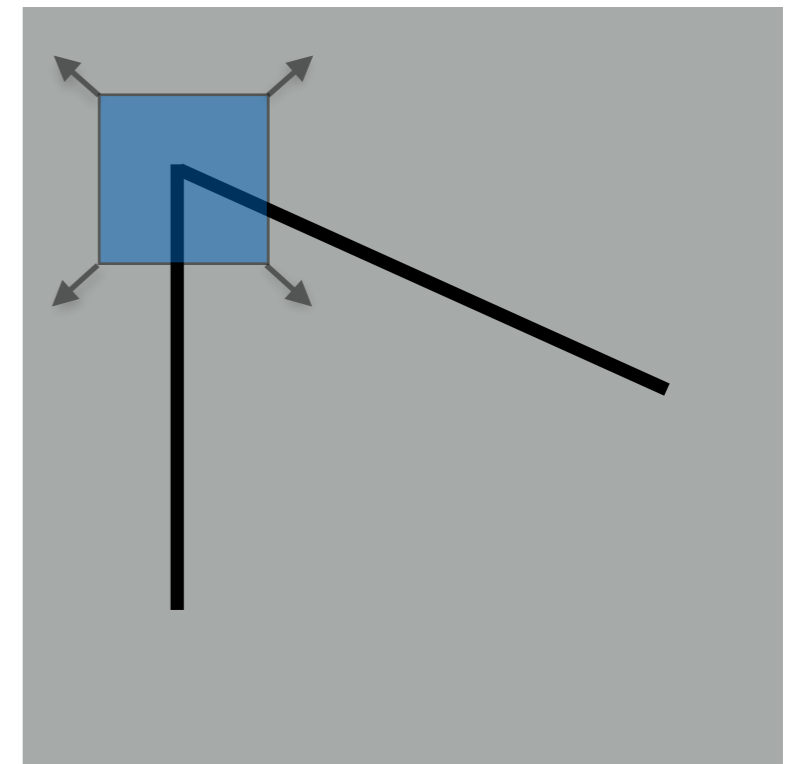
$E(u, v)$ = amount of change when you shift the window by (u, v)



$E(u, v)$ is small
for all shifts



$E(u, v)$ is small
for some shifts



$E(u, v)$ is small
for no shifts

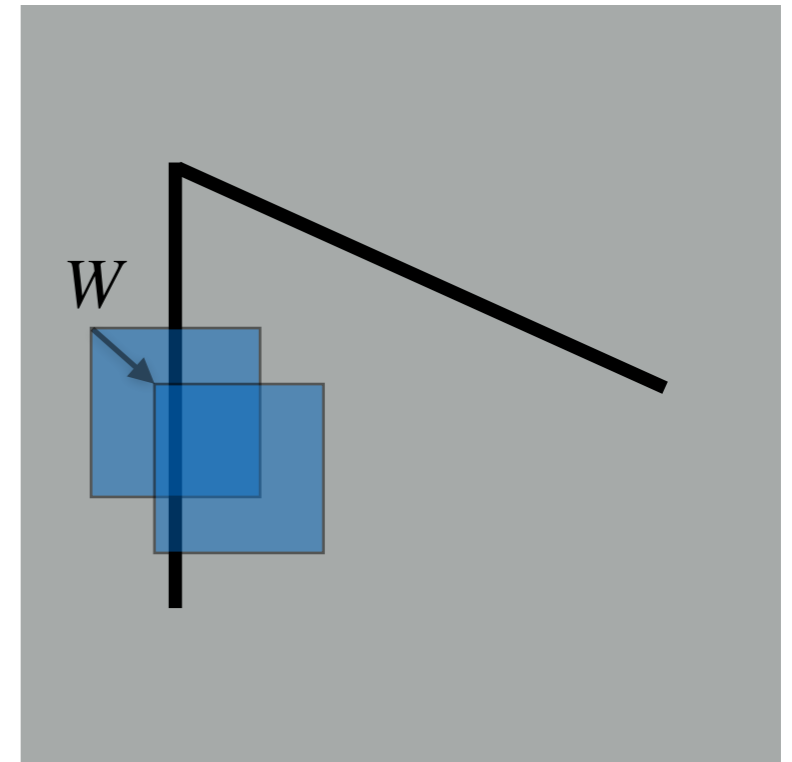
What do we want $\min_{u,v} E(u, v)$ to be?

Corner Detection

Consider shifting the window W by (u, v)

- how do the pixels in W change?
- compare each pixel before and after by Sum of the Squared Differences (SSD)
- this defines an SSD “error”:

$$E(u, v) = \sum_{(x,y) \in W} [I(x + u, y + v) - I(x, y)]^2$$



Corner Detection: Small Motion Assumption

Consider shifting the window W by (u, v)

- how do the pixels in W change?
- compare each pixel before and after by Sum of the Squared Differences (SSD)
- this defines an SSD “error”:

$$E(u, v) = \sum_{(x,y) \in W} [I(x+u, y+v) - I(x, y)]^2$$

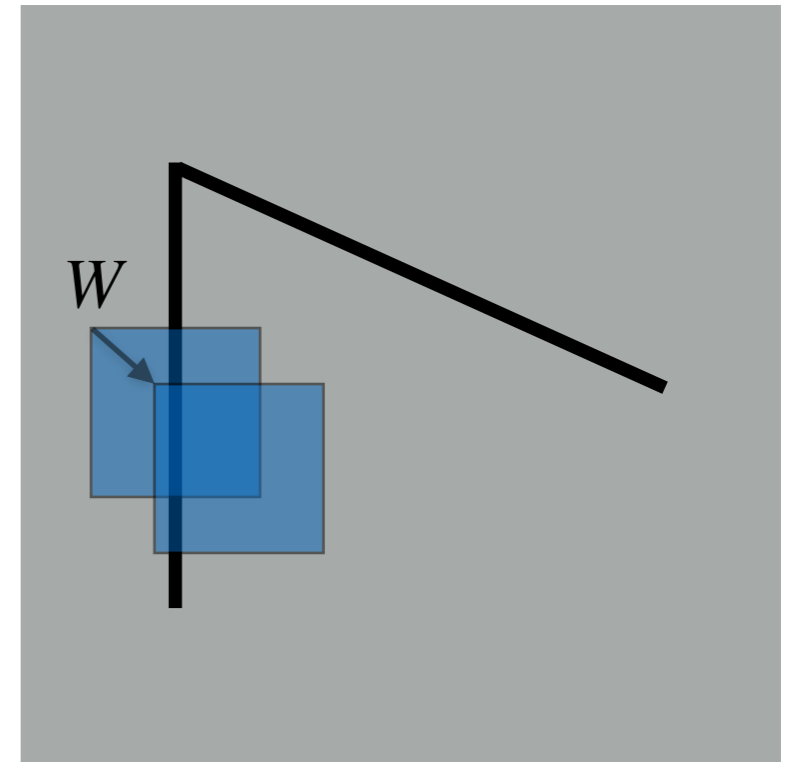
Taylor Series expansion of $I(x, y)$:

$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

If the motion (u, v) is small, then the first order approximation is good:

$$I(x+u, y+v) \approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v = I(x, y) + (I_x \ I_y) \begin{pmatrix} u \\ v \end{pmatrix}$$

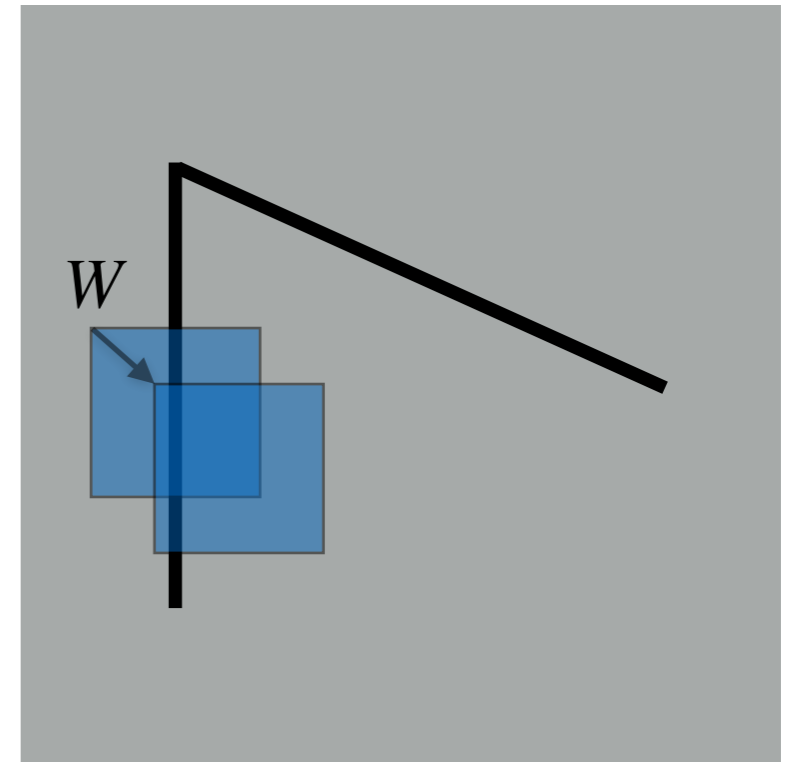
$$\text{shorthand } I_x = \frac{\partial I}{\partial x}$$



Corner Detection: Small Motion Assumption

Consider shifting the window W by (u, v)

- how do the pixels in W change?
- compare each pixel before and after by Sum of the Squared Differences (SSD)
- this defines an SSD “error”:



$$E(u, v) = \sum_{(x,y) \in W} [I(x+u, y+v) - I(x, y)]^2$$

$$\approx \sum_{(x,y) \in W} \left[I(x, y) + (I_x \ I_y) \begin{pmatrix} u \\ v \end{pmatrix} - I(x, y) \right]^2$$

$$I(x+u, y+v) \approx I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v = I(x, y) + (I_x \ I_y) \begin{pmatrix} u \\ v \end{pmatrix}$$

shorthand $I_x = \frac{\partial I}{\partial x}$

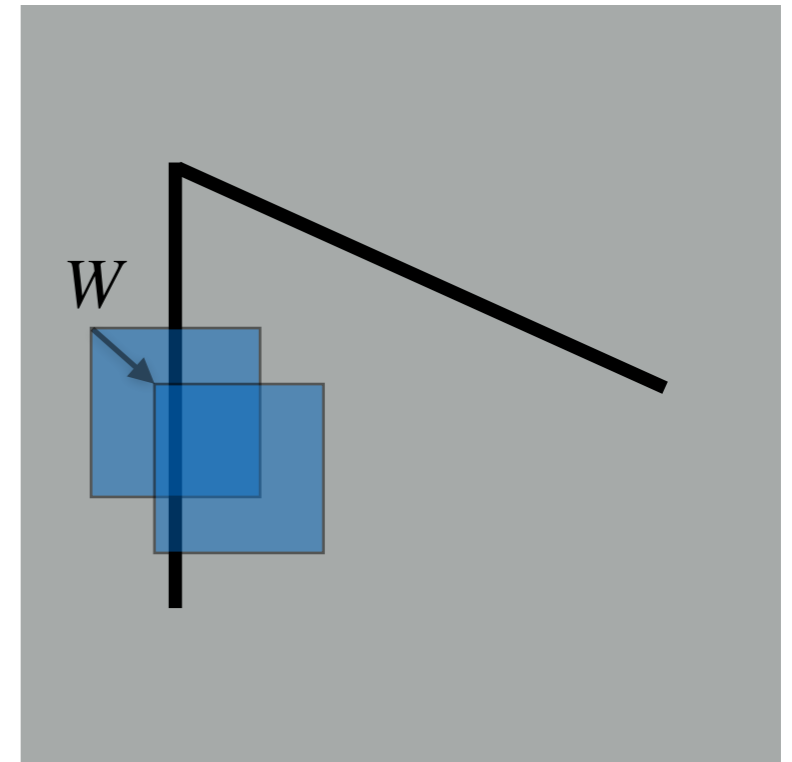
plug back in

Corner Detection

Consider shifting the window W by (u, v)

- how do the pixels in W change?
- compare each pixel before and after by Sum of the Squared Differences (SSD)
- this defines an SSD “error”:

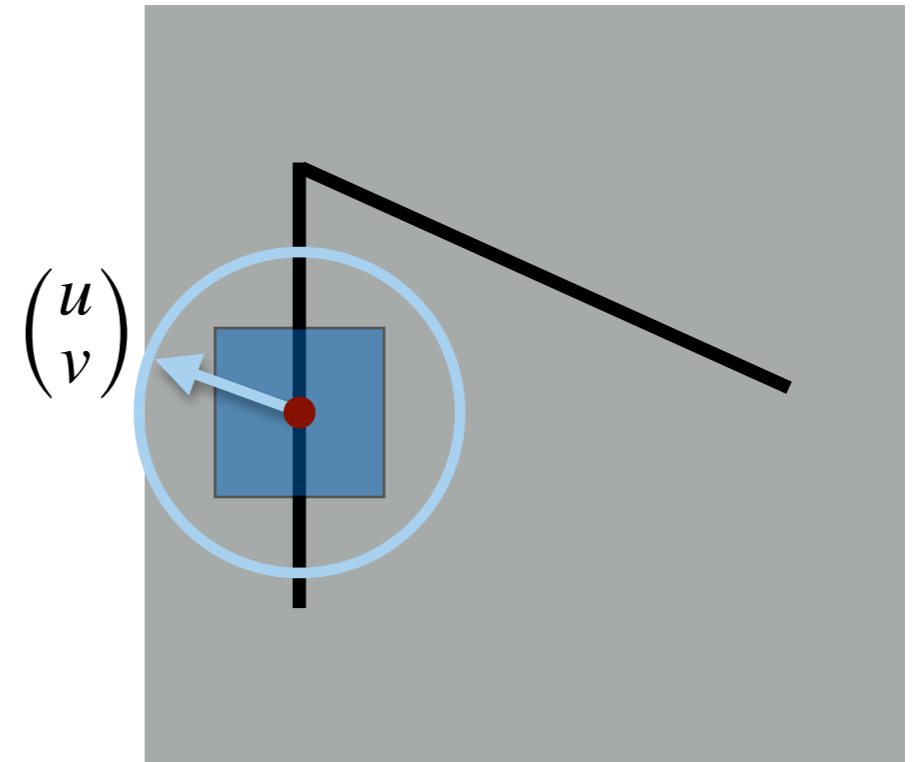
$$\begin{aligned} E(u, v) &= \sum_{(x,y) \in W} [I(x+u, y+v) - I(x, y)]^2 \\ &\approx \sum_{(x,y) \in W} \left[I(x, y) + (I_x \ I_y) \begin{pmatrix} u \\ v \end{pmatrix} - I(x, y) \right]^2 \\ &= \sum_{(x,y) \in W} \left[(I_x \ I_y) \begin{pmatrix} u \\ v \end{pmatrix} \right]^2 \end{aligned}$$



Corner Detection: Structure Tensor

This can be rewritten:

$$E(u, v) \approx \sum_{(x,y) \in W} \left[(I_x \ I_y) \begin{pmatrix} u \\ v \end{pmatrix} \right]^2$$
$$= \sum_{(x,y) \in W} (u \ v) \underbrace{\begin{pmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{pmatrix}}_{\text{"structure tensor" } H} \begin{pmatrix} u \\ v \end{pmatrix}$$

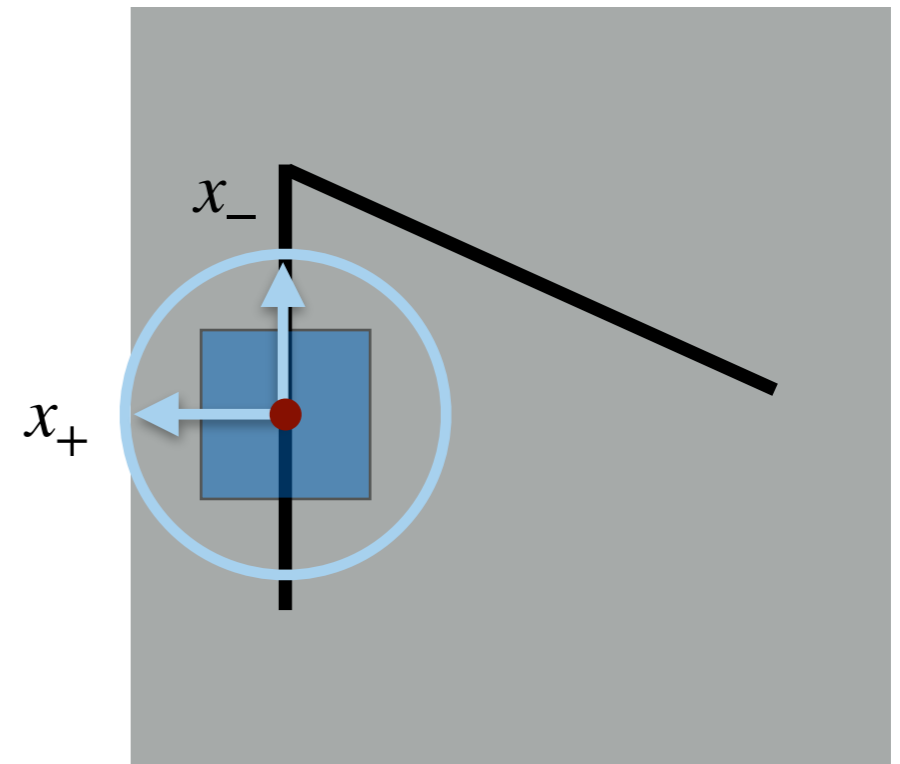


For the example above:

- You can move the center of the window to anywhere on the blue unit circle
- Which directions will result in the largest and smallest E values?
- We can find these directions by looking at the eigenvectors of H

This can be rewritten:

$$\begin{aligned}
 E(u, v) &\approx \sum_{(x,y) \in W} \left[(I_x \ I_y) \begin{pmatrix} u \\ v \end{pmatrix} \right]^2 \\
 &= \sum_{(x,y) \in W} (u \ v) \underbrace{\begin{pmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{pmatrix}}_{\text{"structure tensor" } H} \begin{pmatrix} u \\ v \end{pmatrix}
 \end{aligned}$$



Eigenvalues and eigenvectors of H:

- Define shifts with the smallest and largest change (E value)
- x_+ = direction of largest increase in E
- λ_+ = amount of increase in direction x_+
- x_- = direction of smallest increase in E
- λ_- = amount of increase in direction x_-

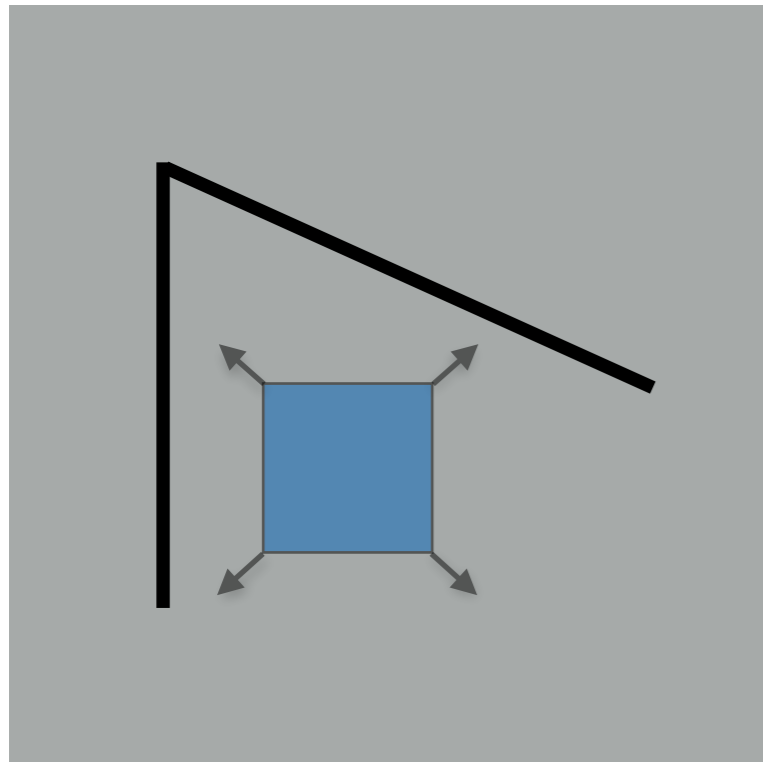
$$Hx_+ = \lambda_+ x_+$$

$$Hx_- = \lambda_- x_-$$

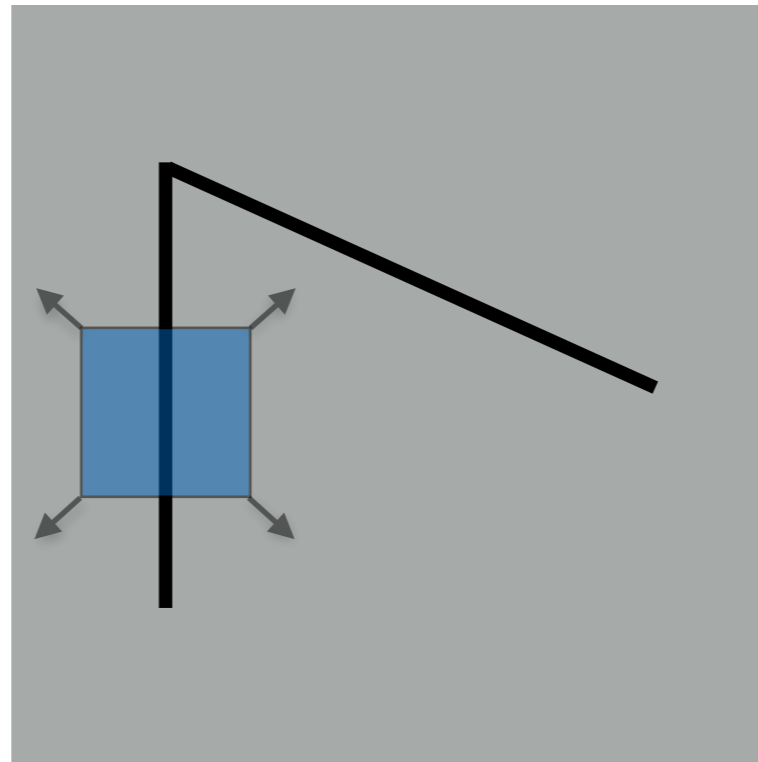
Corner Detection

Define:

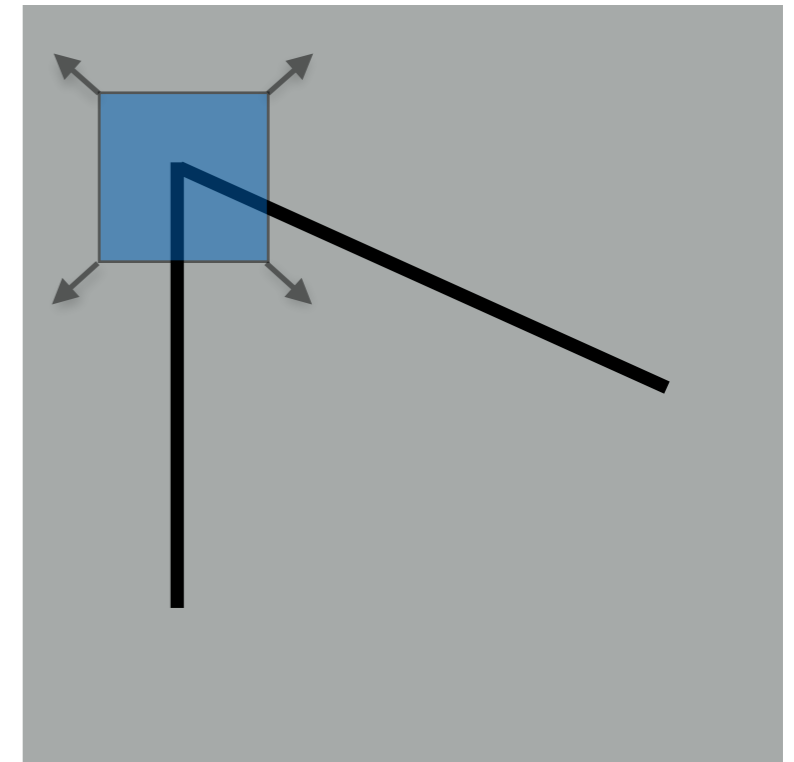
$E(u, v)$ = amount of change when you shift the window by (u, v)



$E(u, v)$ is small
for all shifts



$E(u, v)$ is small
for some shifts

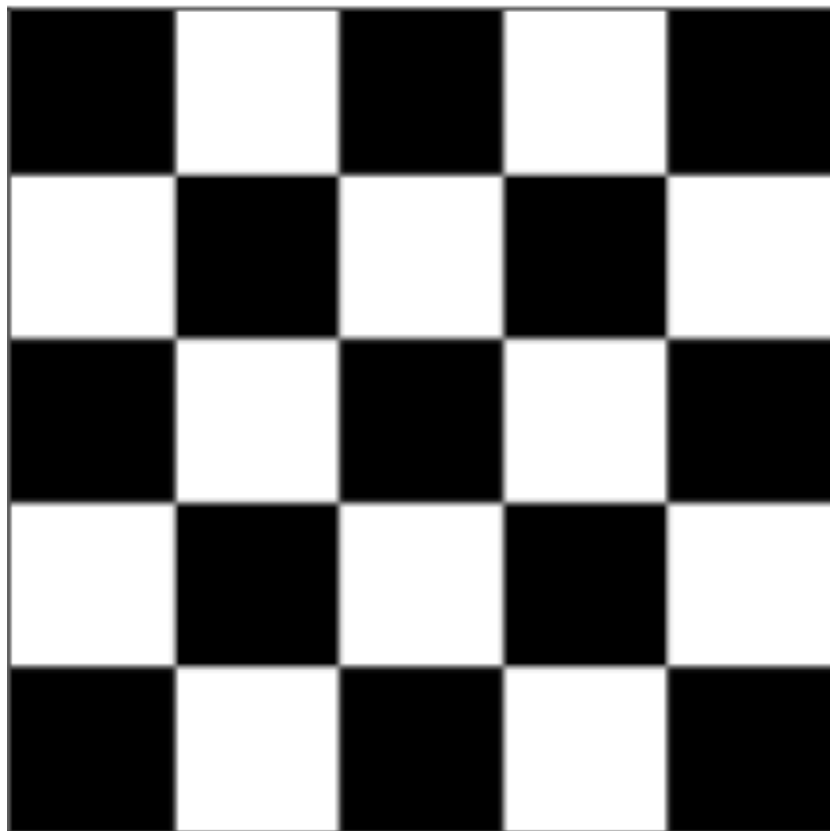


$E(u, v)$ is small
for no shifts

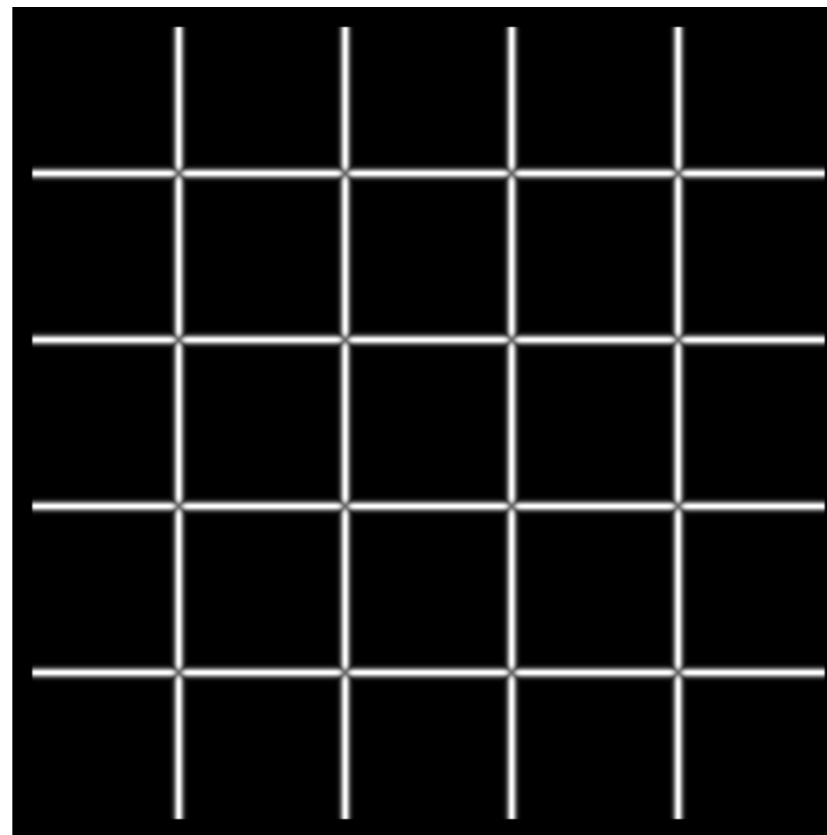
We want $\min_{u,v} E(u, v)$ to be large: maximize λ_{-}

Corner Detection Recipe

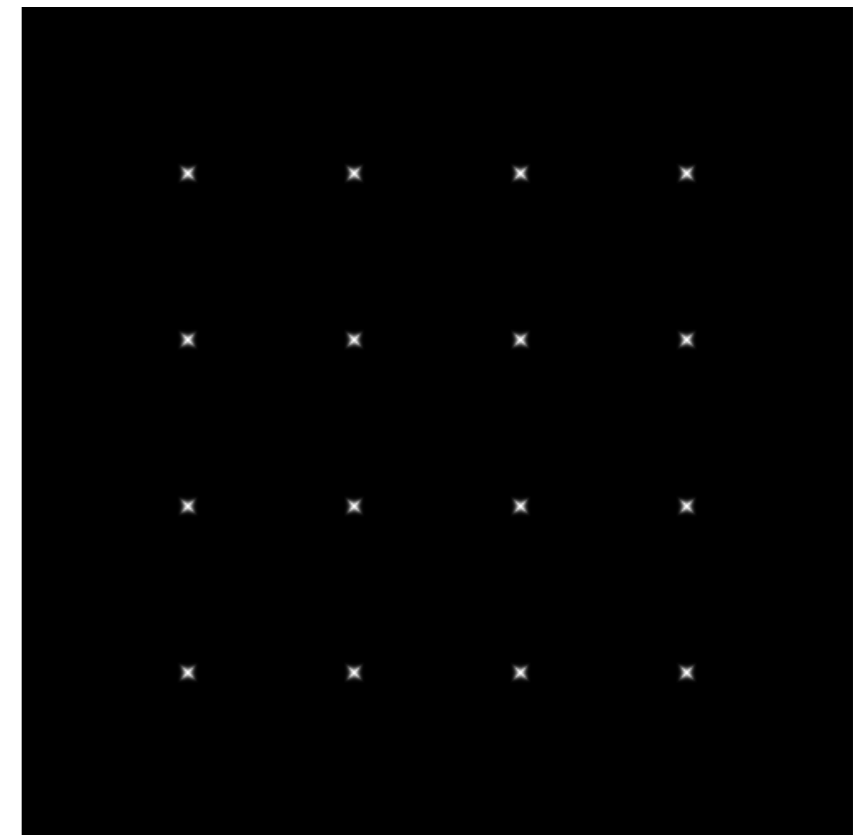
- Compute the gradient at each point in the image.
- Create the H matrix from the entries in the gradient.
- Compute the eigenvalues.
- Find points with large response ($\lambda_- > \text{threshold}$).
- Choose those points where λ_- is a local maximum as features.



I



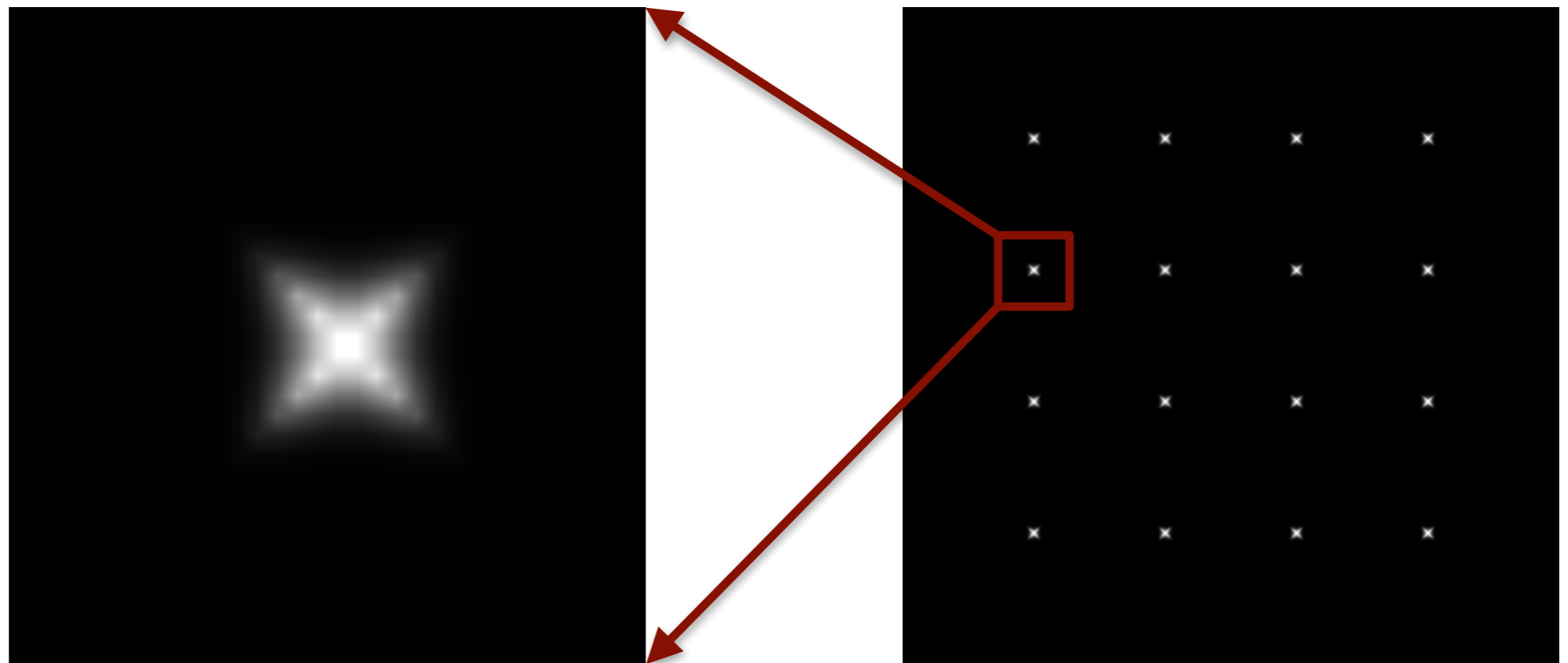
λ_+



λ_-

Corner Detection Recipe

- Compute the gradient at each point in the image.
- Create the H matrix from the entries in the gradient.
- Compute the eigenvalues.
- Find points with large response ($\lambda_- > \text{threshold}$).
- Choose those points where λ_- is a local maximum as features.



λ_-

Corner Detection: Harris Operator

- Thresholding λ_- is also known as “Shi-Tomasi corners” or “Good Features to Track” (Shi & Tomasi, CVPR 1994)
- This is a variant of the “Harris operator” for corner detection

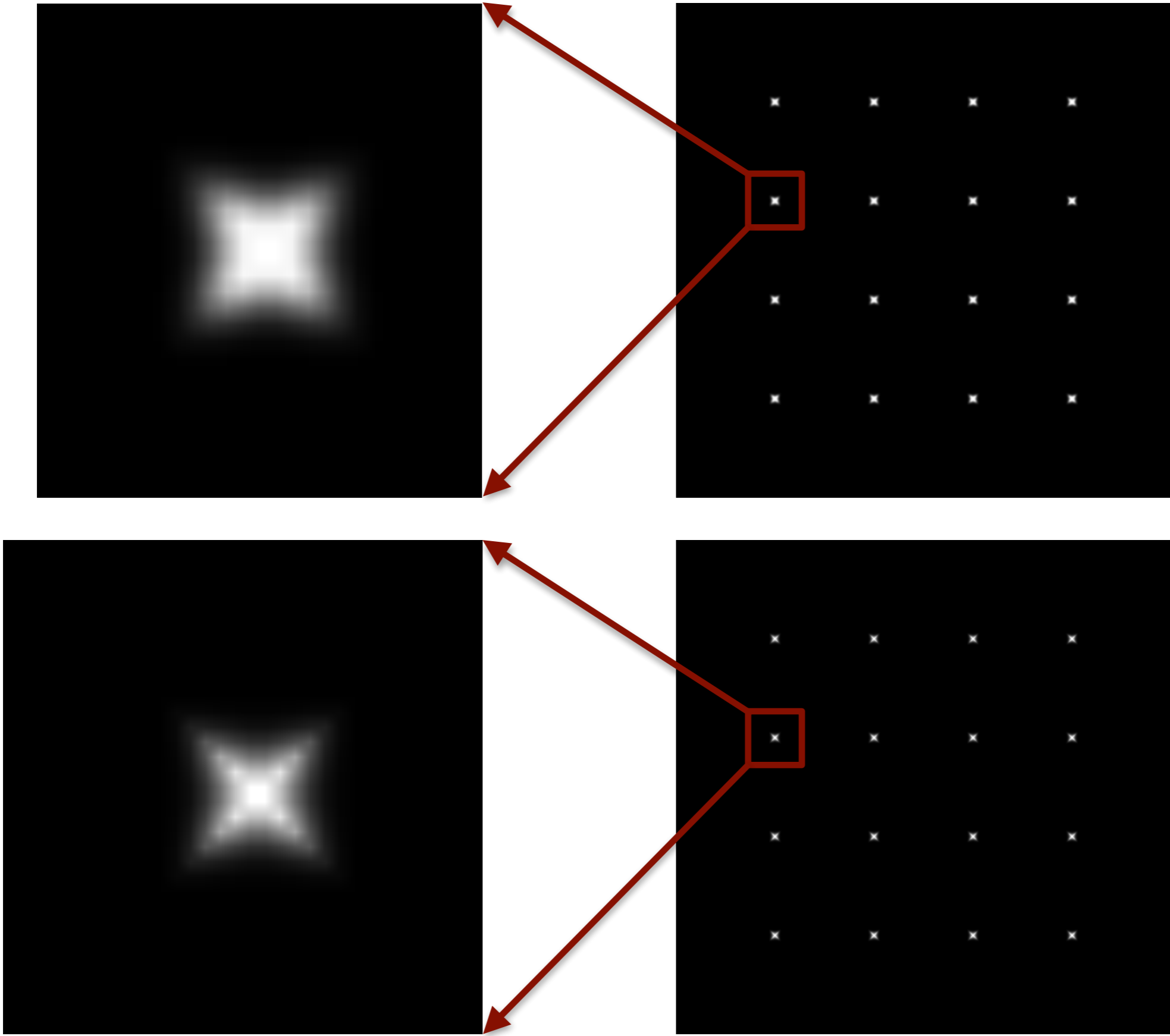
$$f = \frac{\lambda_- \lambda_+}{\lambda_- + \lambda_+}$$
$$= \frac{\text{determinant}(H)}{\text{trace}(H)}$$

- The trace is the sum of the diagonals, i.e., $\text{trace}(H) = h_{11} + h_{22}$
- Very similar to λ_- , but less expensive (no square root)
- Called the “Harris Corner Detector” or “Harris Operator” (Harris and Stephens, AVC 1988)
- Lots of other detectors, this is one of the most popular

Corner Detection: Harris Operator

Harris
Operator

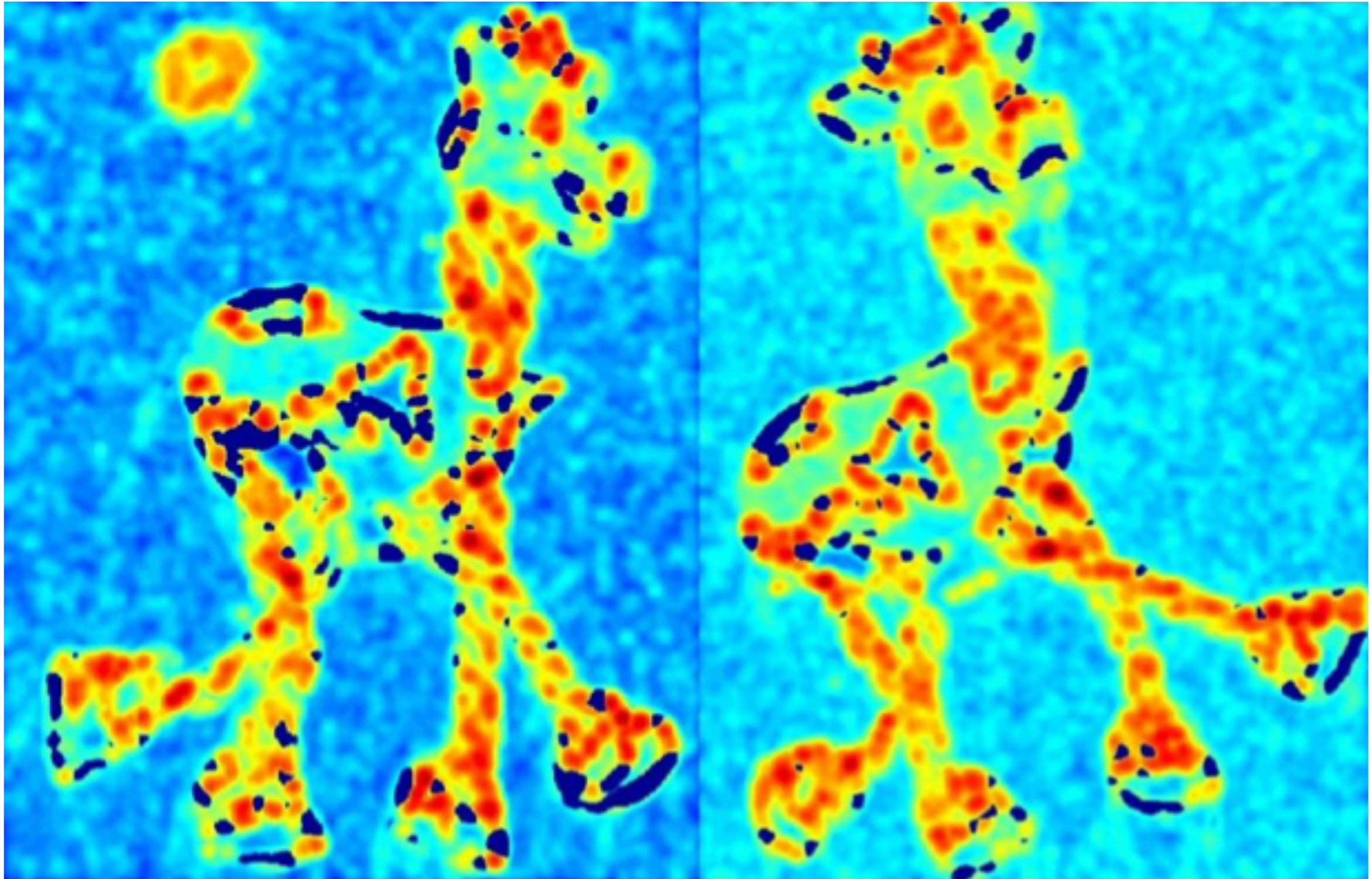
λ_-



Harris Detector Example

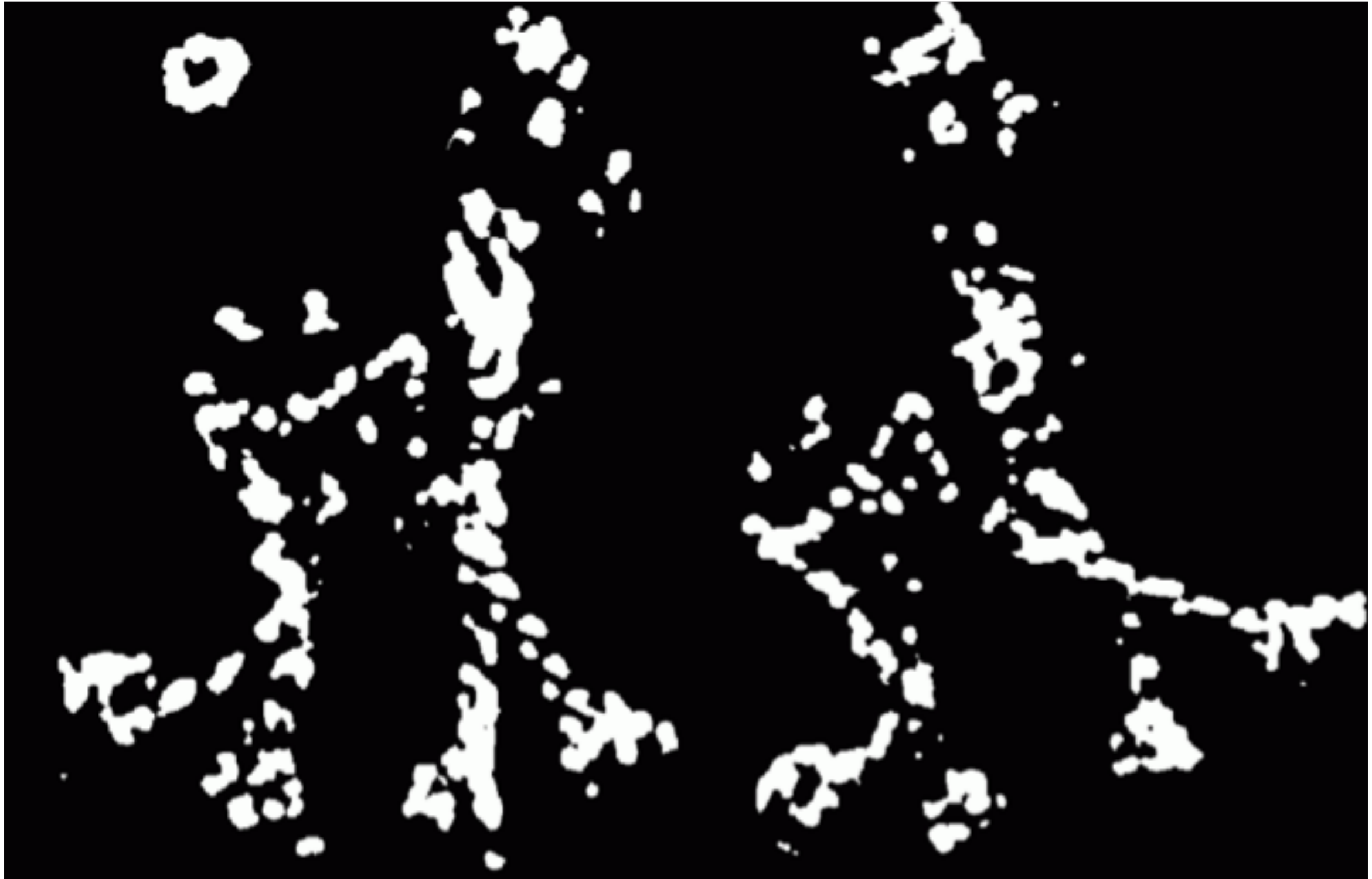


Harris Detector Example



Harris Corner Response

Harris Detector Example



Thresholded Harris Corner Response

Harris Detector Example



Local Maxima of Harris Corner Response

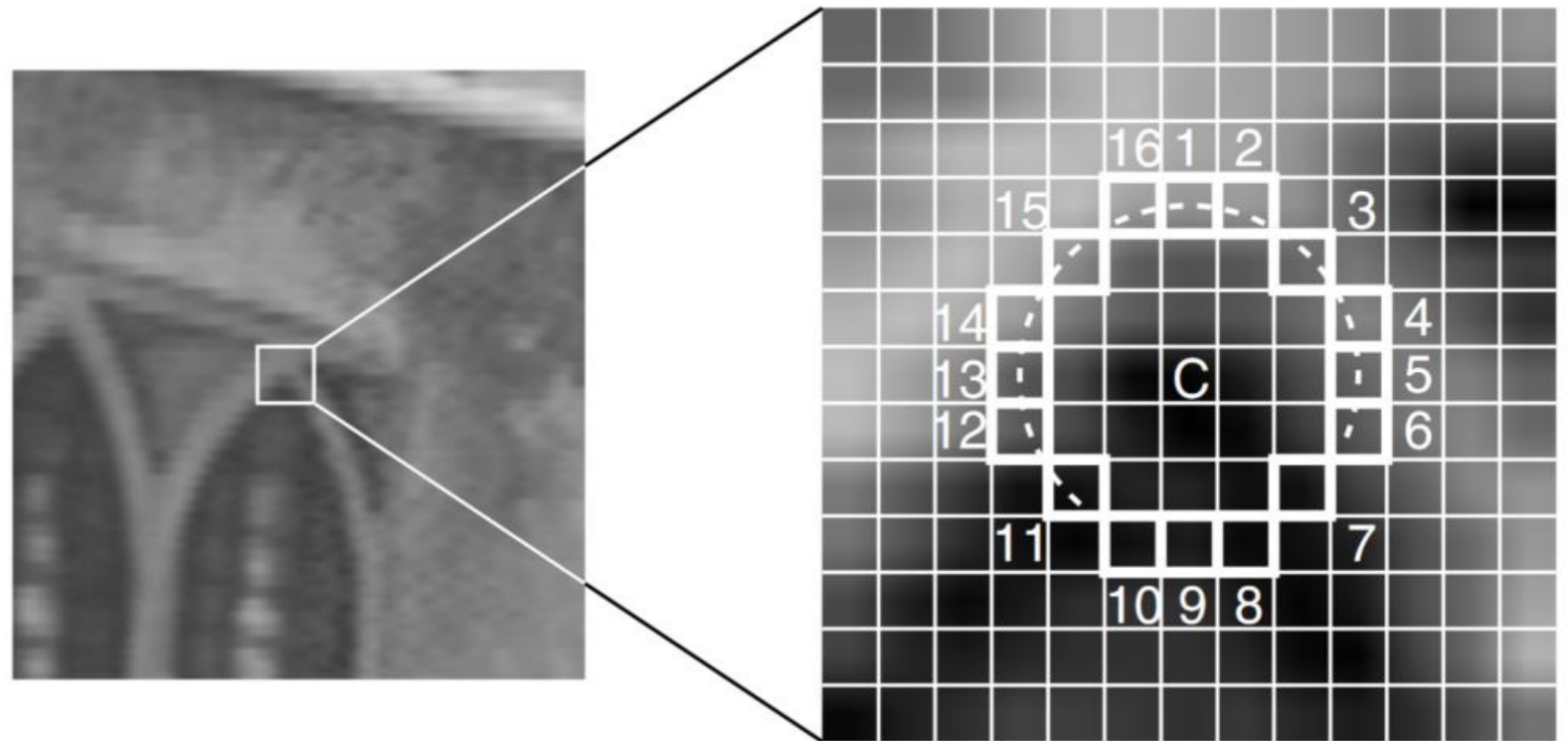
Harris Detector Example



Harris Corner Response

FAST Detector

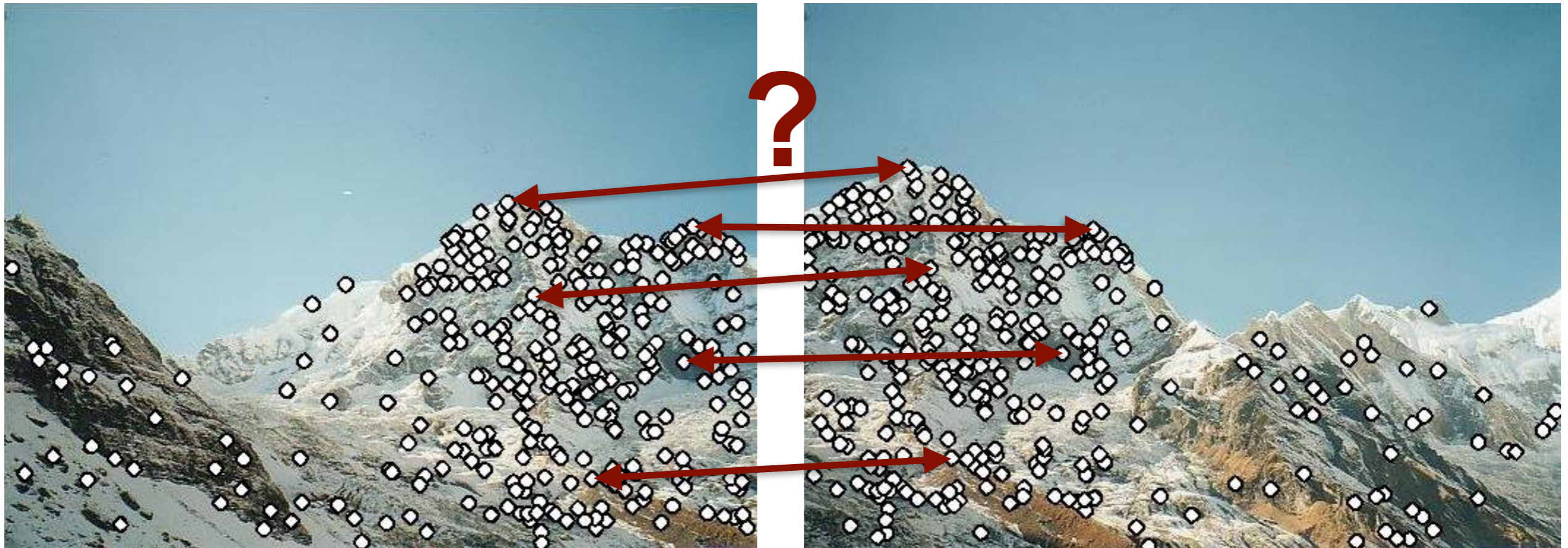
- Features from Accelerated Segment Test
- Check relation of brightness values to center pixel along circle
- Specific number of contiguous pixels brighter or darker than center
- Very fast corner detection



Keypoint Descriptors

Keypoint Descriptors

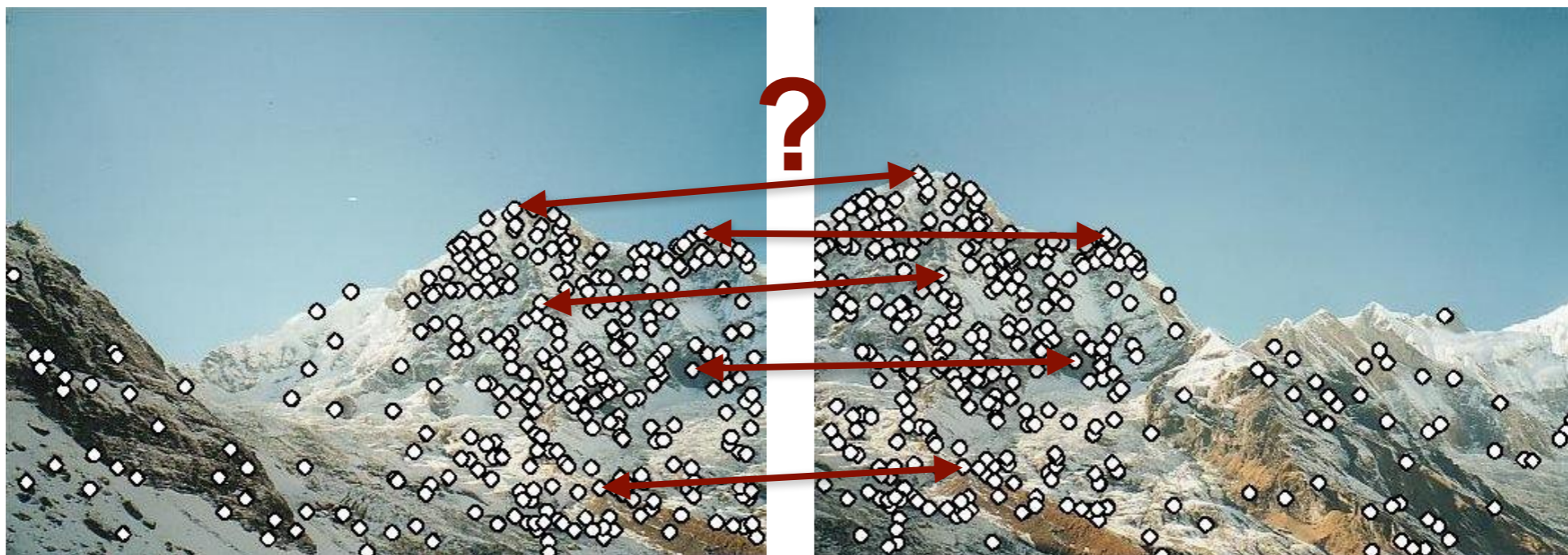
- We know how to detect good points.
- Next question: How to match them?



- Idea: extract distinctive descriptor vector from a local patch around the keypoint.

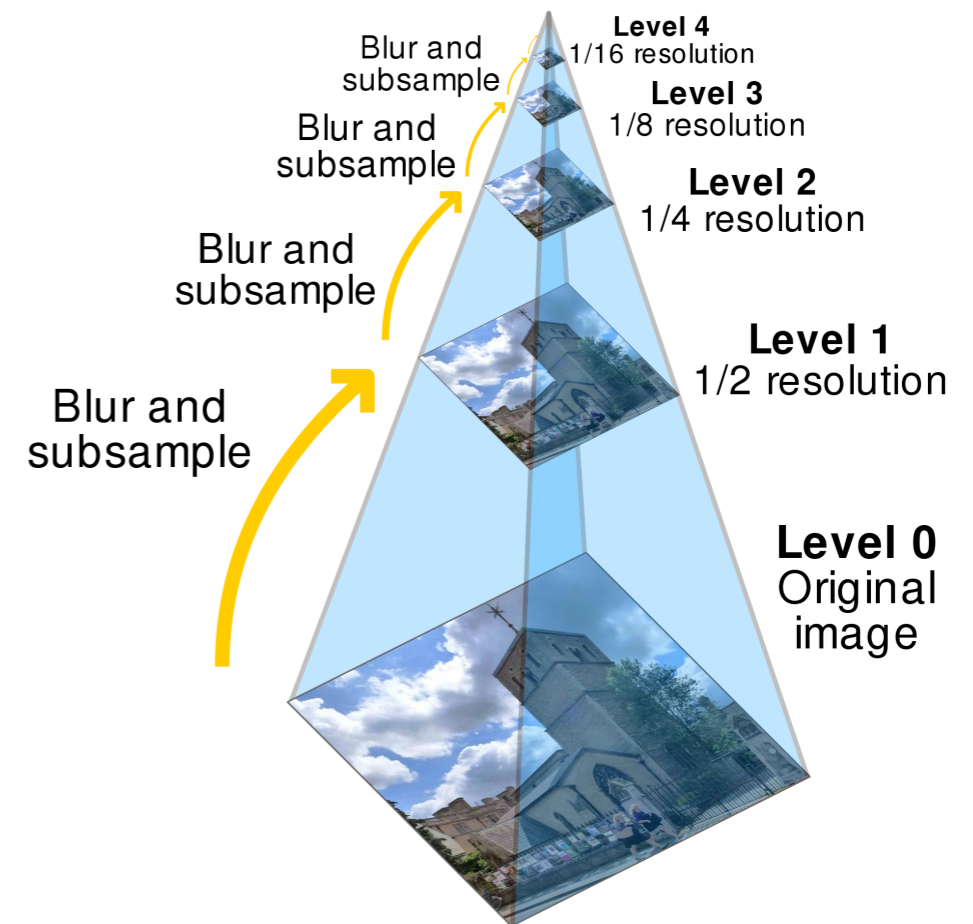
Invariance

- Goal: match keypoints regardless of image transformation.
 - This is called transformational invariance.
- Most keypoint detection and description methods are designed to be invariant to:
 - Translation, 2D rotation, scale
- They can usually also handle:
 - Limited 3D rotations (SIFT works up to about 60 degrees)
 - Limited affine transformations (some are fully affine invariant)
 - Limited illumination/contrast changes



Invariance

- Make sure your detector is invariant.
 - Harris is invariant to translation and rotation.
 - Scale is trickier:
 - Image pyramids
 - Scale selection for blobs (f.e. SIFT)
 - Keypoints at multiple scales for same location
- Design an invariant feature descriptor
 - A descriptor captures the information in a region around the detected feature point.
 - The simplest descriptor: a square window of pixels
 - What's this invariant to?
 - Let's look at some better approaches...



2D Rotation Invariance

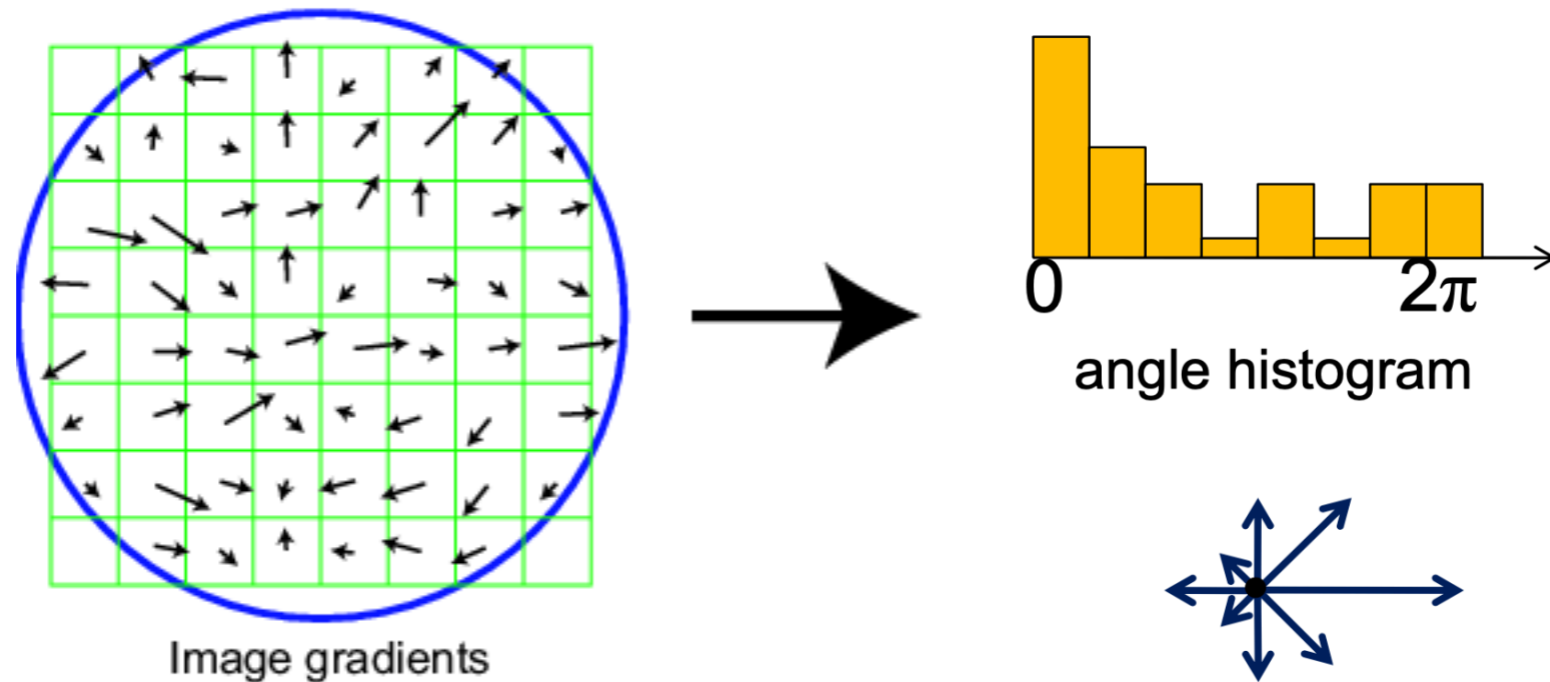
- Idea: align the descriptor with a dominant 2D orientation
- Example approach: Use the eigenvector of H corresponding to larger eigenvalue



Figure by Matthew Brown

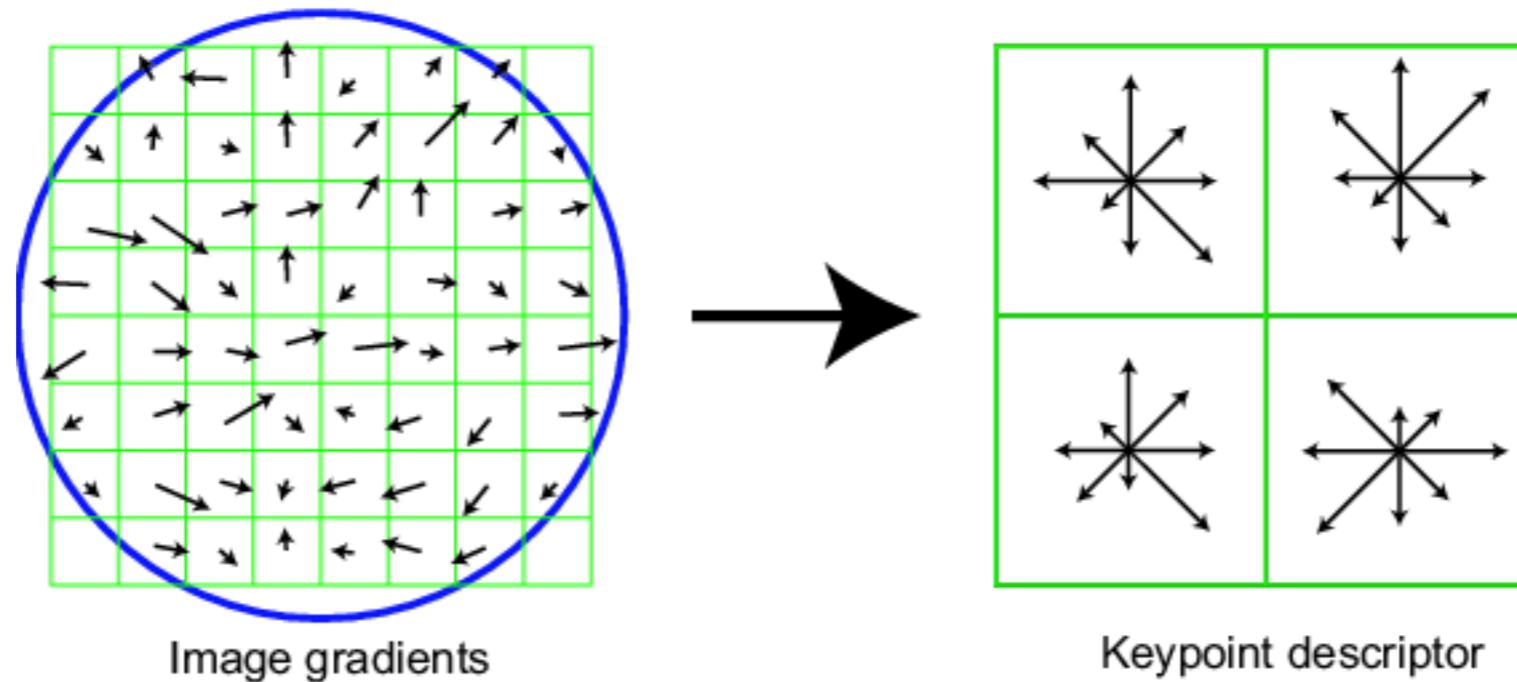
2D Rotation Invariance: SIFT

- Take 16x16 square window around detected feature
- Compute edge orientation (angle of the gradient) for each pixel
- Throw out weak edges (threshold gradient magnitude)
- Create histogram of surviving edge orientations
- Select strong local orientation maxima and create one or more descriptors



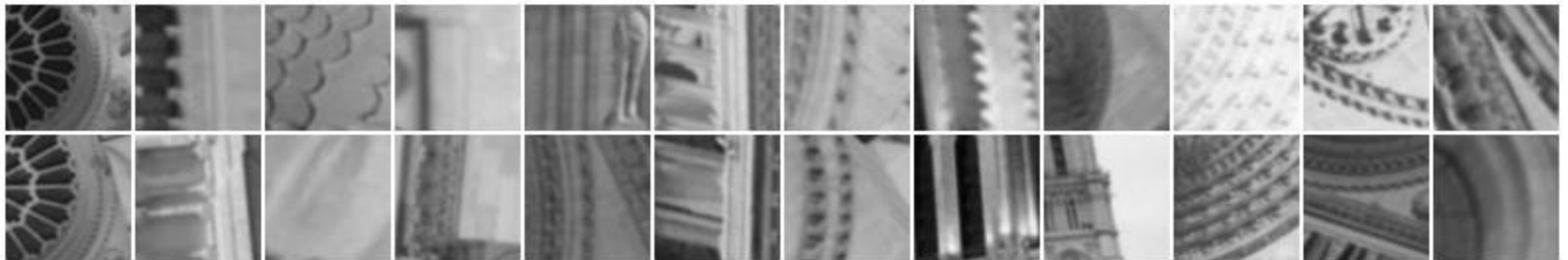
SIFT Descriptor

- Divide the 16x16 window into a 4x4 grid of cells (2x2 case shown below)
- Compute an orientation histogram for each cell
- 16 cells * 8 orientations = 128 dimensional descriptor
- Many important details, e.g.:
 - Trilinear interpolation for binning: viewpoint invariance
 - Descriptor normalization: brightness invariance



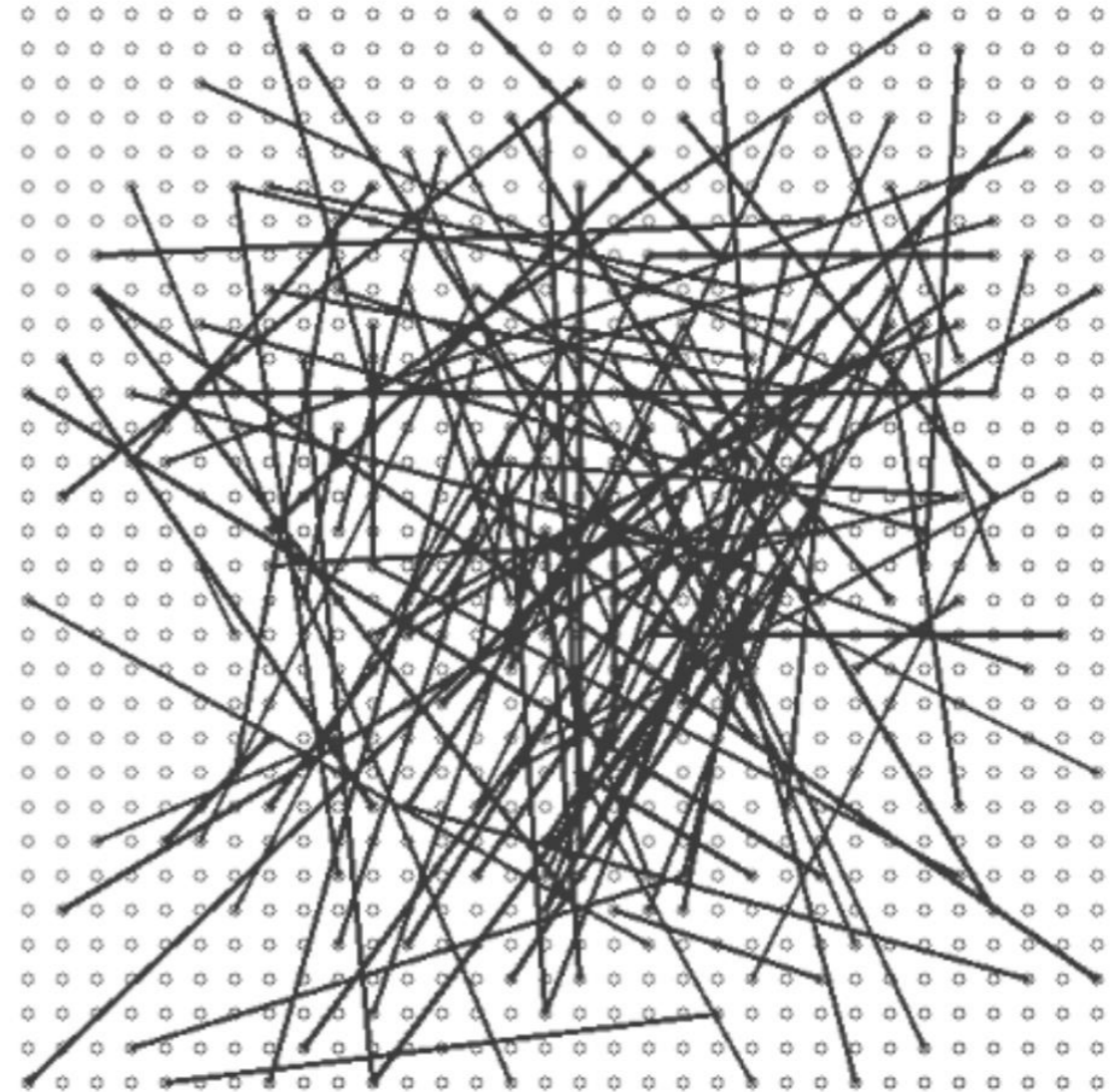
Properties of SIFT

- “Scale-Invariant Feature Transform” (Lowe, IJCV 2004)
- Can handle changes in viewpoint
 - Up to about 60 degree out of plane rotation
- Can handle significant changes in illumination
 - Sometimes even day vs. night
- Fast and efficient—can run in real time (but still costly)
- Lots of code available, CPU, GPU, ... (US patent recently expired)
- Many alternatives since (SURF, KAZE, learning-based, ...), but still popular.
- But: false positive matches still possible



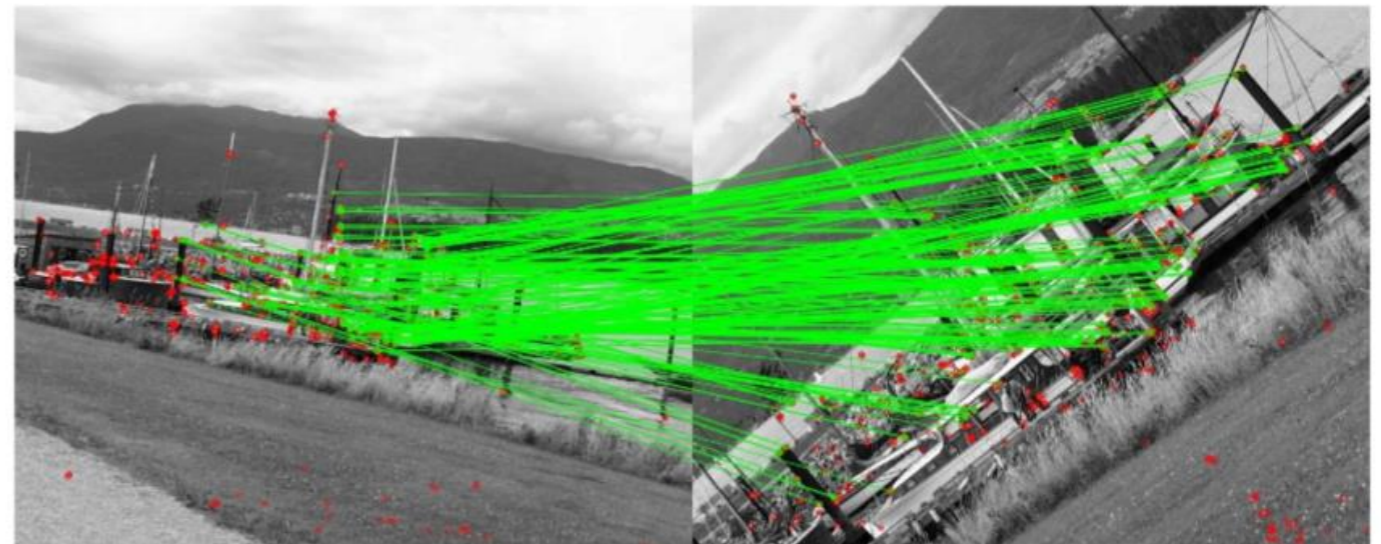
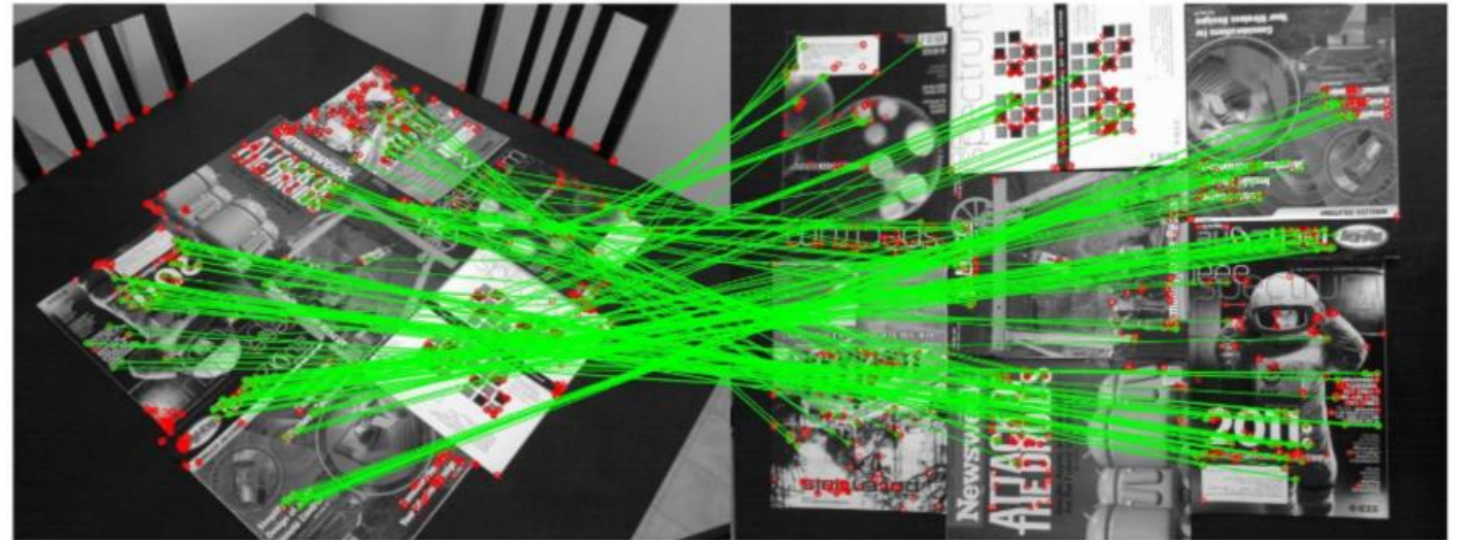
Binary Descriptors: BRIEF

- “Binary Robust Independent Elementary Features” (Calonder et al. ECCV 2010)
- Binary descriptor from intensity comparisons at sample positions
- Very efficient to compute
- Fast matching distance through Hamming distance (xor and popcount CPU instructions)



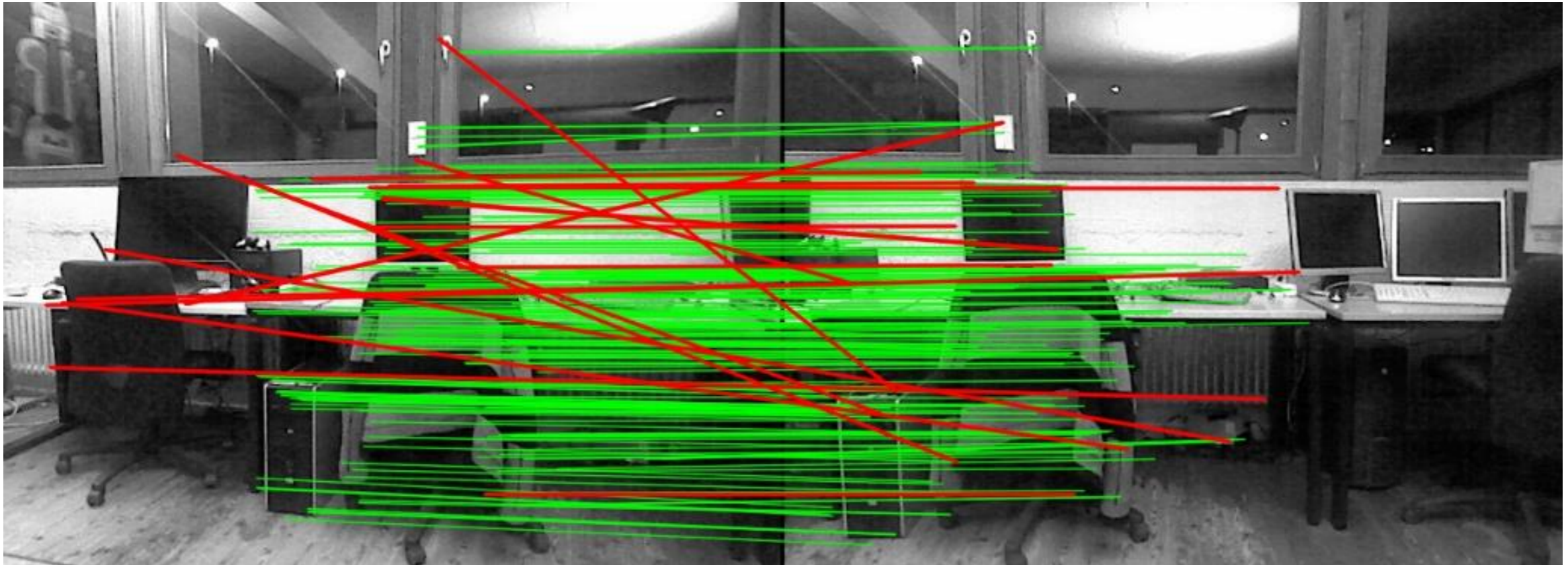
Binary Descriptors: ORB

- “Oriented Fast and Rotated BRIEF” (Rublee et al. ICCV 2011)
- Combination of FAST detector and BRIEF descriptor
- Rotation-invariant BRIEF: Estimate dominant orientation from patch moments (intensity centroid)
- Improved binary pattern
- Very popular for SLAM / VO



Keypoint Matching

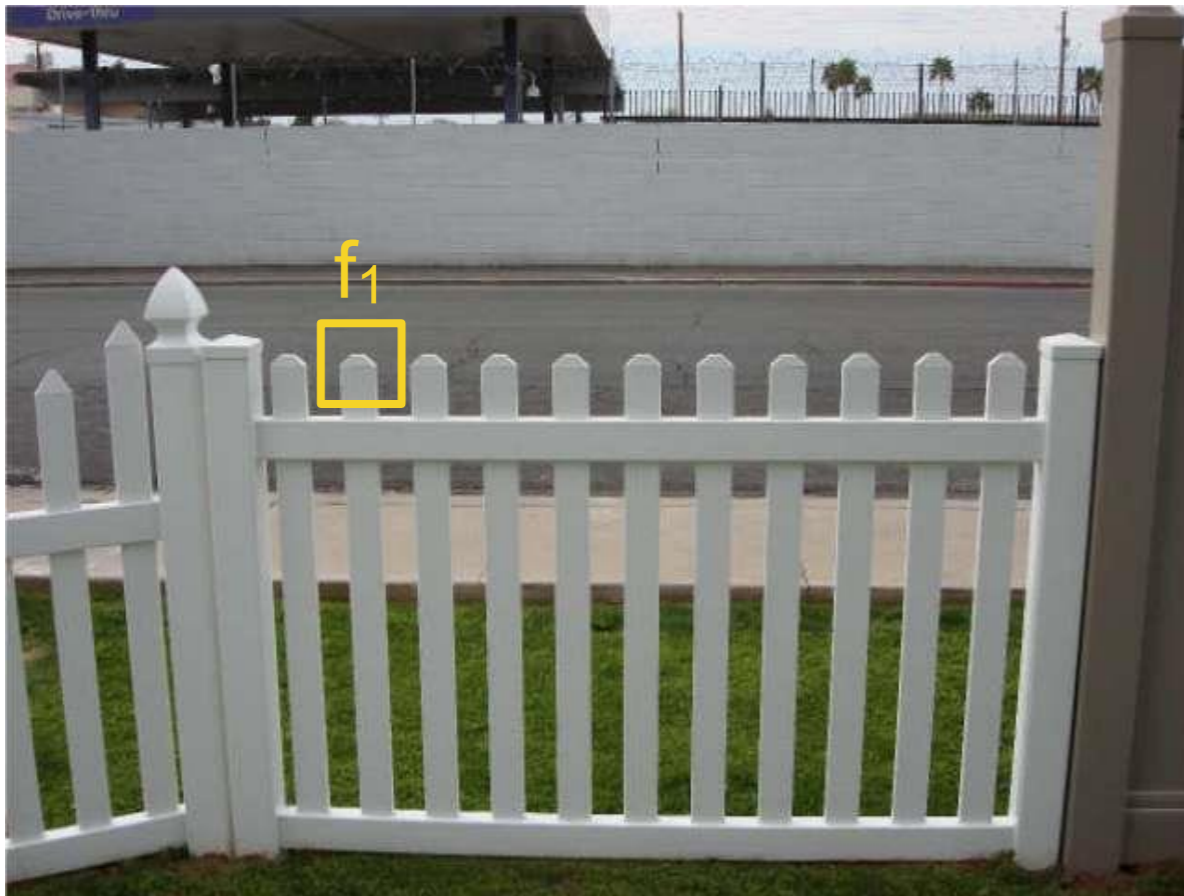
Keypoint Matching



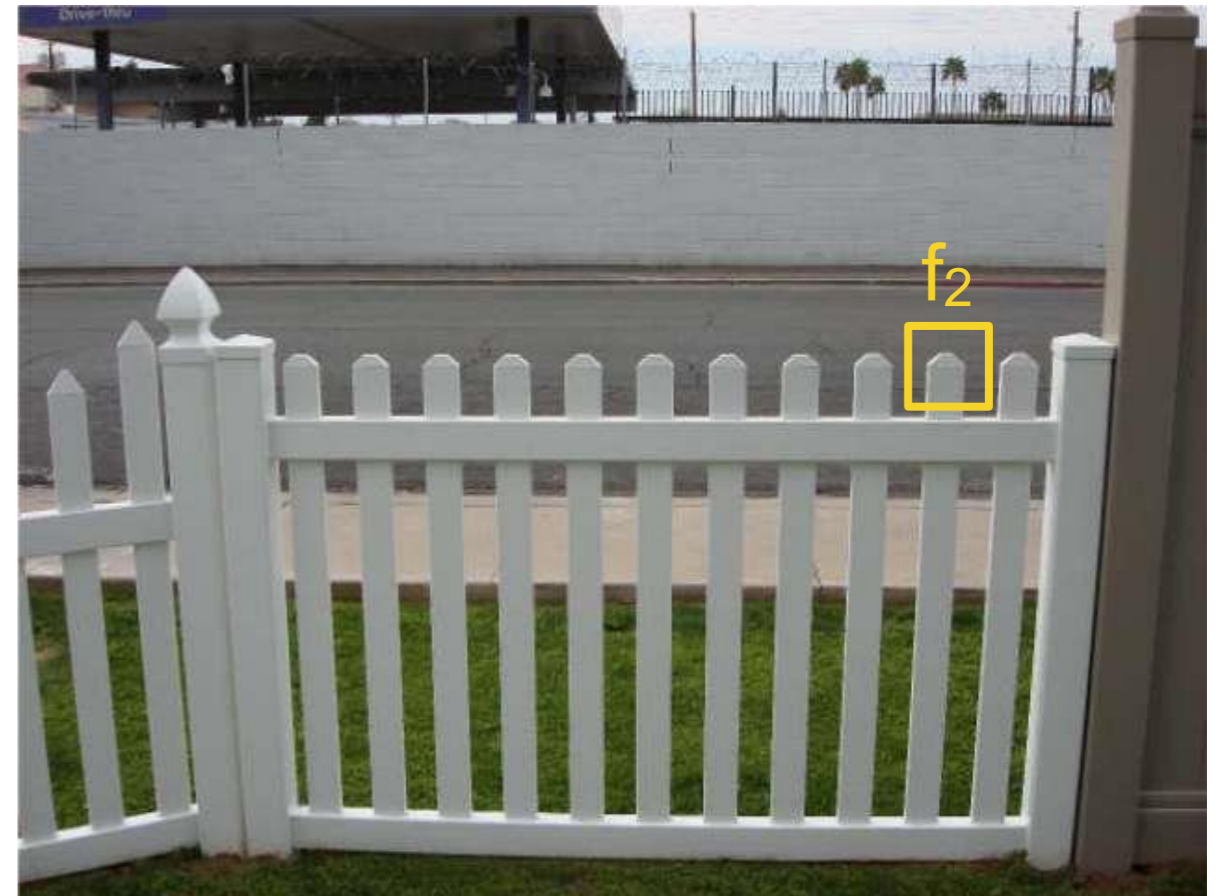
- Main idea: Match keypoints with similar descriptors

Matching Distance

- How to define the difference between two descriptors f_1, f_2 ?
- Simple approach is to assign keypoints with minimal sum of square differences $SSD(f_1, f_2)$ between entries of the two descriptors



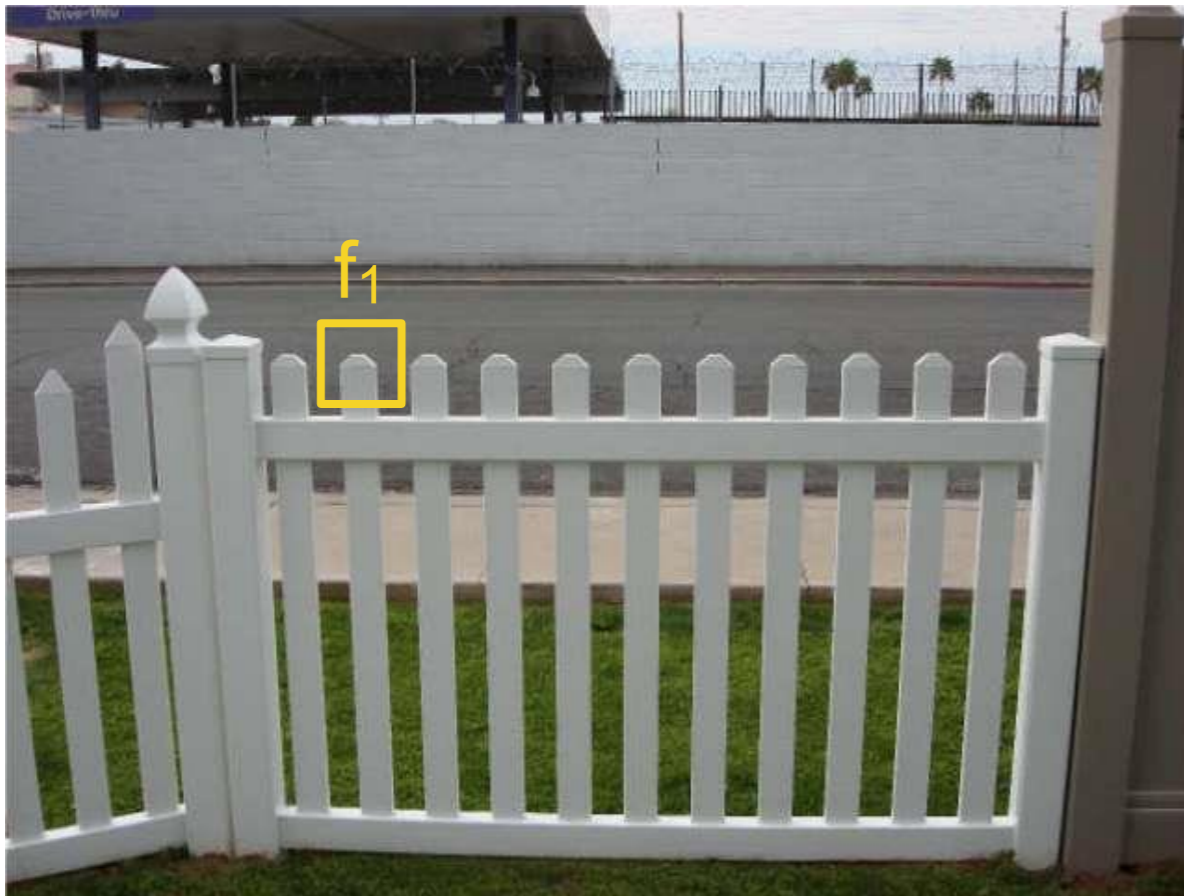
I_1



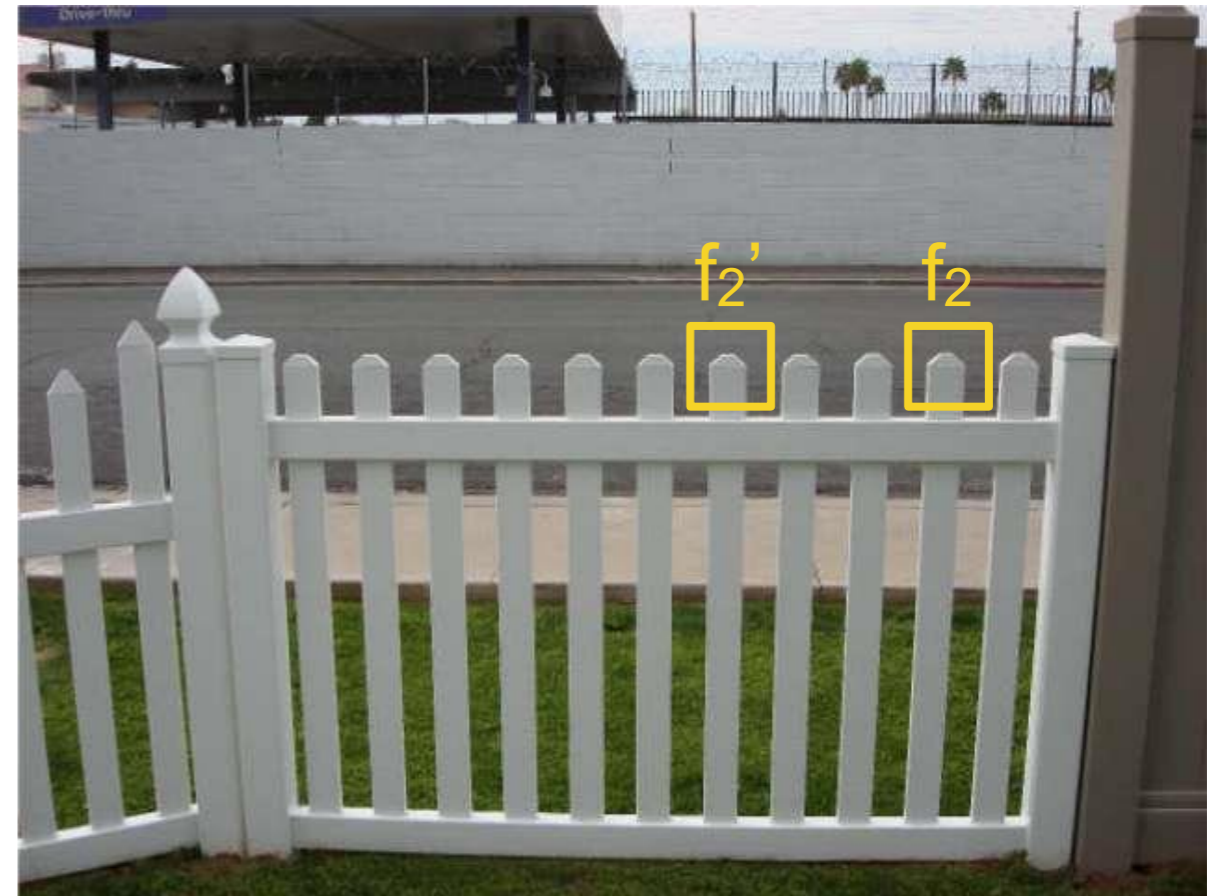
I_2

Matching Distance

- Better approach: best to second best ratio distance = $\text{SSD}(f_1, f_2) / \text{SSD}(f_1, f_2')$
- f_2 is best SSD match to f_1 in I_2
- f_2' is second best SSD match to f_1 in I_2
- Match should be “unique”

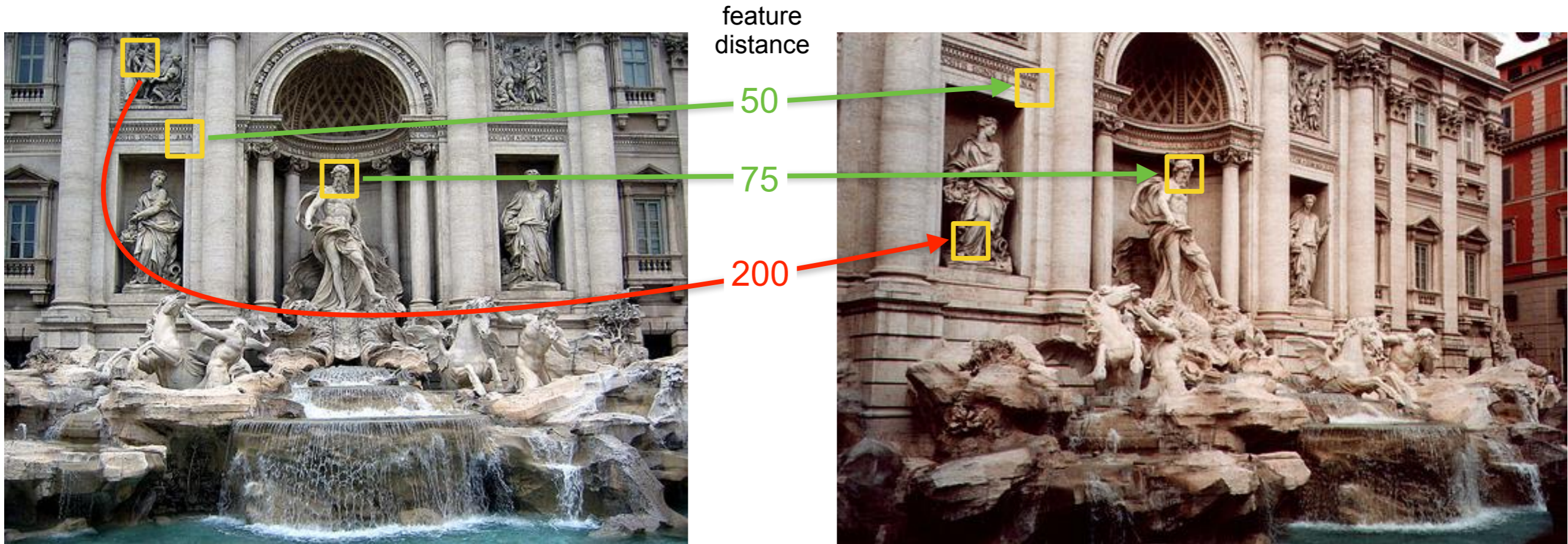


I_1



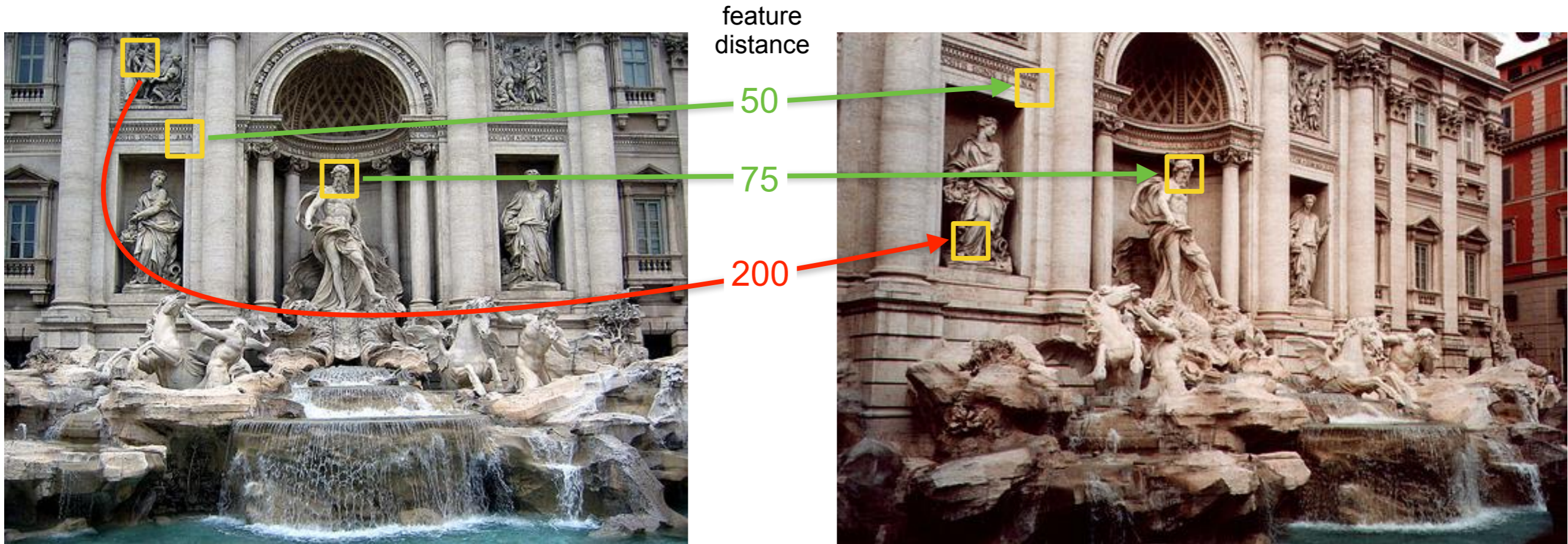
I_2

Eliminating Bad Matches



- Only accept matches with distance smaller a threshold.
- How to choose the threshold?

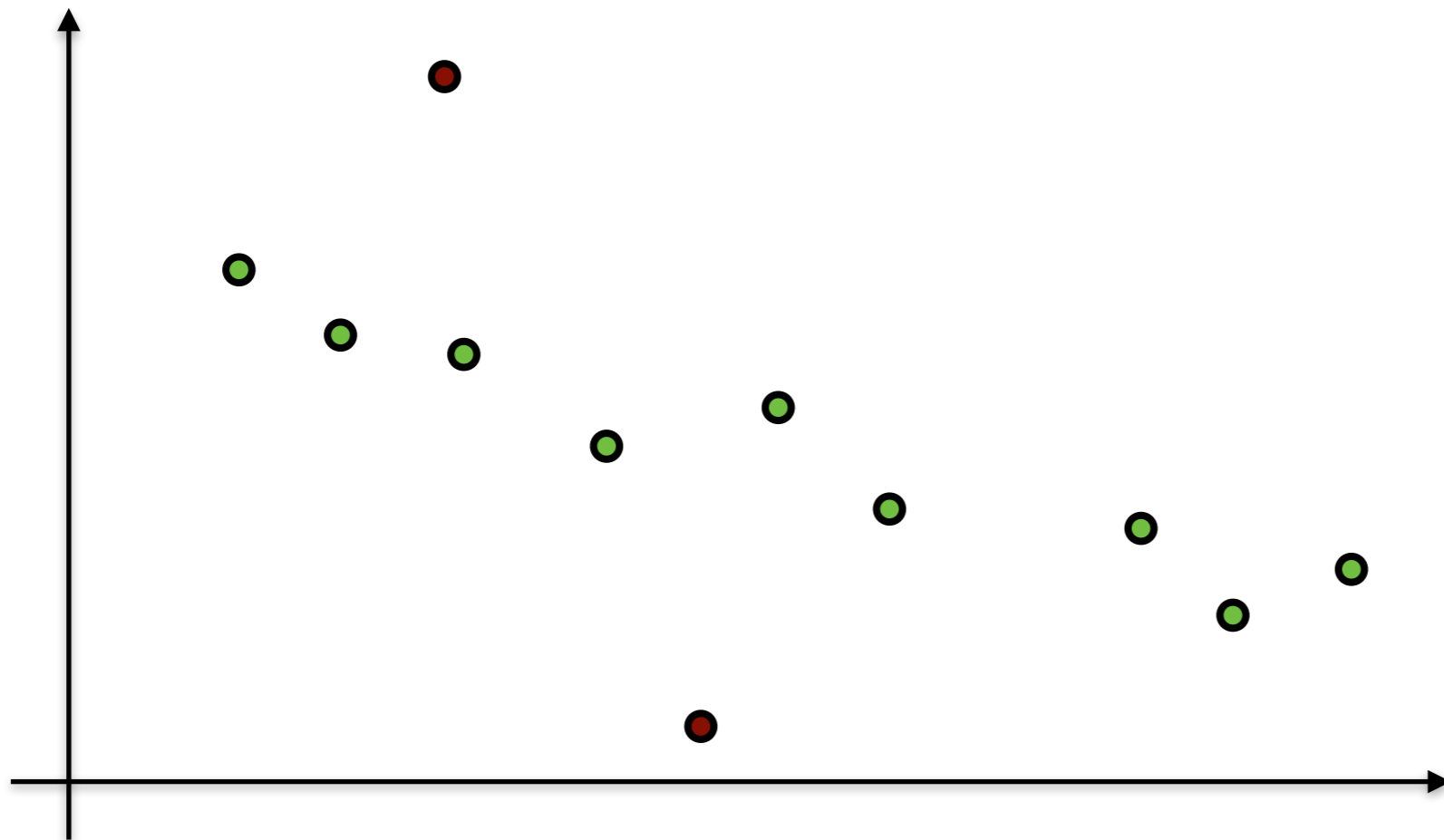
True/False Positives



- Choice of threshold affects performance
 - Too restrictive: less false positives, but also less true positives
 - Too lax: more true positives, but also more false positives
- Can we do more? Yes: Robust model fitting with RANSAC!

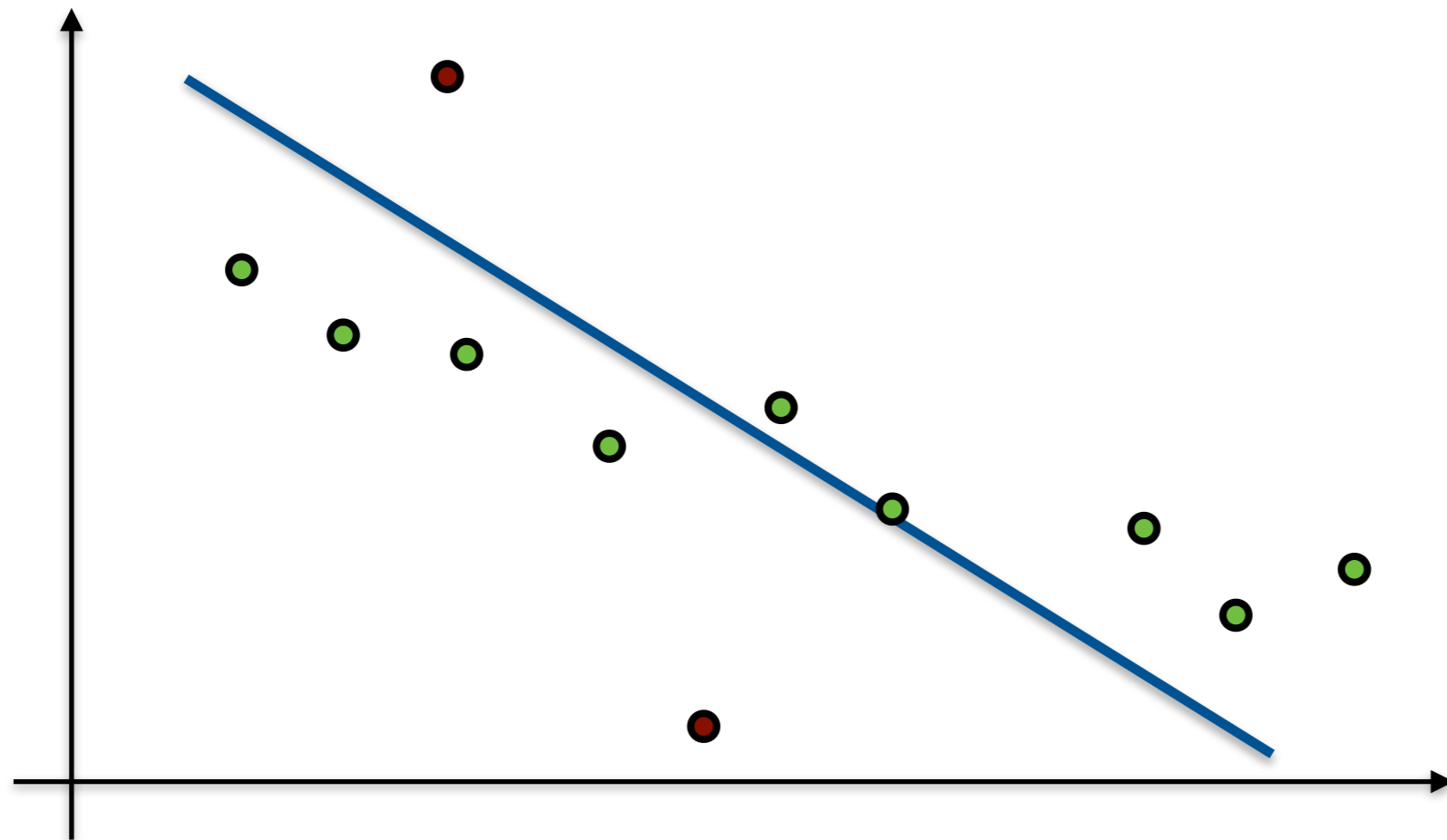
Random Sample Consensus (RANSAC)

- Model fitting in presence of noise and outliers
- Example: fitting a line through 2D points



Random Sample Consensus (RANSAC)

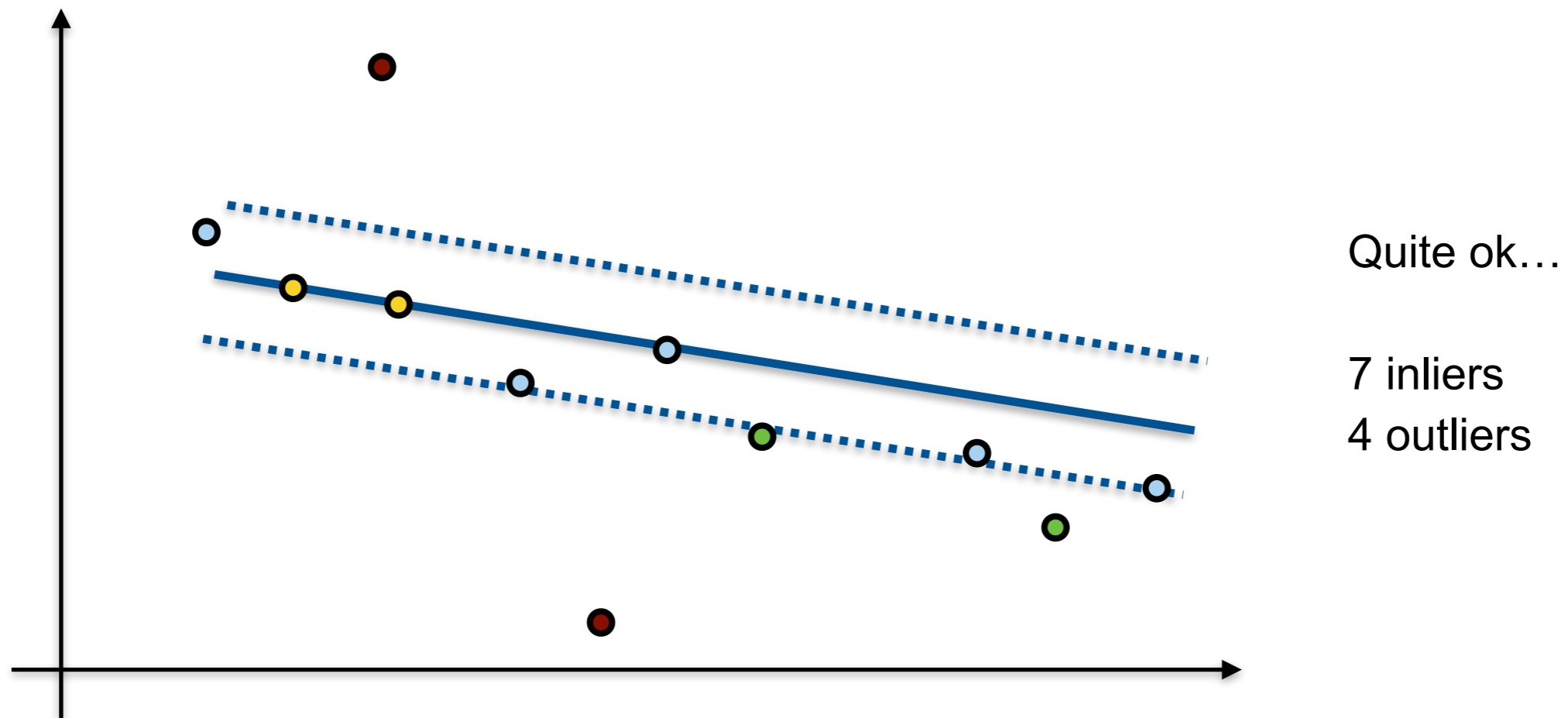
- Least-squares solution, assuming constant noise for all points



Bad!

Random Sample Consensus (RANSAC)

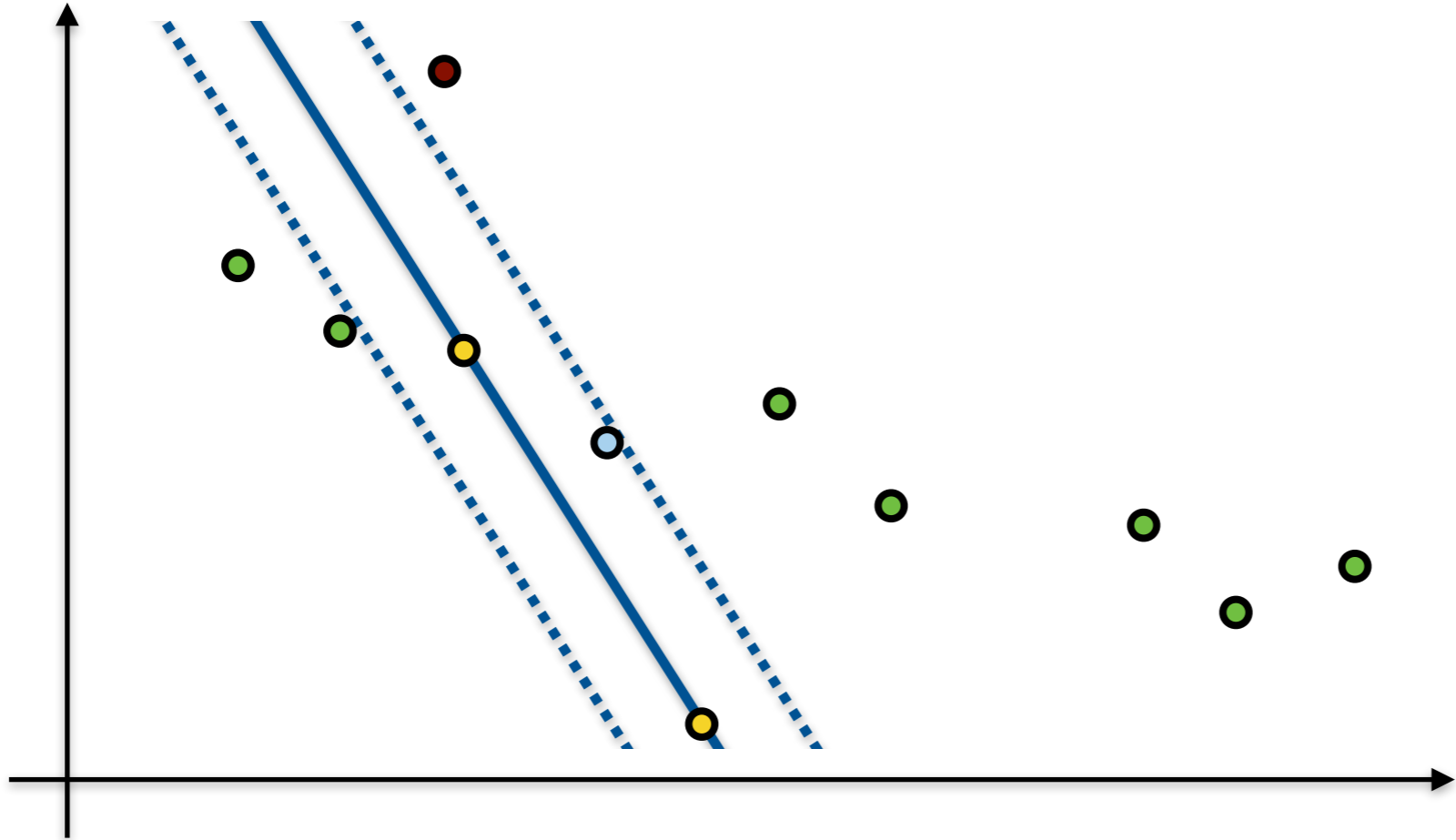
- We only need 2 points to fit a line. Let's try 2 random points.



Random Sample Consensus (RANSAC)



- Let's try 2 other random points.

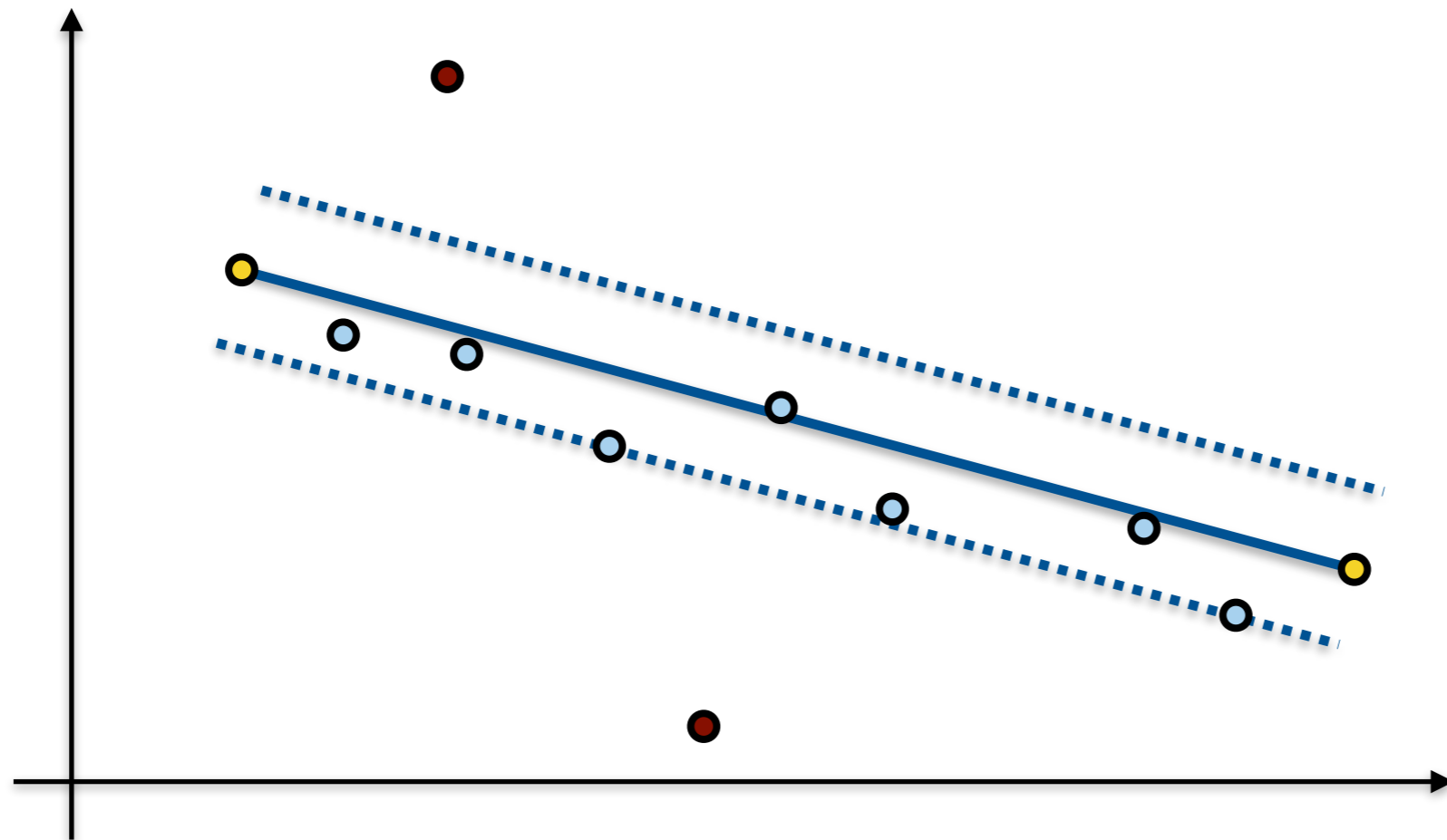


Quite bad..

3 inliers
8 outliers

Random Sample Consensus (RANSAC)

- Let's try yet another 2 random points.



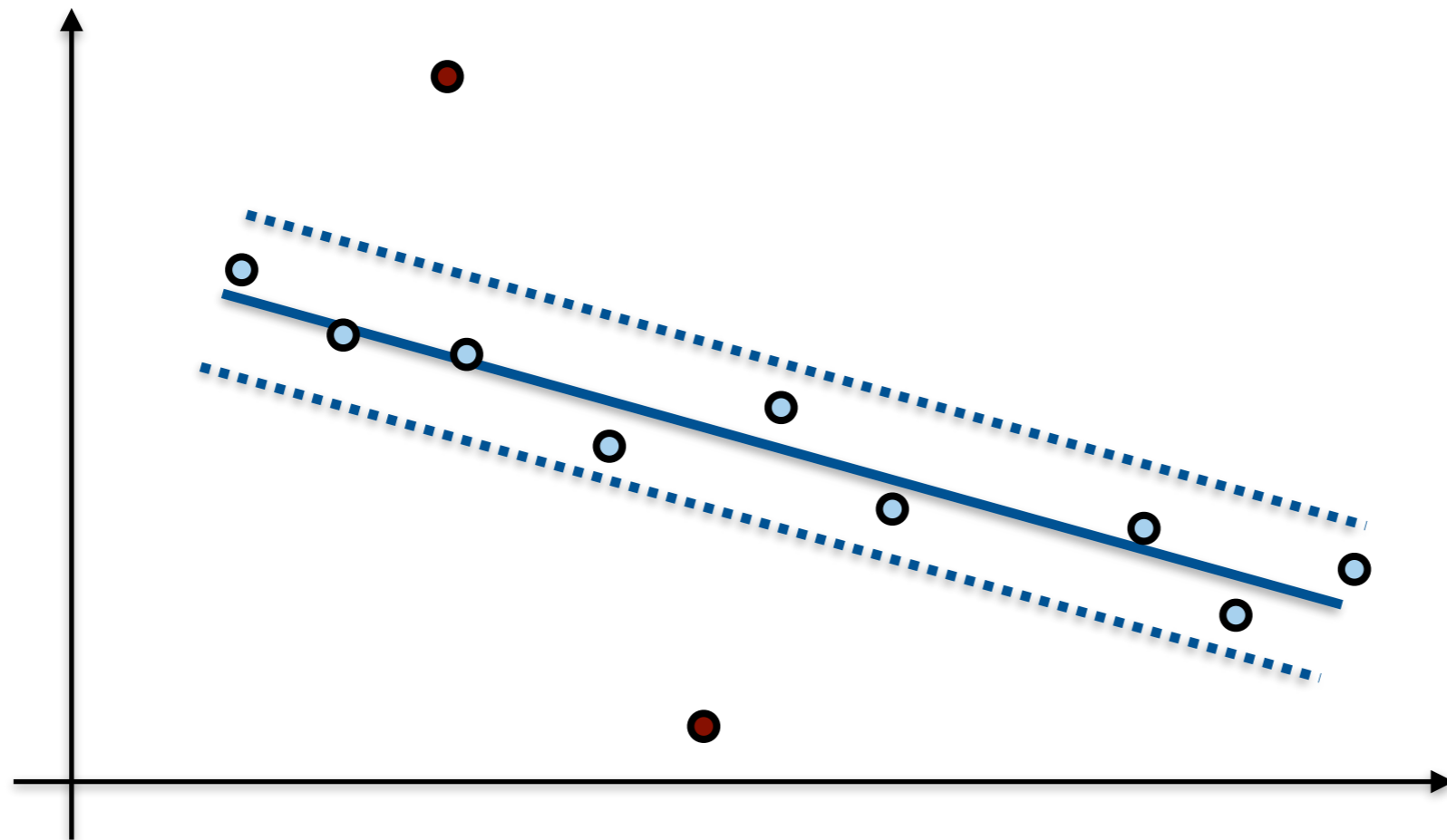
Quite good!

9 inliers

2 outliers

Random Sample Consensus (RANSAC)

- Let's use the inliers of the best trial so far to perform least squares fitting.



Even better!

RANSAC Algorithm

- RANdom SAmples Consensus algorithm formalizes this idea
- Algorithm:

Input: data D , min. required #data points for fitting s , success probability p , outlier ratio ϵ

Output: inlier set

1. Compute required number of iterations $N = \frac{\log(1 - p)}{\log(1 - (1 - \epsilon)^s)}$
2. For N iterations do:
 1. Randomly select a subset of s data points
 2. Fit model on the subset
 3. Count inliers and keep model/subset with largest number of inliers
3. Refit model using found inlier set

RANSAC Number of Iterations

N for $p = 0.99$

| | Required points s | Outlier ratio ε | | | | | | |
|------------------|---------------------|-----------------------------|------|------|------|------|------|-------|
| | | 10 % | 20 % | 30 % | 40 % | 50 % | 60 % | 70 % |
| Line | 2 | 3 | 5 | 7 | 11 | 17 | 27 | 49 |
| Plane | 3 | 4 | 7 | 11 | 19 | 35 | 70 | 169 |
| Essential Matrix | 8 | 9 | 26 | 78 | 272 | 1177 | 7025 | 70188 |

Epipolar Constraint

Epipolar Constraint on the Image Plane

- The epipolar constraint can be computed for a pair of matched 2D image points when the **relative camera pose is known**, here encoded by R, T (e.g. stereo pair)

- The **epipolar plane** is spanned by \mathbf{x}_L and the two camera centers O_L and O_R

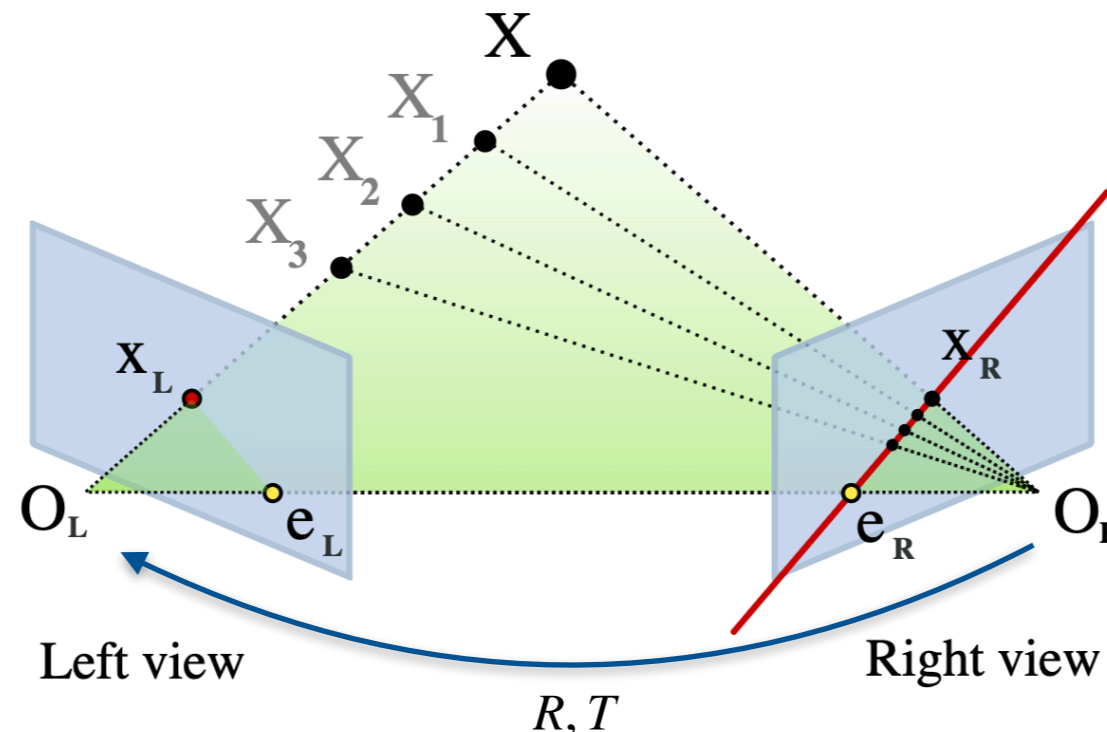
- The **epipolar line** is the intersection of the epipolar plane and the right image plane

- The **epipolar constraint** encodes that \mathbf{x}_R must lie on the epipolar line in the right image

$$\mathbf{x}_L^\top \hat{T} R \mathbf{x}_R = 0$$

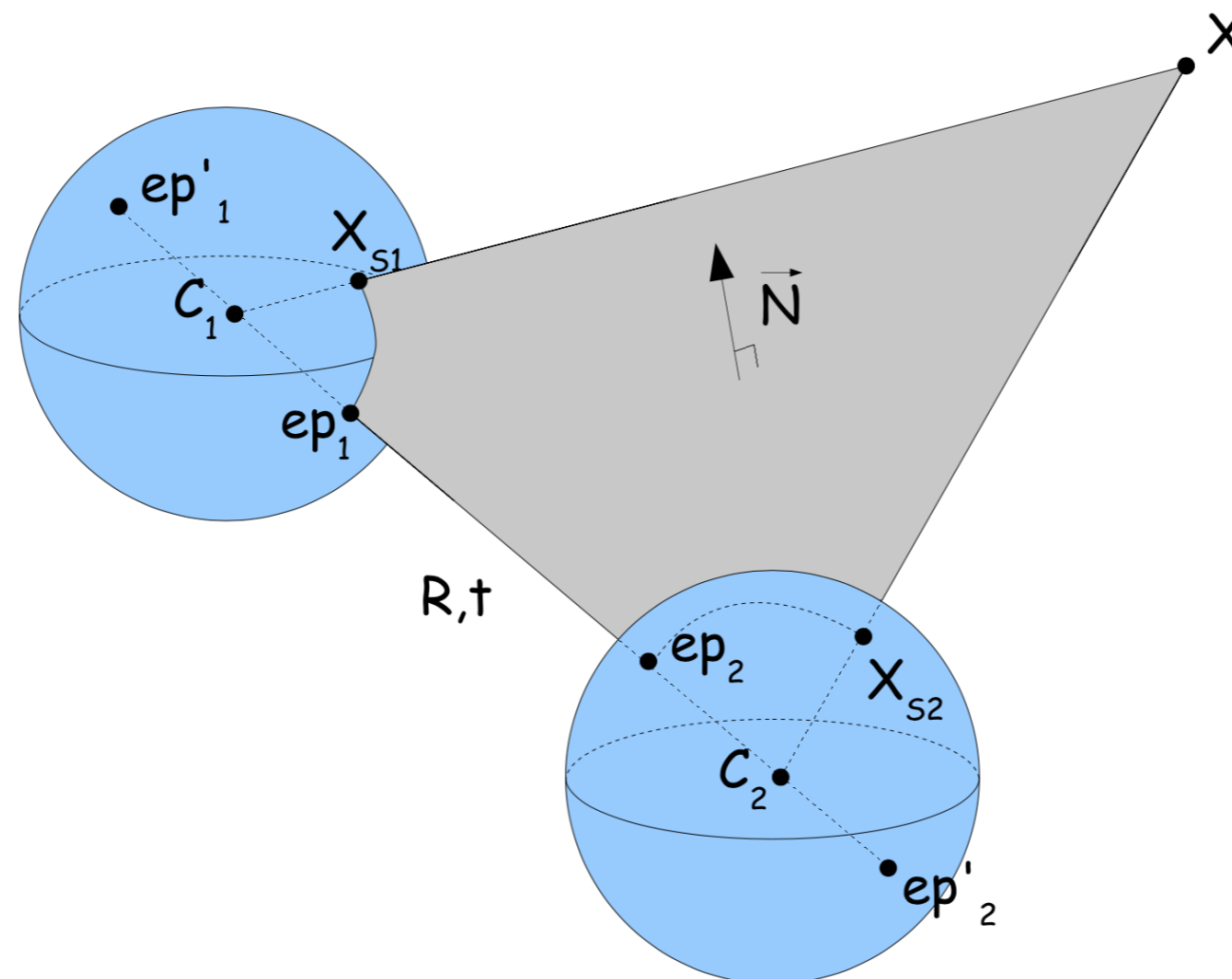
- $E = \hat{T} R$ is called the **Essential Matrix**

- Note that the epipolar constraint is not sufficient to guarantee a correct match. A wrong match may still lie elsewhere on the epipolar line. However, in many cases outliers can be correctly filtered.



Epipolar Constraint on a Sphere

- Epipolar constraint is also possible in 3D with unit-vectors.
- Epipolar line generalises to epipolar curve.



Motion Estimation

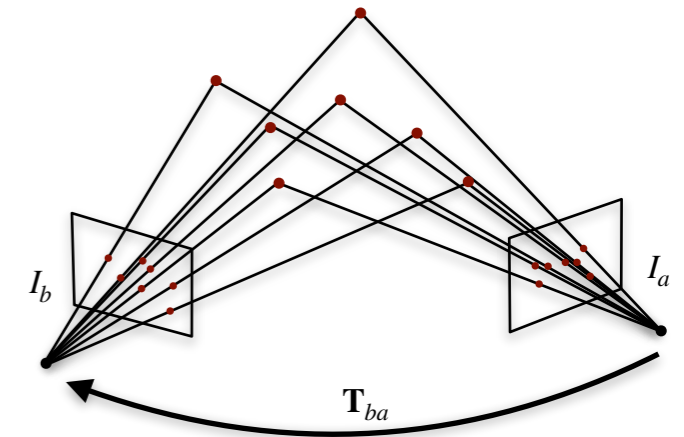
Motion Estimation from Point Correspondences

- **2D-to-2D**

- Reprojection error:

$$E(\mathbf{T}_{ba}, \mathcal{X}) = \sum_{i=1}^N \left\| \mathbf{y}_{a,i} - \pi(\mathbf{X}_i) \right\|^2 + \left\| \mathbf{y}_{b,i} - \pi(\mathbf{T}_{ba}(\mathbf{X}_i)) \right\|^2$$

- Linear Algorithm: 8-point, 5-point

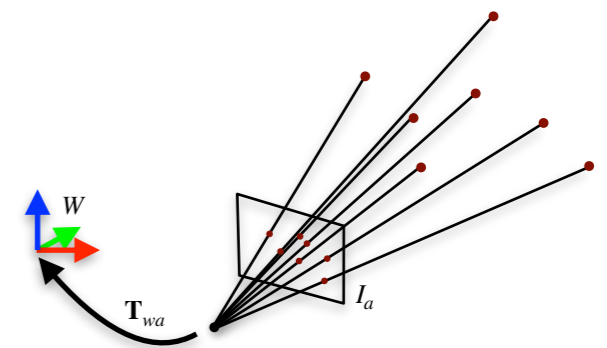


- **2D-to-3D**

- Reprojection error:

$$E(\mathbf{T}_{wa}, \mathcal{X}) = \sum_{i=1}^N \left\| \mathbf{y}_{a,i} - \pi(\mathbf{T}_{wa}^{-1}(\mathbf{X}_i)) \right\|^2$$

- Linear Algorithm: DLT PnP

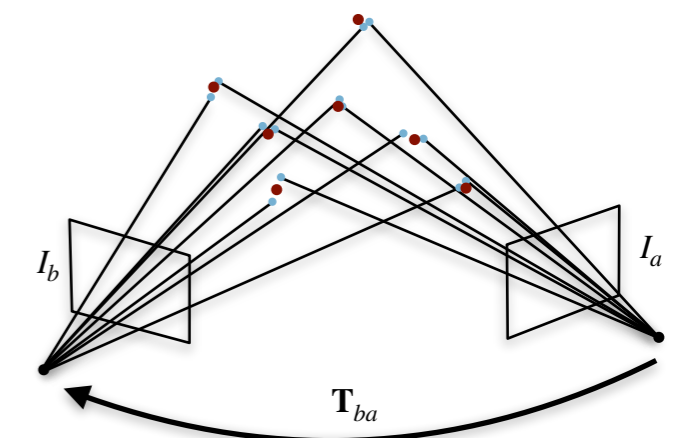


- **3D-to-3D**

- 3D geometric error:

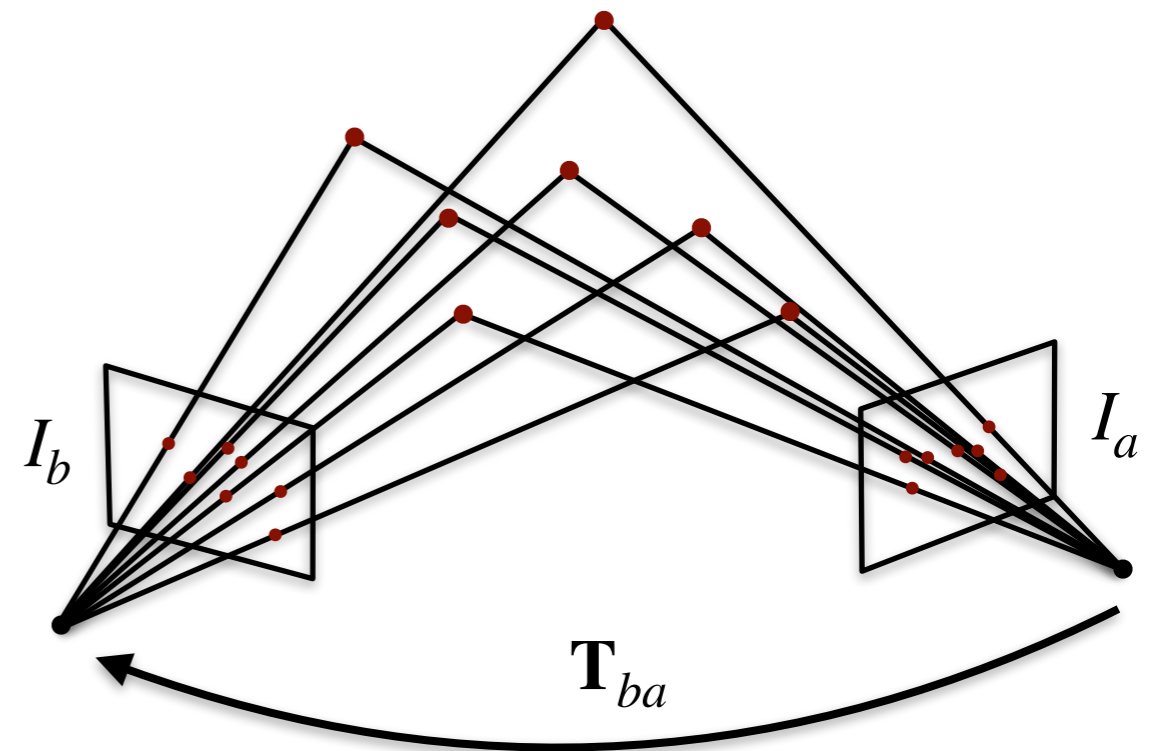
$$E(\mathbf{T}_{ab}) = \sum_{i=1}^N \left\| \mathbf{Y}_{a,i} - \mathbf{T}_{ab}(\mathbf{Y}_{b,i}) \right\|^2$$

- Linear Algorithm: Arun, Horn



2D-to-2D Motion Estimation

- Given corresponding image point observations
$$\mathcal{Y}_a = \{\mathbf{y}_{a,1}, \dots, \mathbf{y}_{a,N}\}$$
$$\mathcal{Y}_b = \{\mathbf{y}_{b,1}, \dots, \mathbf{y}_{b,N}\}$$
of unknown 3D points $\mathcal{X} = \{\mathbf{X}_1, \dots, \mathbf{X}_N\}$ (expressed in camera frame a)determine relative motion \mathbf{T}_{ba} between the frames



- Reprojection Error (Bundle Adjustment):

$$E(\mathbf{T}_{ba}^b, \mathcal{X}) = \sum_{i=1}^N \left\| \mathbf{y}_{a,i} - \pi(\mathbf{X}_i) \right\|^2 + \left\| \mathbf{y}_{b,i} - \pi(\mathbf{T}_{ba}(\mathbf{X}_i)) \right\|^2$$

Non-linear optimization requires good initialization. Non-convex, non-unique (scale ambiguity).

- Algebraic approach based on epipolar geometry to recover relative pose (up to scale) without explicitly recovering 3D point location: 8-point, 5-point algorithm
- Applications:
 - Filtering pairwise feature matches with RANSAC
 - Monocular SLAM / SfM initialisation

- Given a set of 3D points in world frame W

$$\mathcal{X} = \{\mathbf{X}_1, \dots, \mathbf{X}_N\}$$

and corresponding image observations

$$\mathcal{Y}_a = \{\mathbf{y}_{a,1}, \dots, \mathbf{y}_{a,N}\}$$

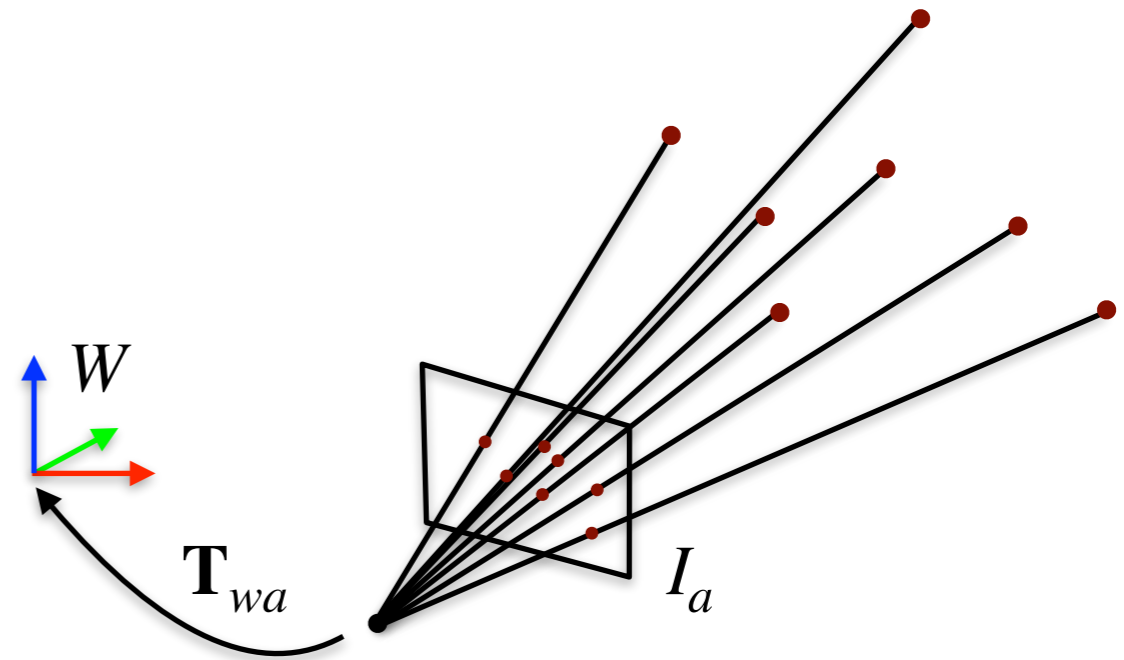
determine camera pose \mathbf{T}_{wa} in world frame

- Reprojection Error (Pose-only Bundle Adjustment):

$$E(\mathbf{T}_{wa}, \mathcal{X}) = \sum_{i=1}^N \left\| \mathbf{y}_{a,i} - \pi(\mathbf{T}_{wa}^{-1}(\mathbf{X}_i)) \right\|^2$$

Non-linear optimization requires good initialization. Non-convex.

- A.k.a. Perspective-n-Points (PnP) problem, many approaches exist, e.g.
 - Direct linear transform (DLT)
 - EPnP (Lepetit et al., An accurate $O(n)$ Solution to the PnP problem, IJCV 2009)
 - OPnP (Zheng et al., Revisiting the PnP Problem: A Fast, General and Optimal Solution, ICCV 2013)
- Applications:
 - Localize camera in local keypoint map (with RANSAC)



- Given corresponding 3D points in two camera frames

$$\mathcal{Y}_a = \{ \mathbf{Y}_{a,1}, \dots, \mathbf{Y}_{a,N} \}$$

$$\mathcal{Y}_b = \{ \mathbf{Y}_{b,1}, \dots, \mathbf{Y}_{b,N} \}$$

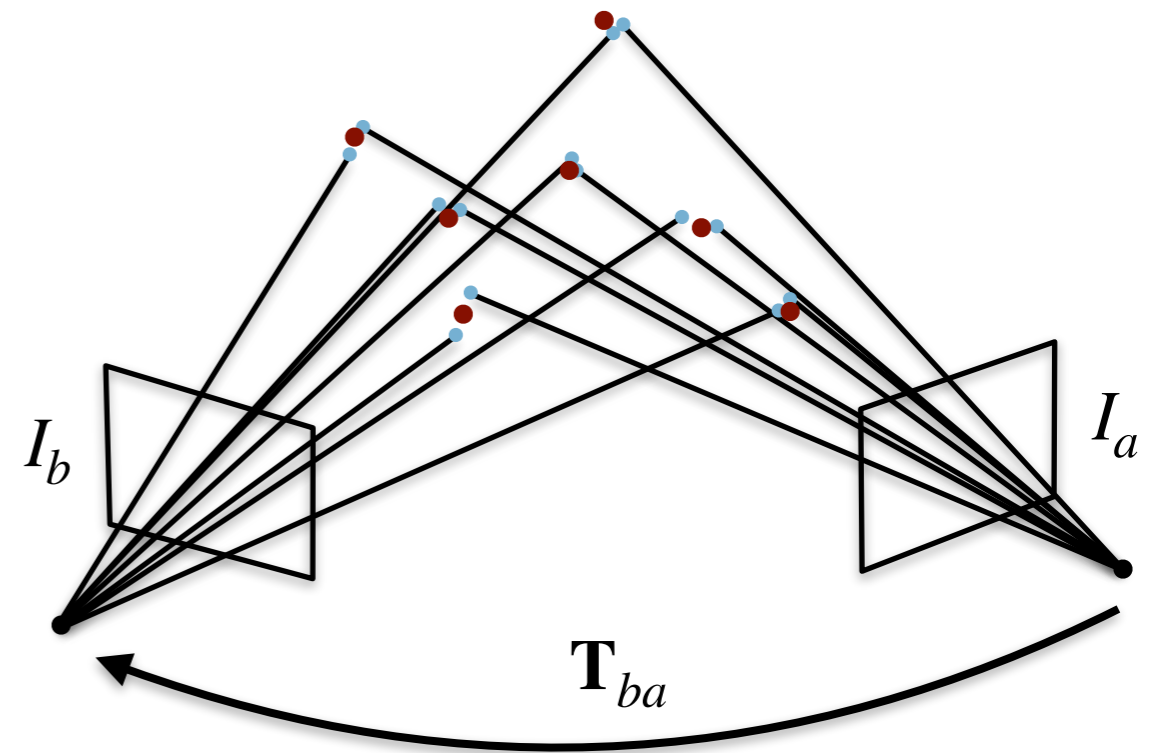
determine the relative camera pose \mathbf{T}_{ab}

- 3D geometric error:

$$E(\mathbf{T}_{ab}) = \sum_{i=1}^N \left\| \mathbf{Y}_{a,i} - \mathbf{T}_{ab}(\mathbf{Y}_{b,i}) \right\|^2$$

Corresponds to least-squares point cloud alignment.

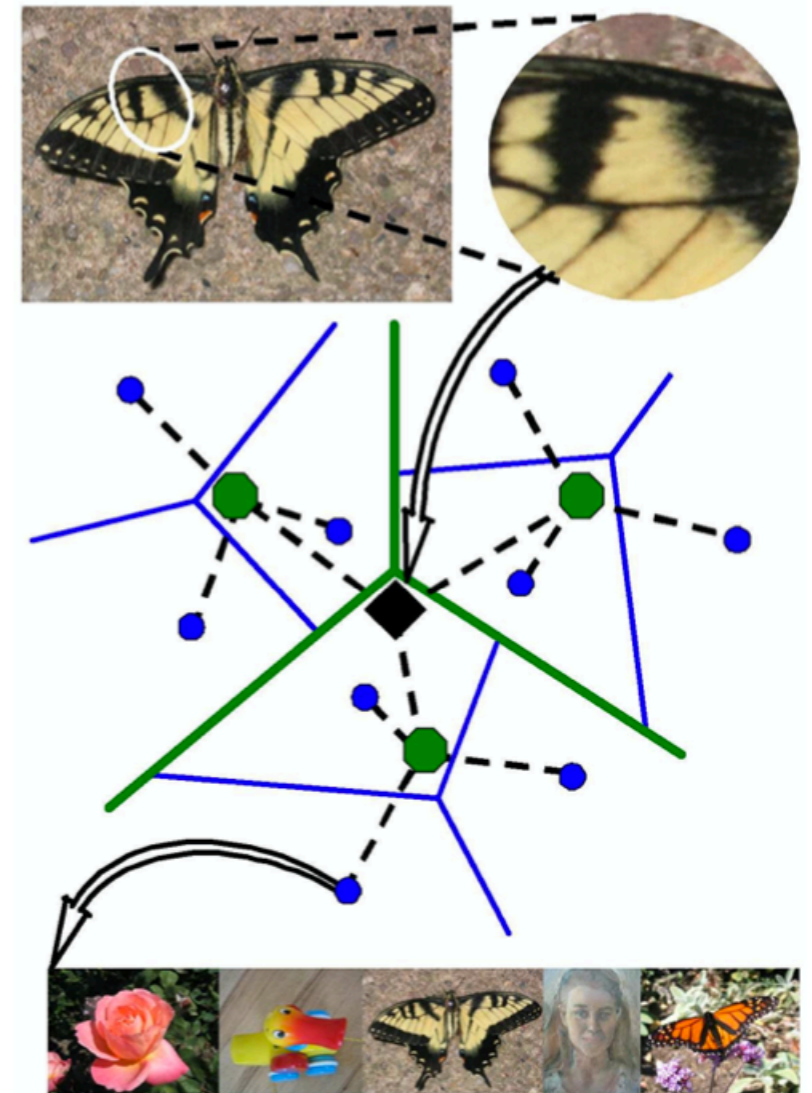
- Closed-form solutions available, e.g. Arun et al., 1987
- Applications:
 - Relative pose for calibrated stereo cameras (triangulation of 3D points) or RGB-D cameras (measured depth)
 - Loop-closure correction (variant with scale estimate available for monocular SLAM)



Place Recognition

Place Recognition with Bag of Words

- Place recognition aims to find similar images to a given query image
 - SfM: Which images to match?
 - SLAM: Detect loops
- Idea: Discretize the feature-descriptor space by hierarchical clustering in a “vocabulary tree”
 - visual “words” correspond to leaf-nodes
 - words are weighted by distinctiveness: e.g. “inverse document frequency” $\log(N/N_i)$
- Image comparison:
 - each feature is assigned to a word by passing it down the tree
 - for an image, count occurrence of each word (“term frequency”): bag-of-words vector
 - to compare images, compute the distance of (normalized) bag-of-words vectors
 - close bag-of-words vectors correspond to potentially similar images
- Vocabulary tree and weights are built offline from large collection of features
- False positives possible: Combine with geometric, temporal, ... consistency checks



Nistér & Stewenius, CVPR 2006

Efficient Query and Feature Matching

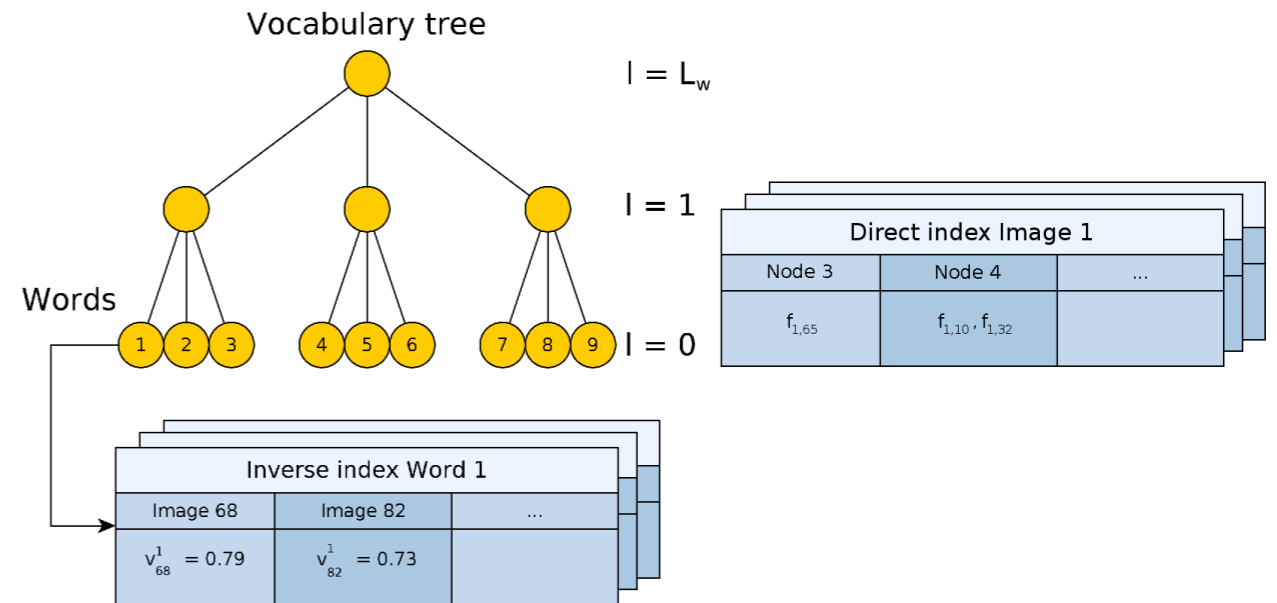
- Image query with “inverse index”:
 - For each word store list of images, and for each image cache the word count.
 - During query, consider only images from the inverse index of each word

- Efficient BoW distance (scoring):
 - BoW vectors are sparse (most entries are 0)

- For normalised vectors, use

$$\|\mathbf{q} - \mathbf{d}\|_p^p = 2 + \sum_{i|q_i \neq 0, d_i \neq 0} \left(|q_i - d_i|^p - |q_i|^p - |d_i|^p \right)$$

- L1-norm is often used ($p = 1$)
- Approximate nearest neighbour feature matching using “direct index”
 - For each image, store occurrence of nodes at each tree level and list of corresponding features
 - When matching images, only consider features from direct image at given level



Gálvez-López & Tardós, T-RO 2012

- Keypoint detection, description and matching is a well researched topic
- Highly performant corner and blob detectors exist
- Descriptors can be invariant to translation, rotation, scale, viewpoint, brightness, ... (e.g. SIFT)
- Binary descriptors are highly efficient and effective (e.g. ORB) and thus popular for real-time
- Keypoint matching by descriptor distance; uniqueness ensured by ratio-to-second-best
- Robust matching based on model fitting using RANSAC
- Motion estimation from 2D-to-2D, 2D-to-3D, 3D-to-3D matches
- Image retrieval and loop-closure detection with bag-of-words

Exercise Sheet 3

In the third exercise sheet you will:

- Implement rotation estimation and descriptor matching for a simple variant of ORB.
- Filter inlier matches for stereo pairs using the epipolar constraint.
- Filter inlier matches for arbitrary pairs using RANSAC with the 5-point algorithm.
- Implement bag-of-words for efficient image retrieval using an inverse index.

