Volume Rendering of Neural Implicit Surfaces

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INTRODUCTION

Background and Related Works

INTRODUCTION

- Volume Rendering
- Neural Radiance Field (NeRF)
    - In nutshell
    - Drawbacks?
- VolSDF Contribution
    - New density representation and reconstruction
        - Moving from generic to geometric density function

RELATED WORKS

- Neural Scene Representation & Rendering
  - Combining neural implicit functions with volume rendering
    - \+ Expressive representation power
    - \+ low memory foot-print
    - − Recovered geometry
    - − Opacity function approximation

RELATED WORKS

- Multi-view 3D Reconstruction
  - Depth-based approaches
    - – Complex pipeline
  - Voxel-based approaches
    - \+ Directly model objects in a volume
    - – Limited to low resolution
    - – Require accurate object masks

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METHODS

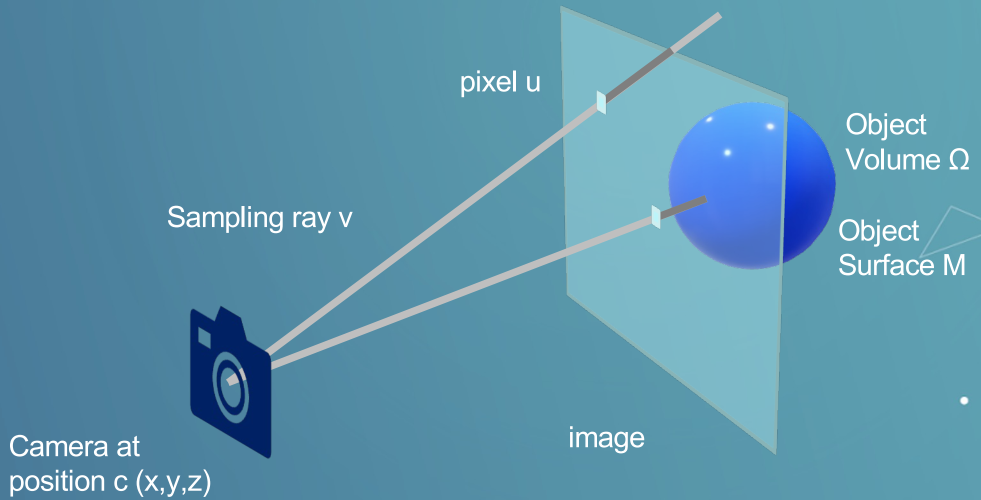
A New Geometric Model Density

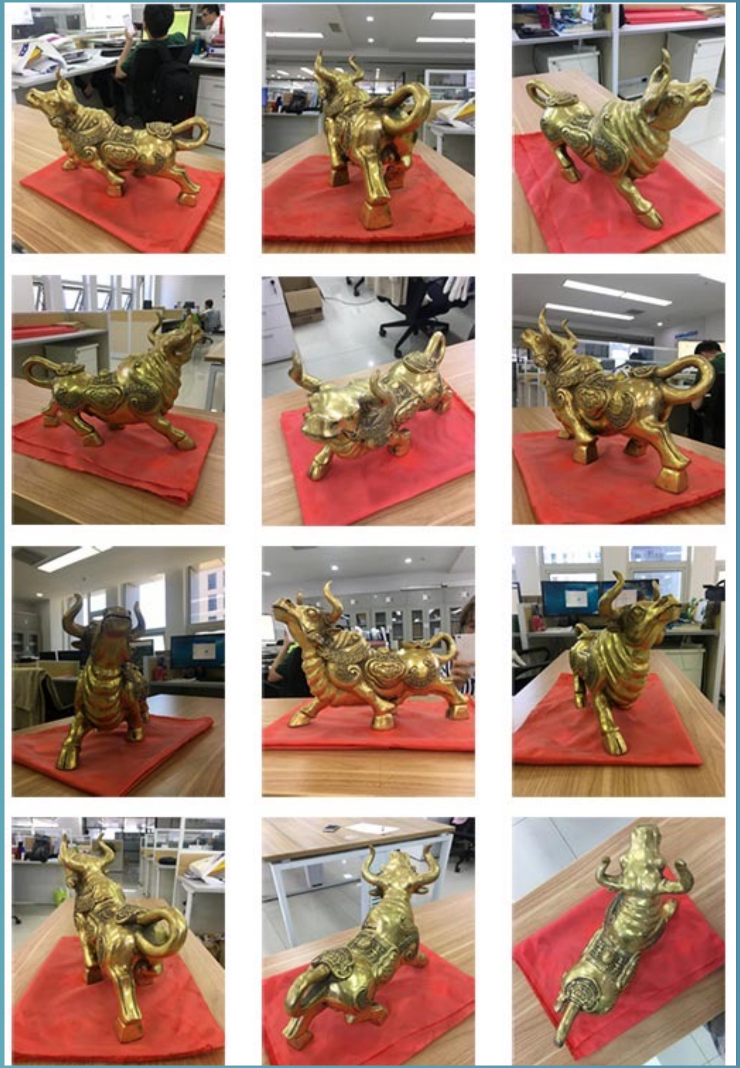
(1/5) Density As Transformed SDF

- What is SDF?
- Drawback of current model
- Improvement of previous model
- Advantages

(1/5) Density As Transformed SDF

- What is SDF?
  - A signed distance function is an n-dimensional implicit function, which associates a scalar value with each point of its n-dimensional domain.





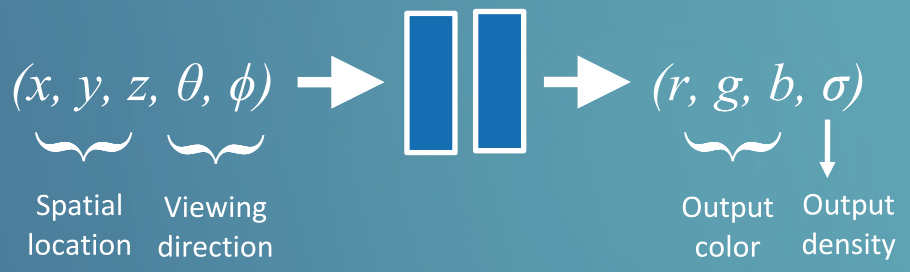
volumetric density

signed distance function

produced rendering

(1/5) Density As Transformed SDF

- What is SDF?
- Drawback of current model



(1/5) Density As Transformed SDF

- What is SDF?
- Drawback of current model

$(x, y, z, \theta, \phi) \rightarrow$  $\rightarrow (r, g, b, \sigma)$

- Geometric volume density at point x

$$\sigma(x) = \alpha \Psi_\beta(-d_\Omega(x)) \quad (2)$$

$$d_\Omega(x) = (-1)^{1_\Omega(x)} \min_{y \in \mathcal{M}} \|x - y\| \quad (1)$$

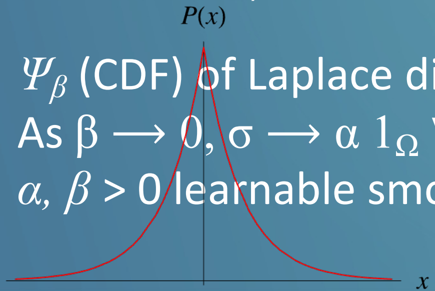
indicator if $x \in$ volume Ω

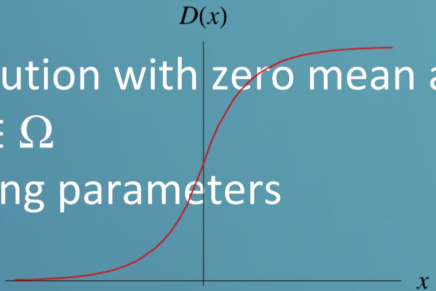
point y at surface M

Ψ_β (CDF) of Laplace distribution with zero mean and β scale

As $\beta \longrightarrow 0, \sigma \longrightarrow \alpha\, 1_\Omega\ \forall\, x \in \Omega$

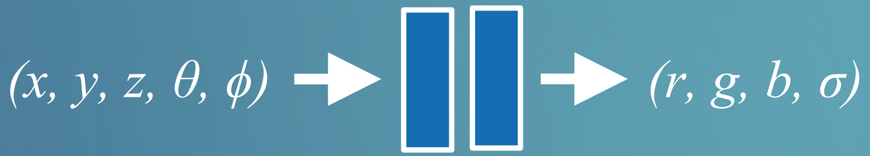
$\alpha, \beta > 0$ learnable smoothing parameters





(1/5) Density As Transformed SDF

- What is SDF?
- Drawback of current model



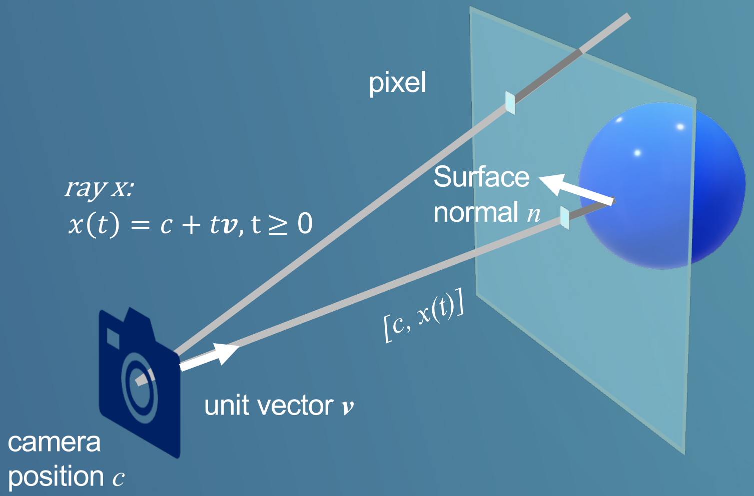
- Geometric volume density

$$\sigma(x) = \alpha \Psi_\beta \left(-d_\Omega(x)\right) \quad (2)$$

- Advantages

(2/5) Volume Rendering of σ

- Volume rendering integral
  - approximating the integrated (i.e., summed) light radiance along this ray reaching the camera



The probability a light particle succeeds traversing the segment [c, x(t)] without bouncing off

$$T(t) = \exp(-\int_0^t \sigma(x(s))ds) \quad (4)$$

The volume rendering equation is the expected light along the ray,

$$I(c, v) = \int_0^\infty L(x(t), n(t), v)\tau(t)dt \quad (7)$$

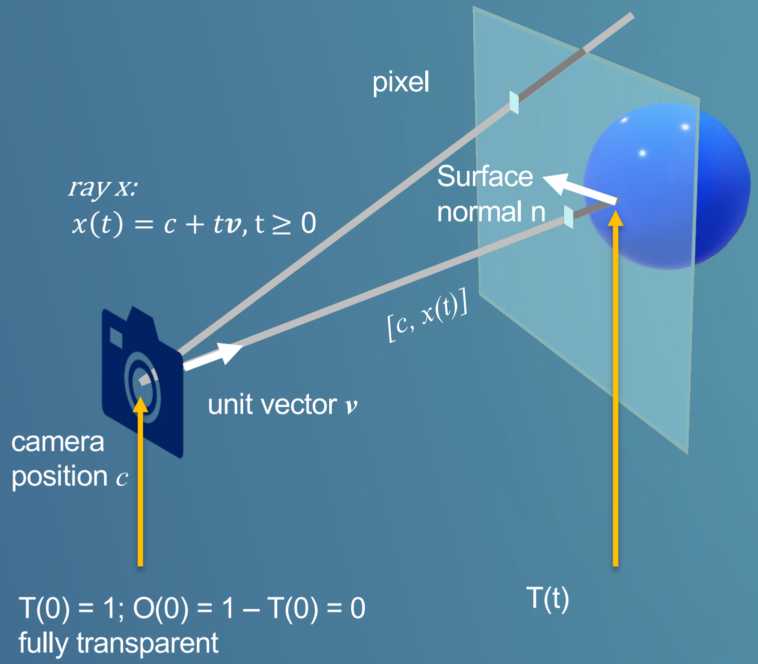
where $L(x, n, v)$ is the radiance field

(2/5) Volume Rendering of σ

- Opacity as CDF:

$$T(t) = \exp(-\int_0^t \sigma(x(s))ds) \quad (4)$$

$$\tau(t) = \sigma(x(t))\ T(t) = \frac{dO}{dt}(t) \quad PDF$$

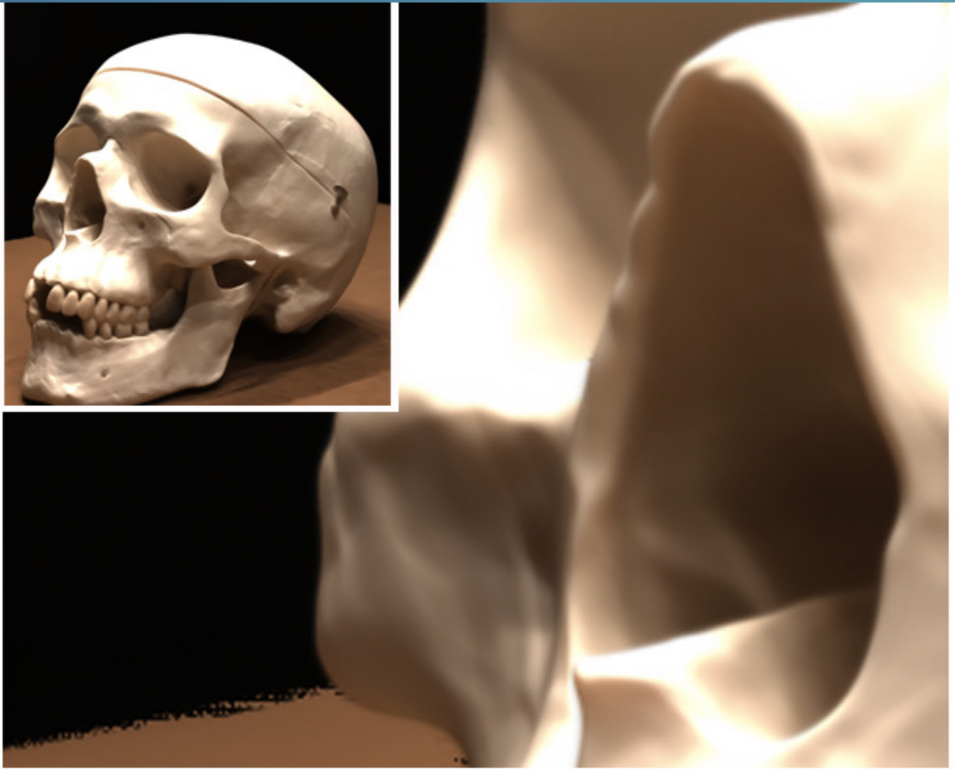


(2/5) Volume Rendering of σ

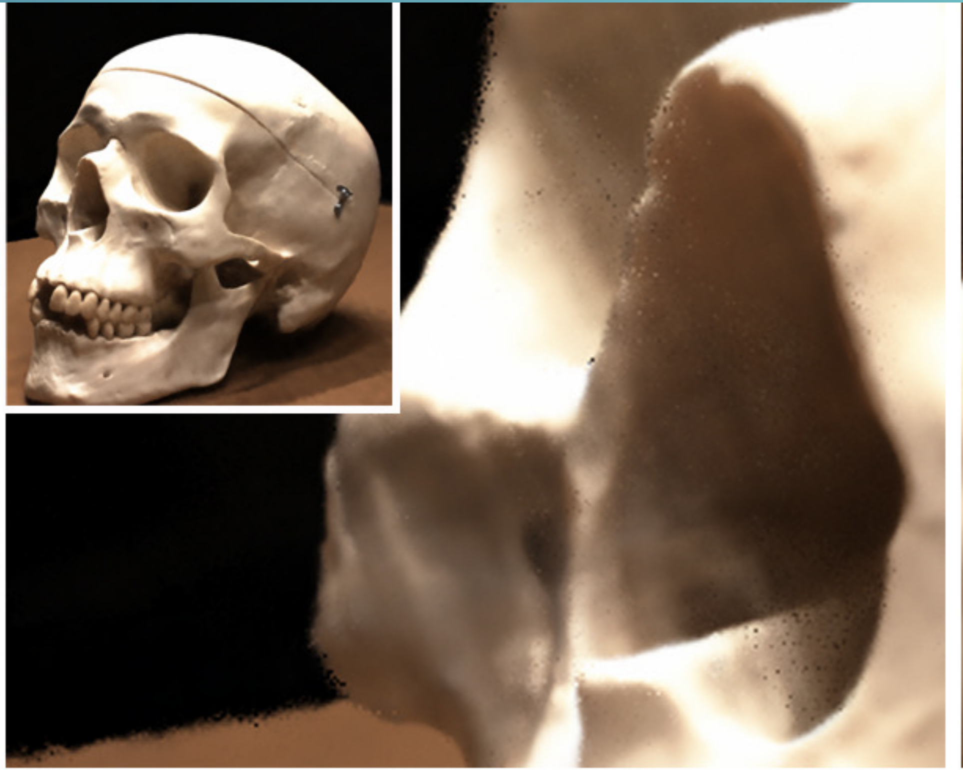
- Sampling τ
  - PDF τ is typically extremely concentrated near the object's boundary
  - the choice of the sample points S has a crucial effect on the approximation quality
  - NeRF: second, coarse network was trained specifically for the approximation of the opacity
  - VolSDF: sampling S is computed by a sampling algorithm based on an error bound for the opacity approximation

(2/5) Volume Rendering of σ

VolSFD

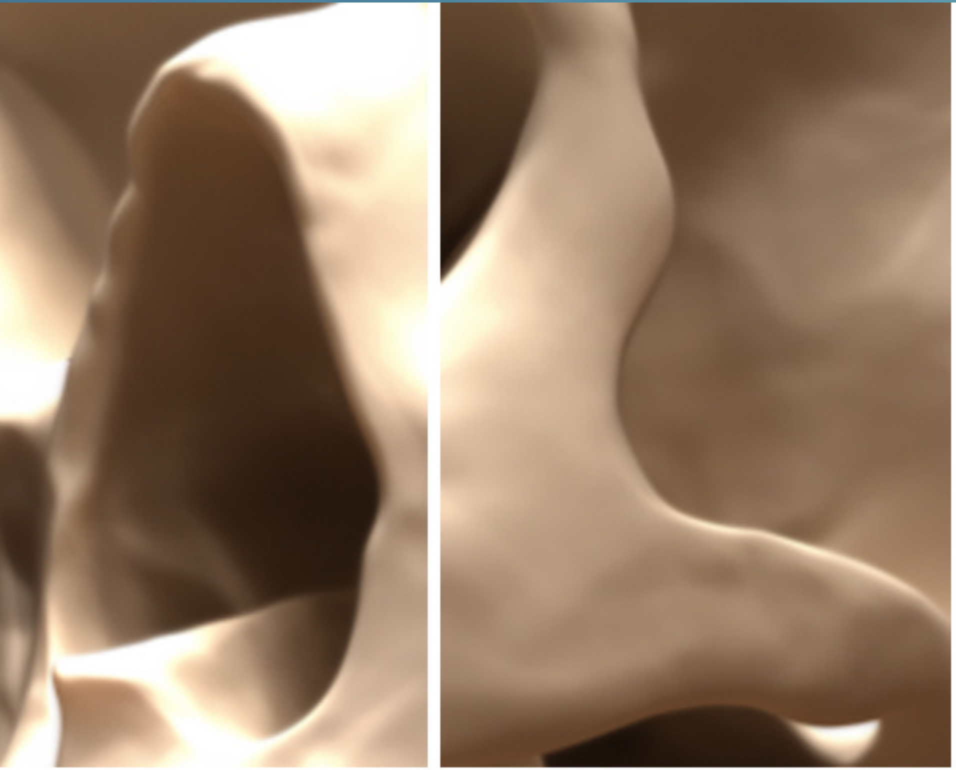


NeRF

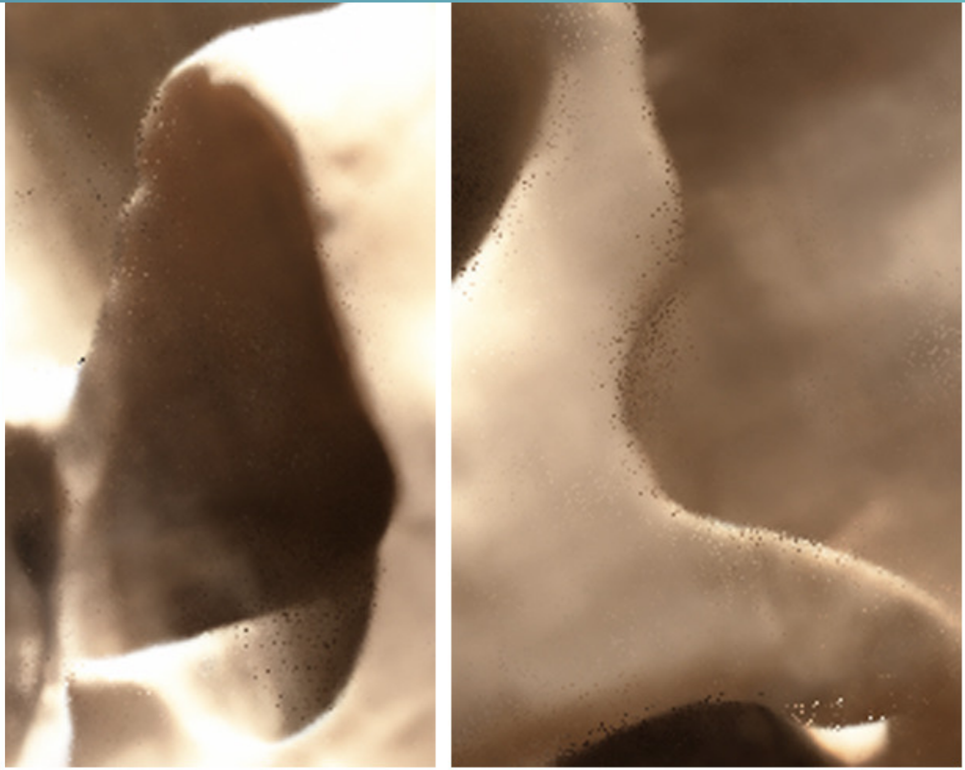


(2/5) Volume Rendering of σ

VolSFD



NeRF



(3/5) Bound on The Opacity Approximation Error

- Transparency for ray x with sample point s:

$$T(t) = \exp(-\int_0^t \sigma(x(s))ds)$$

$$O(t) = 1 - T(t)$$

$$O(t) = 1 - \exp(-\int_0^t \sigma(x(s))ds)$$

$$O(t) = 1 - \exp(-\hat{R}(t))$$

$$\hat{R}(t) = \sum_{i=1}^{k-1} \delta_i \sigma_i + (t - t_k)\sigma_k$$

(3/5) Bound on The Opacity Approximation Error

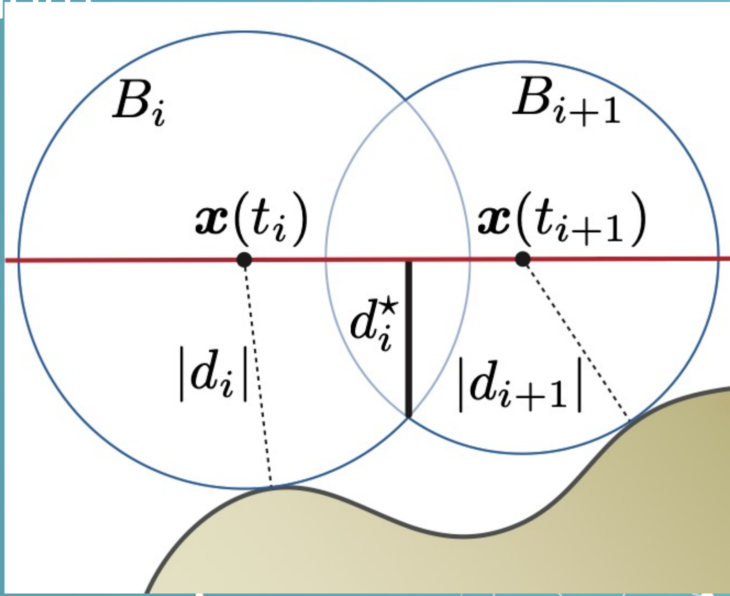
- What about the error in summation?

$$\sigma(x) = \alpha\Psi_\beta(-d_\Omega(x))$$

$$\Psi_\beta = \begin{cases} \frac{1}{2}\exp\left(\frac{s}{\beta}\right), s \leq 0 \\ 1 - \frac{1}{2}\exp\left(-\frac{s}{\beta}\right), s > 0 \end{cases}$$

$$\frac{d}{dt}\Psi_\beta = \frac{1}{2\beta}\exp\left(-\frac{|s|}{\beta}\right)$$

$$\left|\frac{d}{ds}\boldsymbol{\sigma}(x(s))\right| \leq \frac{\alpha}{2\beta}\exp\left(-\frac{d_i^*}{\beta}\right), where\ d_i^* = \min_{\substack{s\in[t_i,t_{i+1}] \\ y\notin B_i\cup B_{i+1}}} \|x(s) - y\|$$



(3/5) Bound on The Opacity Approximation Error

- This bound can be used to derive an error bound for $\hat{R}$

$$\left|\frac{d}{ds}\sigma(x(s))\right| \leq \frac{\alpha}{2\beta}\exp\left(-\frac{d_i^*}{\beta}\right), where\ d_i^* = \min_{\substack{s\in[t_i,t_{i+1}] \\ y\notin B_i \cup B_{i+1}}} \|x(s)-y\|$$

$$\hat{R}(t) = \sum_{i=1}^{k-1} \delta_i \sigma_i + (t - t_k)\sigma_k$$

$$|E(t)| \leq \hat{E}(t) = \frac{\alpha}{4\beta}\left(\sum_{i=1}^{k-1} \delta_i^2 \exp\left(-\frac{d_i^*}{\beta}\right) + (t-t_k)^2 \exp\left(-\frac{d_k^*}{\beta}\right)\right).$$

- So the error of the appr. opacity Ô can be bounded as

$$\left|O(t) - \hat{O}(t)\right| \leq \exp\left(-\hat{R}(t)\right)\left(\exp\left(\hat{E}(t)\right) - 1\right)$$

(3/5) Bound on The Opacity Approximation Error

- Taking the maximum over all intervals furnishes a bound $B_{\mathcal{T},\beta}$ as a function of $\mathcal{T}$ and β

$$\left|O(t) - \hat{O}(t)\right| \leq \exp\left(-\hat{R}(t)\right)\left(\exp\left(\hat{E}(t)\right) - 1\right)$$

$$\max_{t\in[o,M]}\left|O(t) - \hat{O}(t)\right| \leq B_{\mathcal{T},\beta} = \max_{k\in[n-1]}\left\{\exp\left(-\hat{R}(t_k)\right)\left(\exp\left(\hat{E}(t_{k+1})\right) - 1\right)\right\}$$

- Where is $\mathcal{T}$ a set of samples

$$\mathcal{T} = \{t_i\}_{i=1}^{n}, 0 = t_1 < \cdots < t_n = M$$

(4/5) Sampling Algorithm

- Using the bound to compute sampling:
- $I(\boldsymbol{c}, \boldsymbol{v}) = \int_0^\infty L(x(t), n(t), v)\tau(t)dt$
- $I(\boldsymbol{c}, \boldsymbol{v}) \approx \hat{I}_S(\boldsymbol{c}, \boldsymbol{v}) = \sum_{i=1}^{m-1} \hat{\tau}_i L_i$

Algorithm 1: Sampling algorithm.

Input: error threshold $\epsilon > 0$; β

Initialize $\mathcal{T} = \mathcal{T}_0$

Initialize β_+ such that $B_{\mathcal{T},\beta_+} \leq \epsilon$

while $B_{\mathcal{T},\beta} > \epsilon$ *and not max_iter* **do**

  upsample $\mathcal{T}$

  if $B_{\mathcal{T},\beta_+} < \epsilon$ **then**

    Find $\beta_\star \in (\beta, \beta_+)$ so that $B_{\mathcal{T},\beta_\star} = \epsilon$

    Update $\beta_+ \leftarrow \beta_\star$

  end

end

Estimate $\widehat{O}$ using $\mathcal{T}$ and β_+

$\mathcal{S} \leftarrow$ get fresh m samples using $\hat{O}^{-1}$

return $\mathcal{S}$

(4/5) Sampling Algorithm

Setting β as:

$$\beta \geq \frac{\alpha M}{4(n-1)\log(1+\epsilon)}$$

For $n > 0$, $\epsilon > 0$ and $B_{\mathcal{T},\beta} \leq \epsilon$

Here n = 128 was used.

Algorithm 1: Sampling algorithm.

Input: error threshold $\epsilon > 0$; β

Initialize $\mathcal{T} = \mathcal{T}_0$

Initialize β_+ such that $B_{\mathcal{T},\beta_+} \leq \epsilon$

while $B_{\mathcal{T},\beta} > \epsilon$ *and not max_iter* **do**

    upsample $\mathcal{T}$

    if $B_{\mathcal{T},\beta_+} < \epsilon$ **then**

        Find $\beta_\star \in (\beta, \beta_+)$ so that $B_{\mathcal{T},\beta_\star} = \epsilon$

        Update $\beta_+ \leftarrow \beta_\star$

    end

end

Estimate $\widehat{O}$ using $\mathcal{T}$ and β_+

$\mathcal{S} \leftarrow$ get fresh m samples using $\hat{O}^{-1}$

return $\mathcal{S}$

(4/5) Sampling Algorithm

We initialize $\mathcal{T}$ with uniform sampling $\mathcal{T}_0$

Algorithm 1: Sampling algorithm.

Input: error threshold $\epsilon > 0$; β

```
Initialize 𝒯 = 𝒯_0
Initialize β_+ such that B_{𝒯,β_+} ≤ ϵ
while B_{𝒯,β} > ϵ and not max_iter do
    upsample 𝒯
    if B_{𝒯,β_+} < ϵ then
        Find β_⋆ ∈ (β, β_+) so that
          B_{𝒯,β_⋆} = ϵ
        Update β_+ ← β_⋆
    end
end
Estimate Ô using 𝒯 and β_+
𝒮 ← get fresh m samples using Ô^{-1}
return 𝒮
```

(4/5) Sampling Algorithm

pick $\beta_+ > \beta$ so that the error bound satisfies the required ϵ bound

Algorithm 1: Sampling algorithm.

Input: error threshold $\epsilon > 0$; β

Initialize $\mathcal{T} = \mathcal{T}_0$

Initialize β_+ such that $B_{\mathcal{T},\beta_+} \leq \epsilon$

while $B_{\mathcal{T},\beta} > \epsilon$ *and not max_iter* **do**

  upsample $\mathcal{T}$

  if $B_{\mathcal{T},\beta_+} < \epsilon$ **then**

    Find $\beta_\star \in (\beta, \beta_+)$ so that $B_{\mathcal{T},\beta_\star} = \epsilon$

    Update $\beta_+ \leftarrow \beta_\star$

  end

end

Estimate $\widehat{O}$ using $\mathcal{T}$ and β_+

$\mathcal{S} \leftarrow$ get fresh m samples using $\hat{O}^{-1}$

return $\mathcal{S}$

(4/5) Sampling Algorithm

n samples are added to $\mathcal{T}$ to reduce β_+ while keeping $B_{\mathcal{T},\beta}$ within error bound

Algorithm 1: Sampling algorithm.

Input: error threshold $\epsilon > 0$; β

Initialize $\mathcal{T} = \mathcal{T}_0$
Initialize β_+ such that $B_{\mathcal{T},\beta_+} \leq \epsilon$
while $B_{\mathcal{T},\beta} > \epsilon$ *and not max_iter* **do**
    upsample $\mathcal{T}$
    if $B_{\mathcal{T},\beta_+} < \epsilon$ **then**
        Find $\beta_\star \in (\beta, \beta_+)$ so that $B_{\mathcal{T},\beta_\star} = \epsilon$
        Update $\beta_+ \leftarrow \beta_\star$
    end
end
Estimate $\widehat{O}$ using $\mathcal{T}$ and β_+
$\mathcal{S} \leftarrow$ get fresh m samples using $\hat{O}^{-1}$
return $\mathcal{S}$

(4/5) Sampling Algorithm

We use the bisection method (10 max iterations) to search for β_* and update β_+

Algorithm 1: Sampling algorithm.

Input: error threshold $\epsilon > 0$; β

Initialize $\mathcal{T} = \mathcal{T}_0$

Initialize β_+ such that $B_{\mathcal{T},\beta_+} \leq \epsilon$

while $B_{\mathcal{T},\beta} > \epsilon$ *and not max_iter* **do**

  upsample $\mathcal{T}$

  if $B_{\mathcal{T},\beta_+} < \epsilon$ **then**

    Find $\beta_\star \in (\beta, \beta_+)$ so that $B_{\mathcal{T},\beta_\star} = \epsilon$

    Update $\beta_+ \leftarrow \beta_\star$

  end

end

Estimate $\widehat{O}$ using $\mathcal{T}$ and β_+

$\mathcal{S} \leftarrow$ get fresh m samples using $\hat{O}^{-1}$

return $\mathcal{S}$

(4/5) Sampling Algorithm

Run iteratively until $B_{\mathcal{T},\beta} \leq \epsilon$ (5 max iter)

Algorithm 1: Sampling algorithm.

Input: error threshold $\epsilon > 0$; β

Initialize $\mathcal{T} = \mathcal{T}_0$

Initialize β_+ such that $B_{\mathcal{T},\beta_+} \leq \epsilon$

while $B_{\mathcal{T},\beta} > \epsilon$ *and not max_iter* **do**

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  if $B_{\mathcal{T},\beta_+} < \epsilon$ **then**

    Find $\beta_\star \in (\beta, \beta_+)$ so that $B_{\mathcal{T},\beta_\star} = \epsilon$

    Update $\beta_+ \leftarrow \beta_\star$

  end

end

Estimate $\widehat{O}$ using $\mathcal{T}$ and β_+

$\mathcal{S} \leftarrow$ get fresh m samples using $\hat{O}^{-1}$

return $\mathcal{S}$

(4/5) Sampling Algorithm

Use final $\mathcal{T}$ and β_+ to est. opacity $\hat{O}$

Algorithm 1: Sampling algorithm.

Input: error threshold $\epsilon > 0$; β

Initialize $\mathcal{T} = \mathcal{T}_0$

Initialize β_+ such that $B_{\mathcal{T},\beta_+} \leq \epsilon$

while $B_{\mathcal{T},\beta} > \epsilon$ *and not max_iter* **do**

  upsample $\mathcal{T}$

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    Find $\beta_\star \in (\beta, \beta_+)$ so that $B_{\mathcal{T},\beta_\star} = \epsilon$

    Update $\beta_+ \leftarrow \beta_\star$

  end

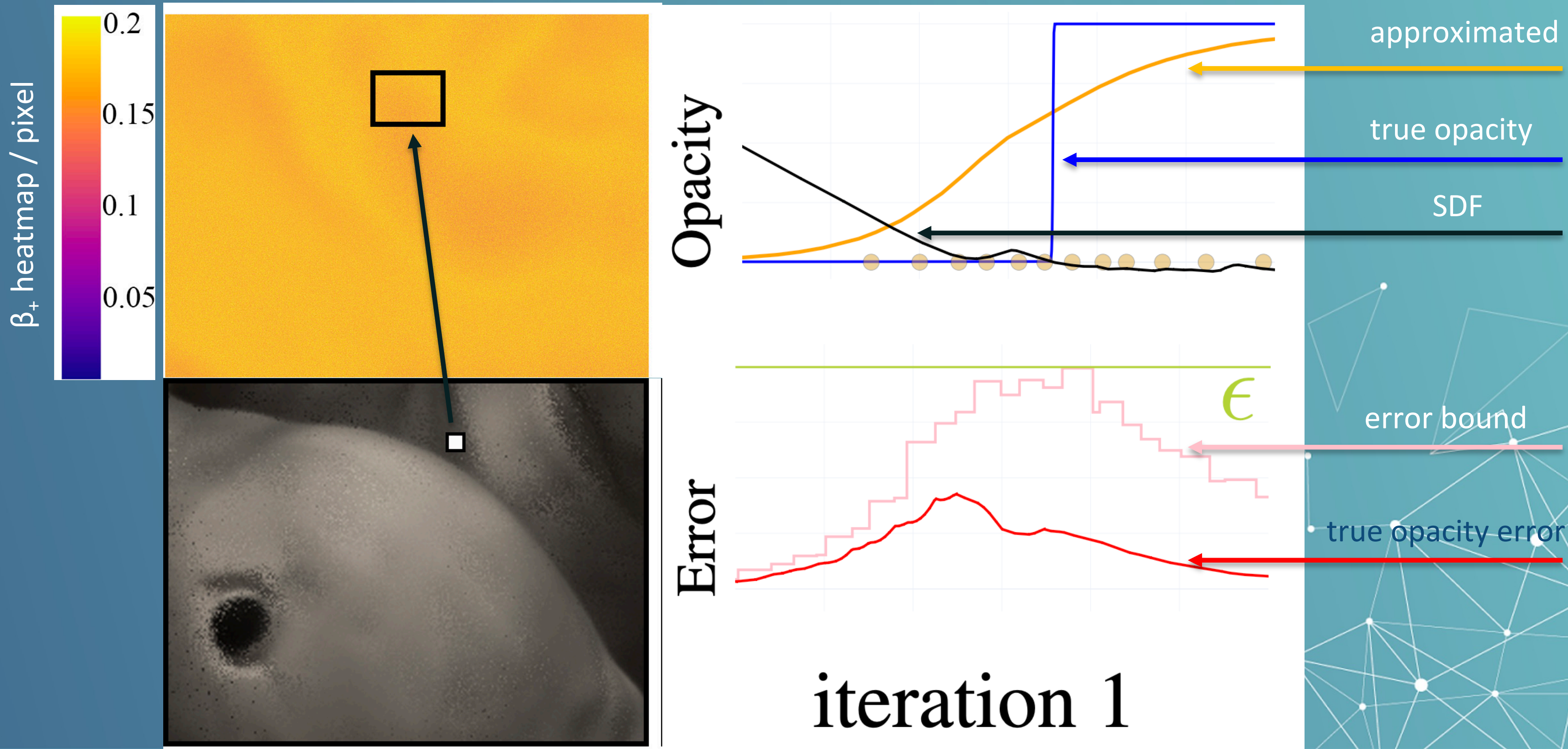
end

Estimate $\hat{O}$ using $\mathcal{T}$ and β_+

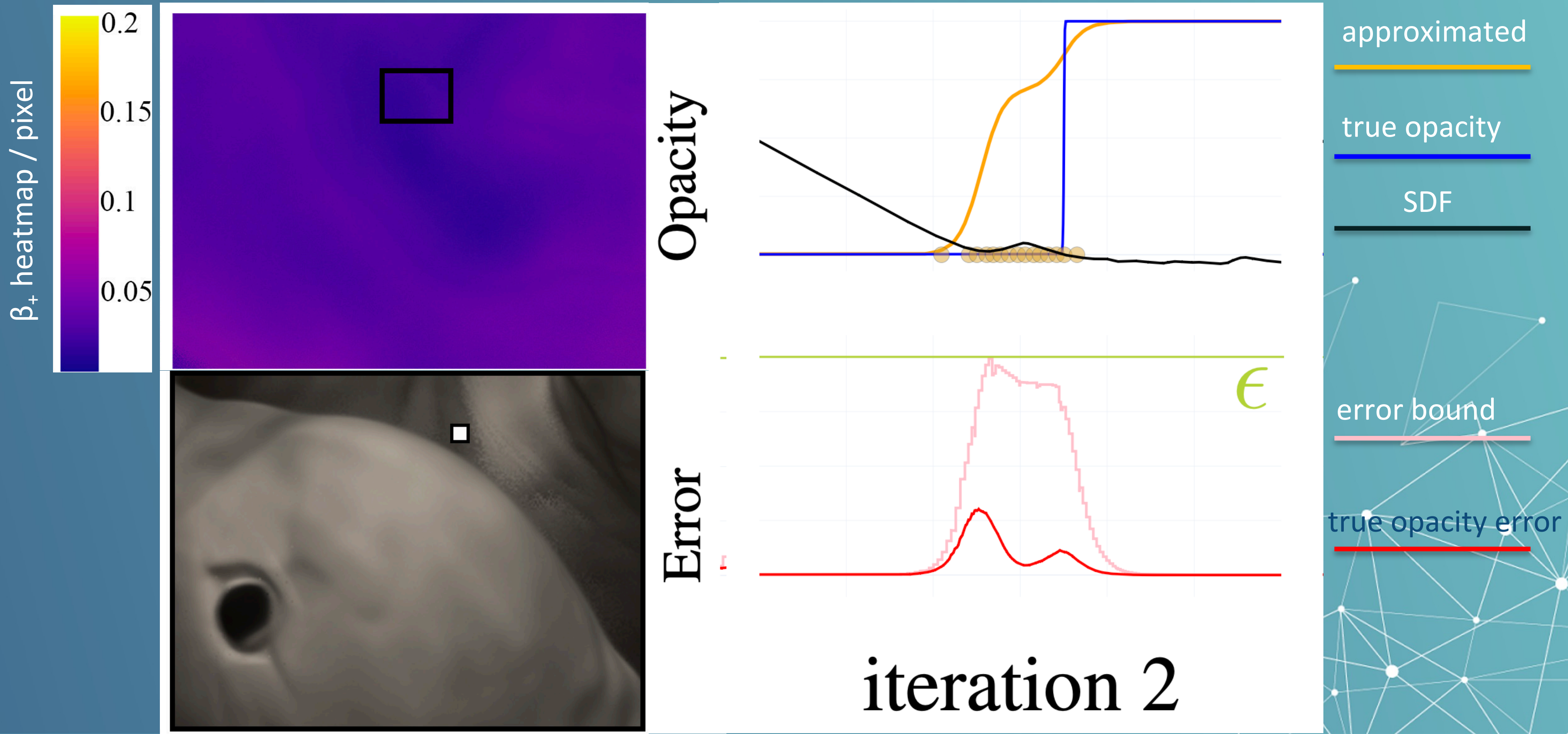
$\mathcal{S} \leftarrow$ get fresh m samples using $\hat{O}^{-1}$

return $\mathcal{S}$

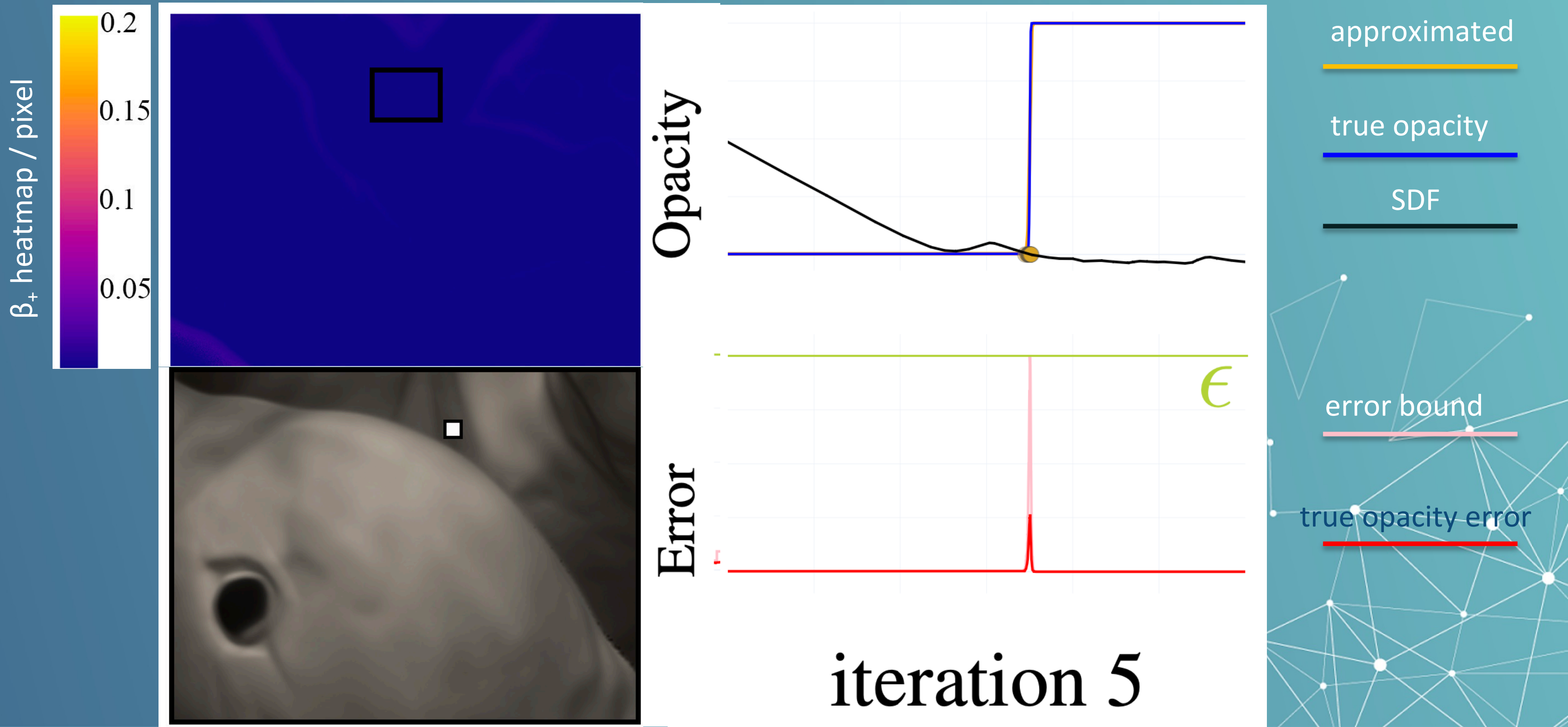
(4/5) Sampling Alogorithm - Qualitative



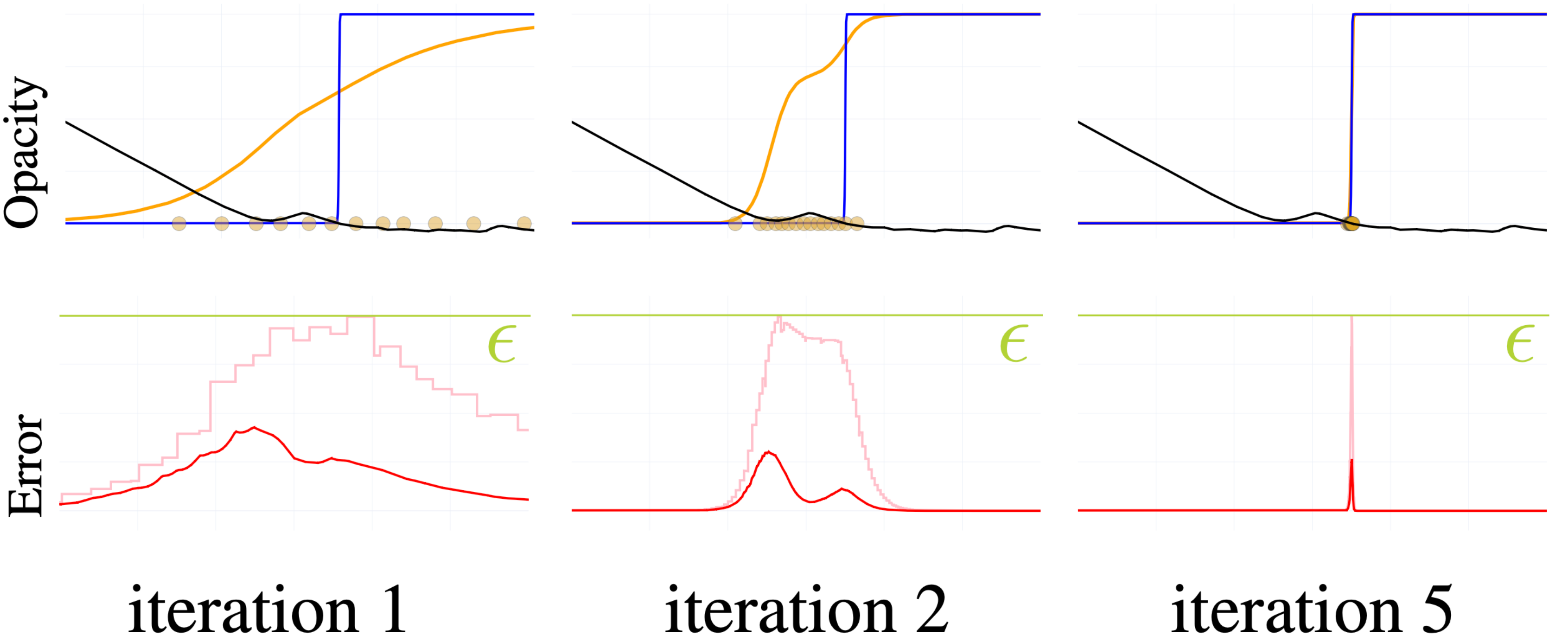
(4/5) Sampling Alogorithm - Qualitative



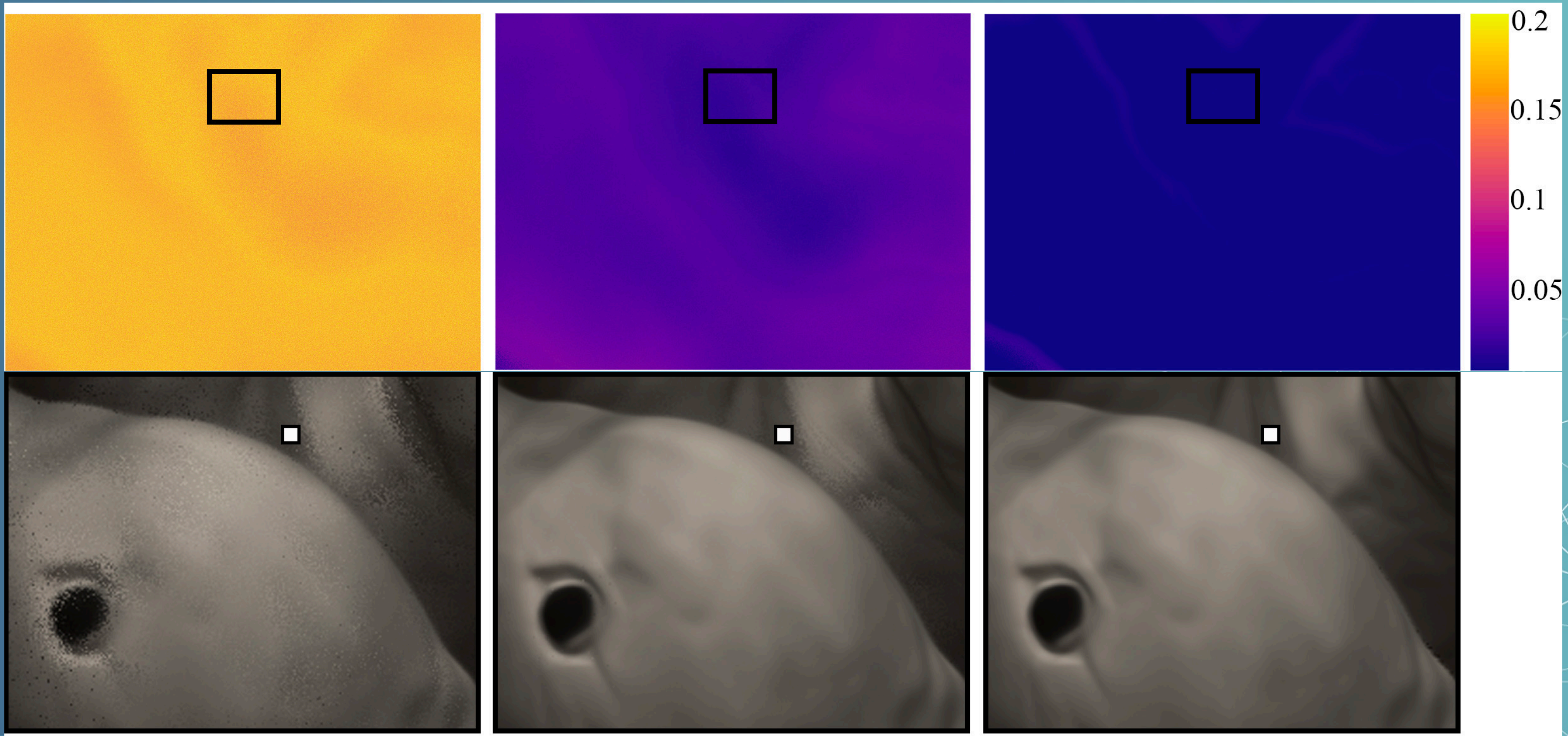
(4/5) Sampling Alogorithm - Qualitative



(4/5) Sampling Alogorithm - Qualitative



(4/5) Sampling Alogorithm - Qualitative



(5/5) Training

- 2x MLP:
  - Approximating the SDF of the learned geometry, and global geometry feature z of dimension 256 :
    - $f_\varphi(x) = (d(x), z(x)) \in \mathbb{R}^{1+256}$
  - Presenting the scene's radiance field with learnable parameters ψ :
    - $L_\psi(x,n,v,z) \in \mathbb{R}^3$
  - Two scalar learnable parameters
    - $\alpha, \beta \in \mathbb{R}$, with $\alpha = \beta^{-1}$
  - Positional enconding for x and v, same as NeRF

(5/5) Training

- For each pixel p a triplet $(I_p\ ,\ c_p\ ,\ v_p)$
  - $I_p \in \mathbb{R}^3$ is its intensity (RGB color)
  - $c_p \in \mathbb{R}^3$ is its camera location
  - $v_p \in \mathbb{R}^3$ is the viewing direction (camera to pixel)
- Training loss:
  - $\mathcal{L}(\theta) = \mathcal{L}_{\mathrm{RGB}}(\theta) + \lambda\, \mathcal{L}_{\mathrm{SDF}}(\varphi)$ (17)
  - $\mathcal{L}_{\mathrm{RGB}}(\theta) = \mathbb{E}_p \left\| I_p - \hat{I}_S(c_p, v_p) \right\|_1$ (18) color loss
  - $\mathcal{L}_{\mathrm{SDF}}(\varphi) = \mathbb{E}_z \left(\|\nabla d(z)\| - 1 \right)^2$ (18) Eikonal loss

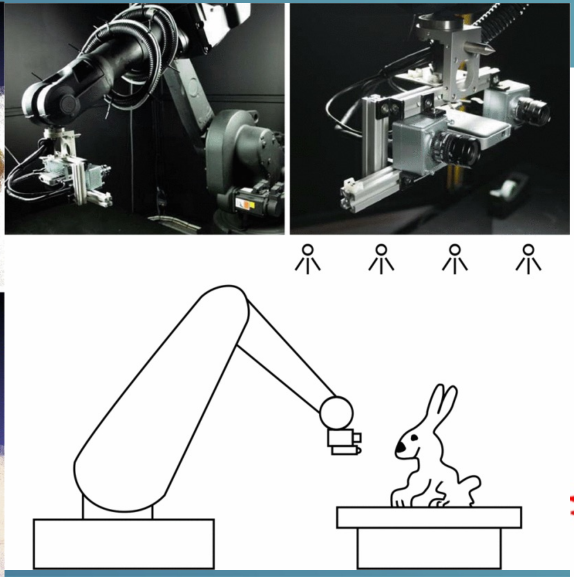
04

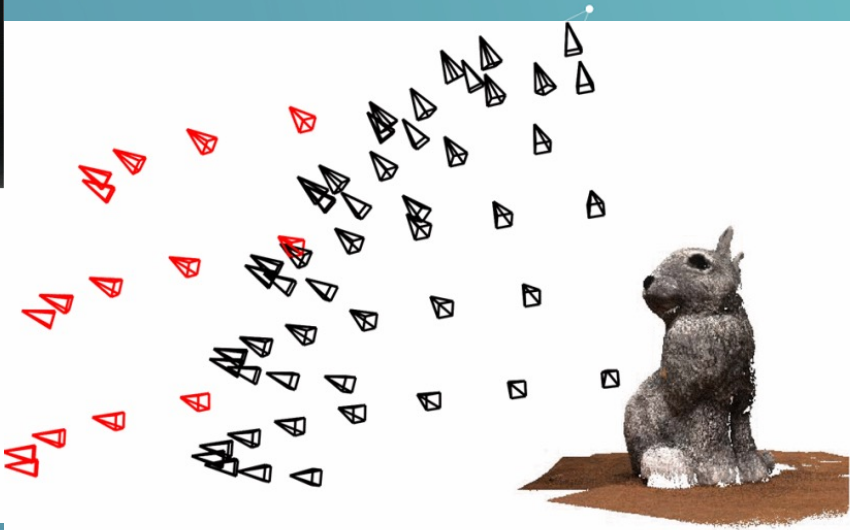
EXPERMENTS

Method evaluation on the challenging task of multiview 3D surface reconstruction

Multi-view 3D reconstruction

- Quantitative results for the **DTU dataset**
- DTU multi-view image; different objects; fixed camera and lighting parameters





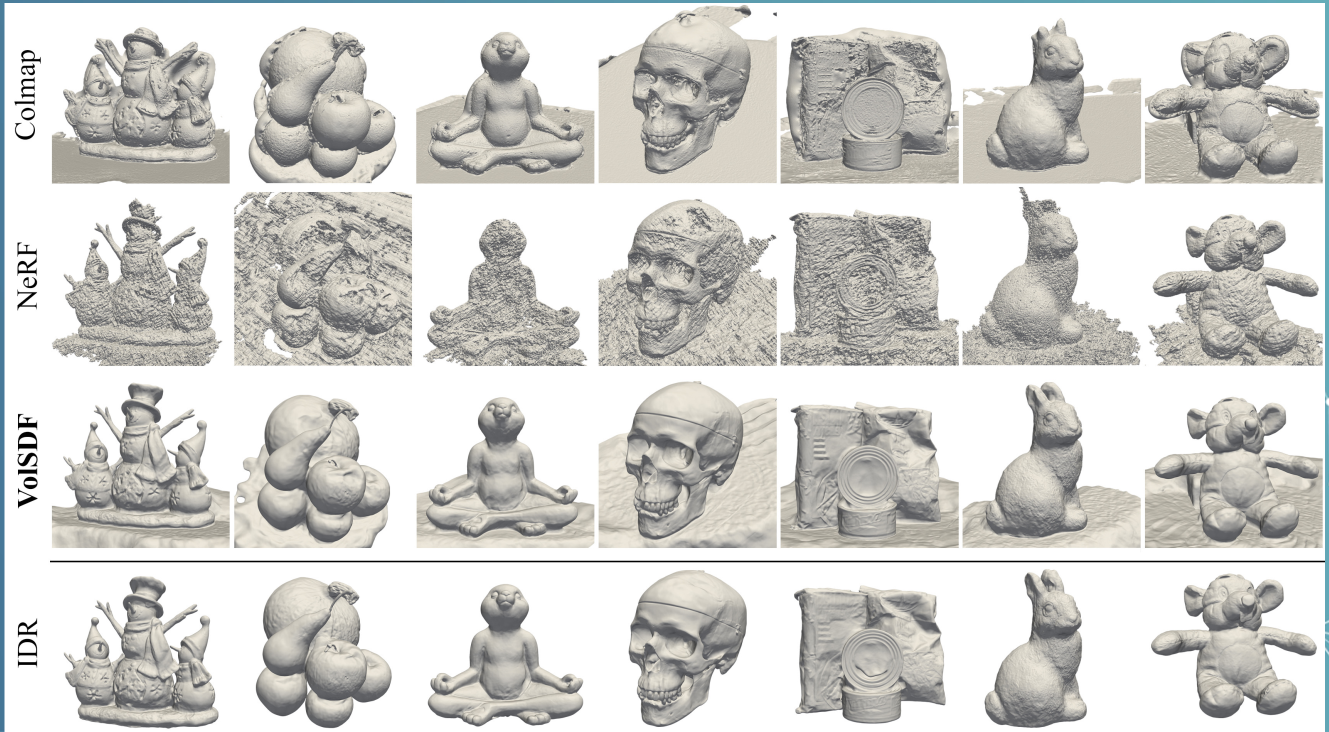
Multi-view 3D reconstruction

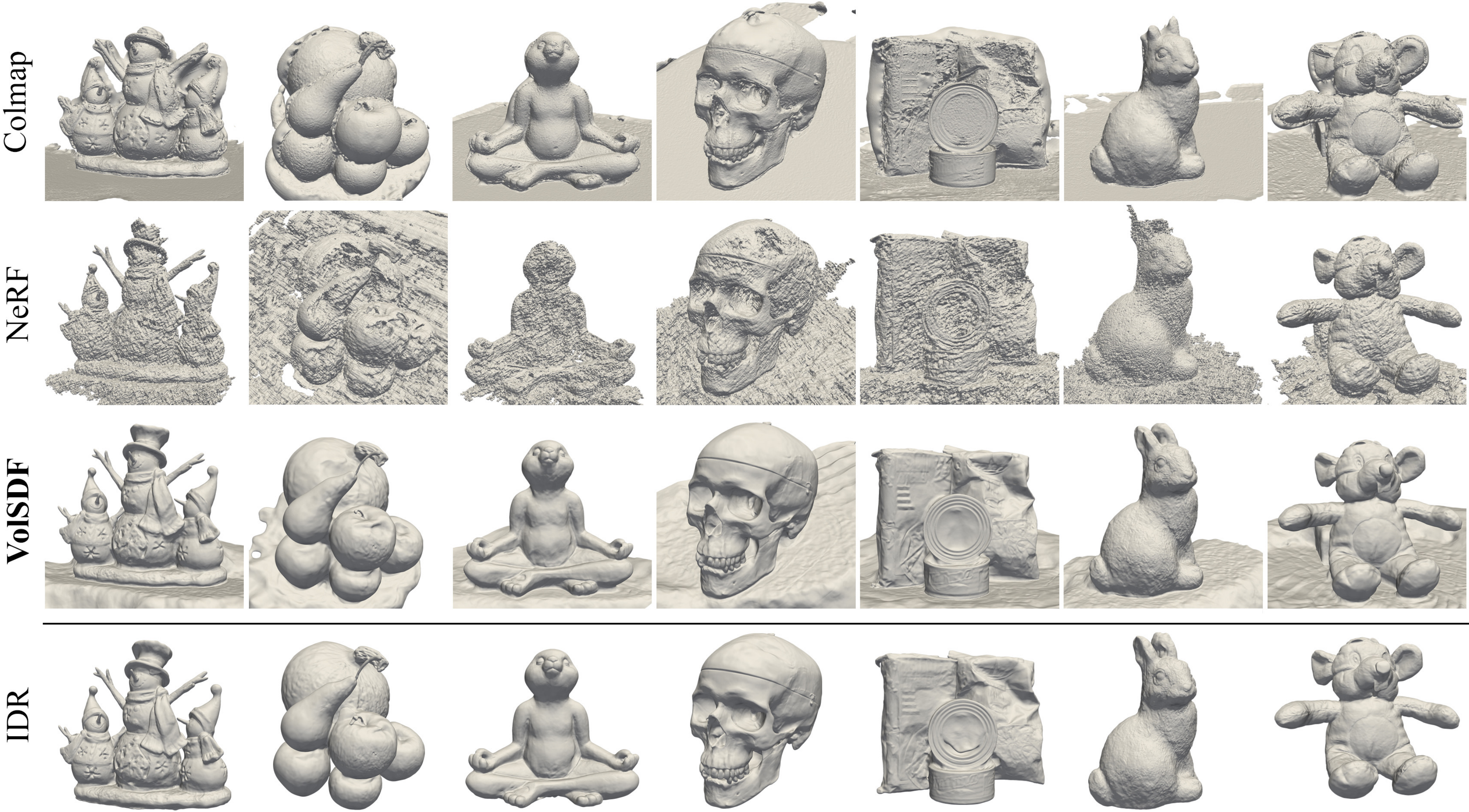
- Quantitative results for the **DTU dataset**
- DTU multi-view image; different objects; fixed camera and lighting parameters

	Scan	24	37	40	55	63	65	69	83	97	105	106	110	114	118	122	**Mean**
Chamfer Distance	**IDR**	1.63	1.87	0.63	0.48	1.04	0.79	0.77	1.33	1.16	0.76	0.67	0.90	0.42	0.51	0.53	0.90
	colmap$_7$	0.45	0.91	0.37	0.37	0.90	1.00	0.54	1.22	1.08	0.64	0.48	0.59	0.32	0.45	0.43	0.65
	colmap$_0$	**0.81**	2.05	**0.73**	1.22	1.79	1.58	1.02	3.05	1.40	2.05	1.00	1.32	0.49	0.78	1.17	1.36
	NeRF	1.92	1.73	1.92	0.80	3.41	1.39	1.51	5.44	2.04	1.10	1.01	2.88	0.91	1.00	0.79	1.89
	VolSDF	1.14	**1.26**	0.81	**0.49**	**1.25**	**0.70**	**0.72**	**1.29**	**1.18**	**0.70**	**0.66**	**1.08**	**0.42**	**0.61**	**0.55**	**0.86**
PSNR	**NeRF**	26.24	25.74	26.79	27.57	31.96	31.50	29.58	32.78	28.35	32.08	33.49	31.54	31.0	35.59	35.51	30.65
	VolSDF	26.28	25.61	26.55	26.76	31.57	31.5	29.38	33.23	28.03	32.13	33.16	31.49	30.33	34.9	34.75	30.38

Multi-view 3D reconstruction

- Qualitative results for the **DTU dataset**




Colmap
NeRF
VolSDF
IDR

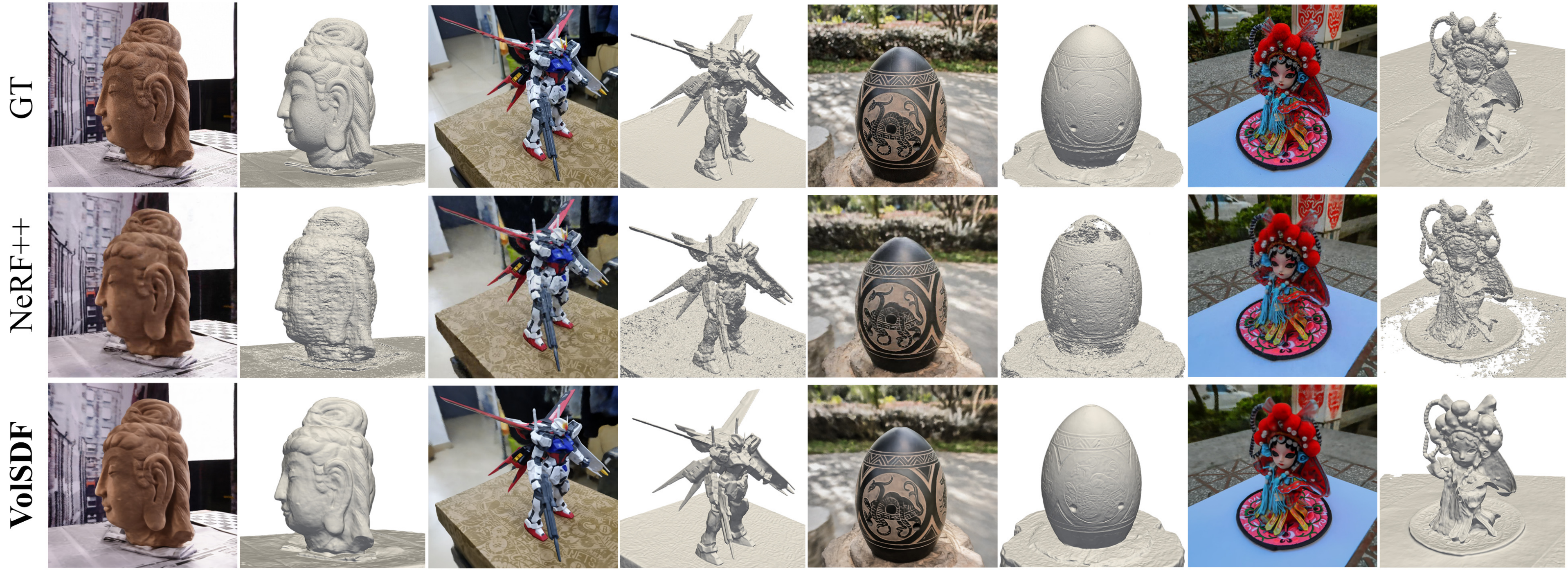
Multi-view 3D reconstruction

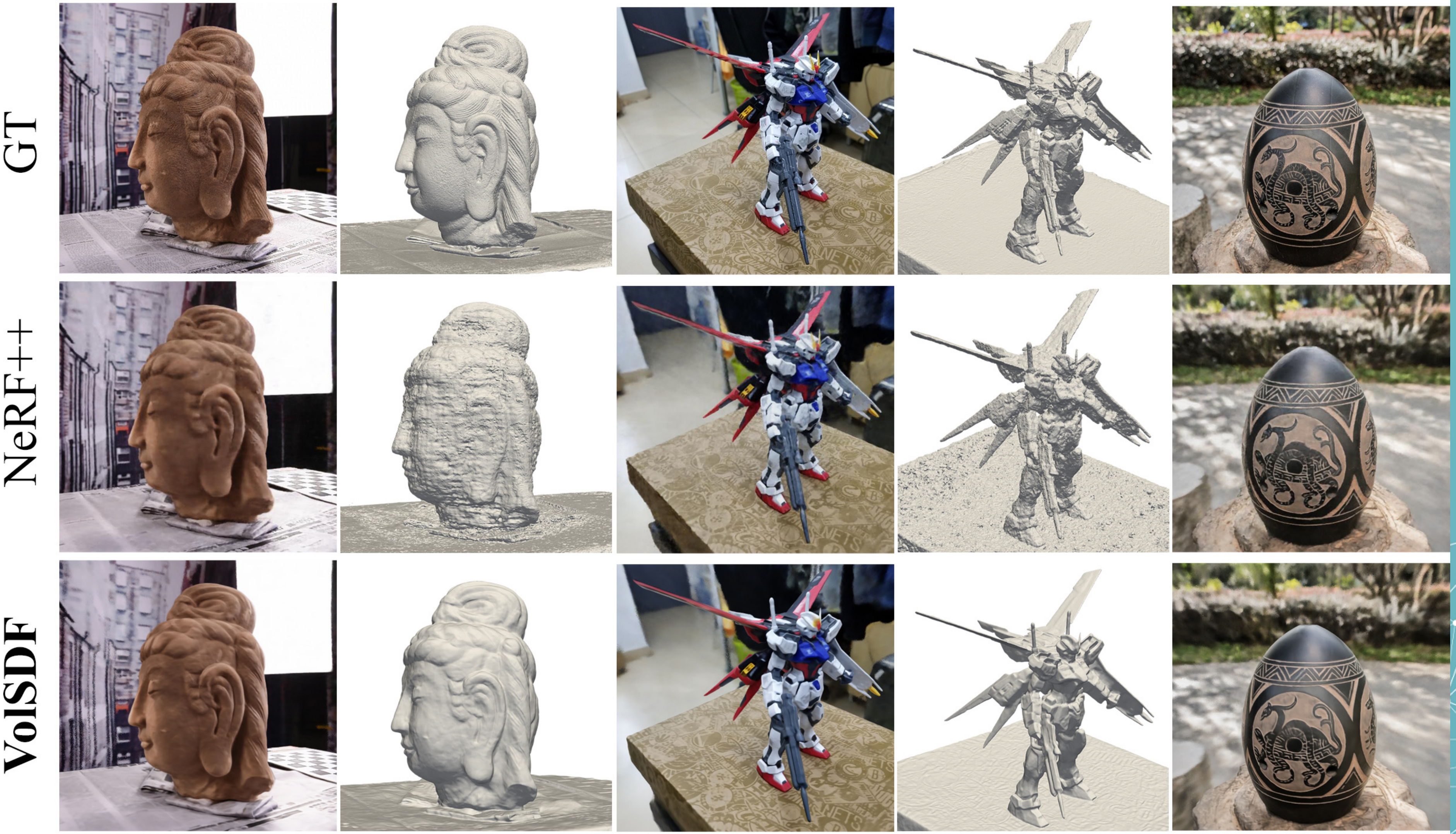
- Quantitative results for the **BlendedMVS** dataset
- Large collection of 113 scenes. High quality GT.
- 9 different scenes were selected

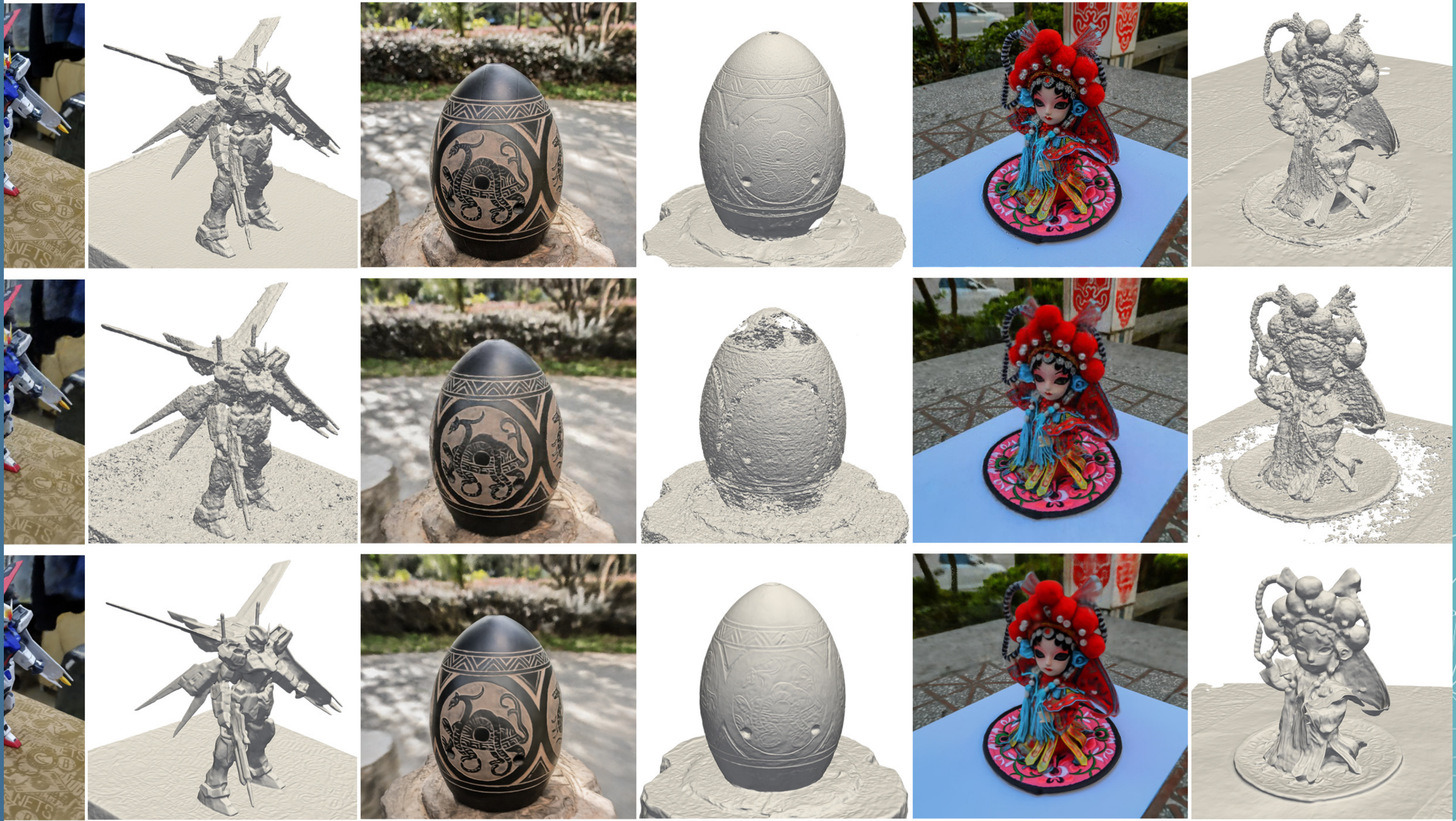
	Scene	Doll	Egg	Head	Angel	Bull	Robot	Dog	Bread	Camera	**Mean**
Chamfer l_1	Our Improvement (%)	54.0	91.2	24.3	75.1	60.7	27.2	47.7	34.6	51.8	51.8
PSNR	**NeRF++**	26.95	27.34	27.23	30.06	26.65	26.73	27.90	31.68	23.44	27.55
	VolSDF	25.49	27.18	26.36	29.79	26.01	26.03	28.65	31.24	22.97	27.08

Multi-view 3D reconstruction

- Qualitative results for the **BlendedMVS** dataset

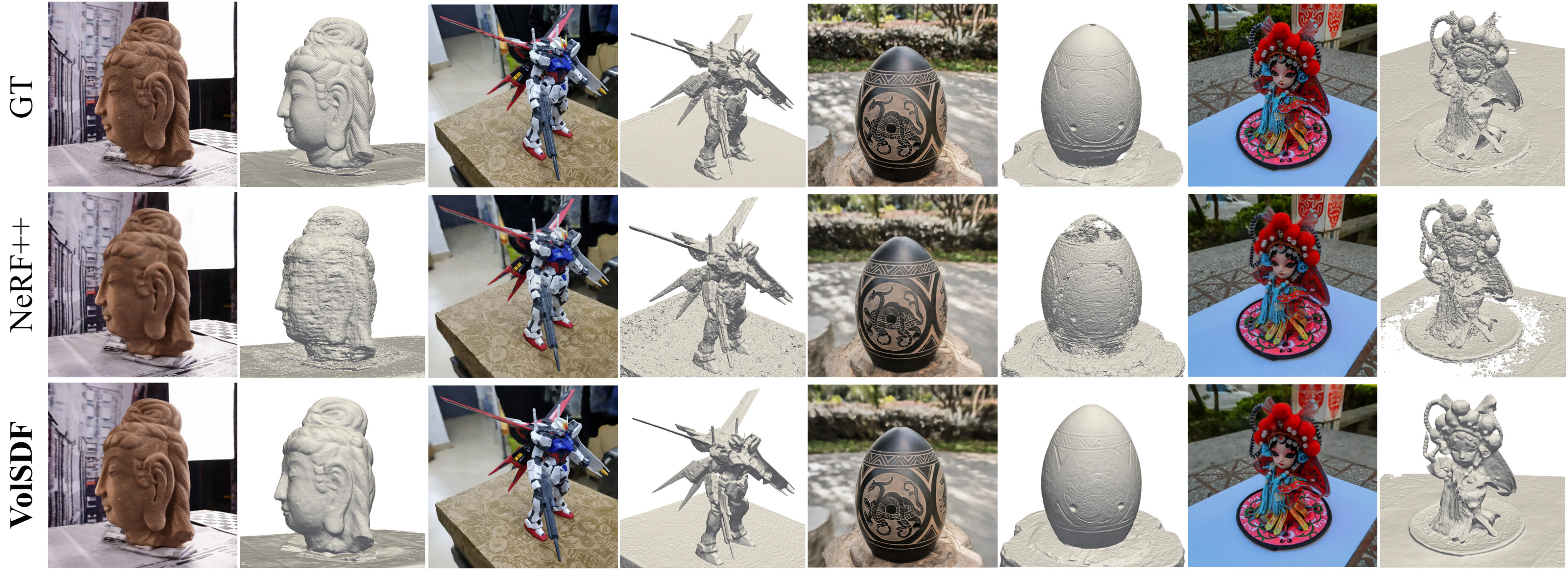



GT
NeRF++
VolSDF



Multi-view 3D reconstruction

- Qualitative results for the **BlendedMVS** dataset



05

CONCLUSION

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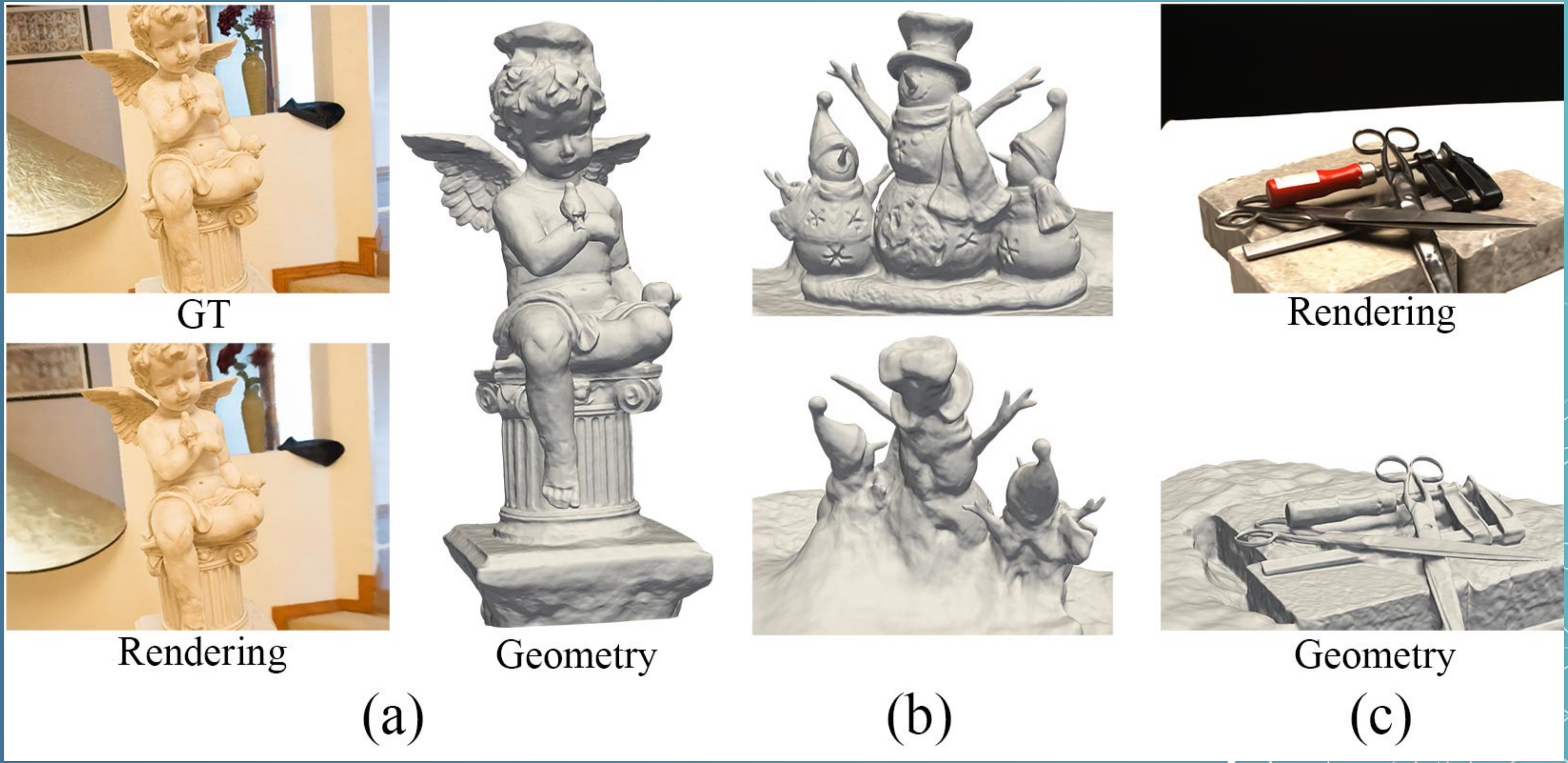
CONCLUSIONS AND REMARKS

- The paper does not have a proof of correctness for the sampling algorithm.
- Representing non-watertight manifolds and/or manifolds with boundaries, such as zero thickness surfaces, is not possible with an SDF
- Assumption of homogeneous density; extending it to more general density models would allow representing a broader class of geometries

CONCLUSIONS AND REMARKS

- High quality geometries can be learned in an unsupervised manner.
- Accurate geometry reconstruction from images can be used for malice purposes.

LIMITATIONS



06
SUPPLEMENTALS

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Thank you.