Part II:

Convex Relaxation and Multiview Reconstruction

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DAGM Tutorial on „Convex Optimization“, Frankfurt 2011

Inverse Problems: Denoising

Noisy input image f

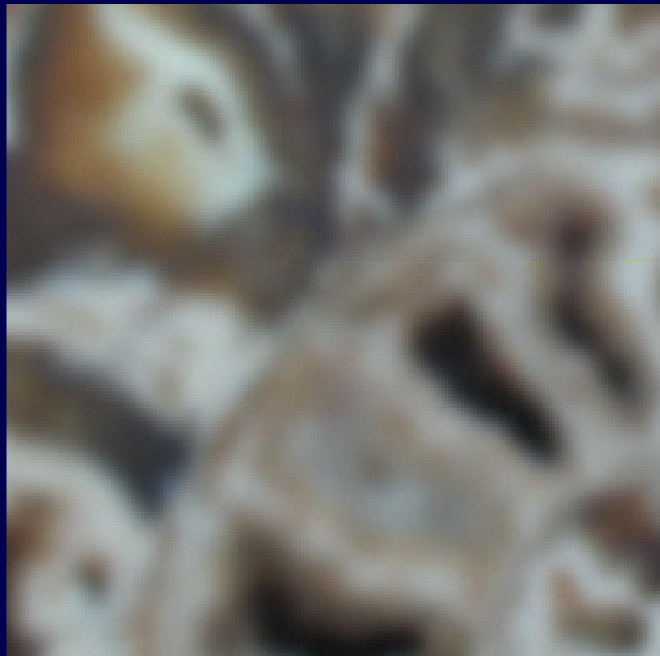
Denoised image u_{den}

$$u_{den} = \arg\min_u \int_\Omega (u - f)^2 \, dx + \lambda \int_\Omega |\nabla u| \, dx$$

Rudin, Osher, Fatemi, Physica D '92

Inverse Problems: Deblurring

Original image

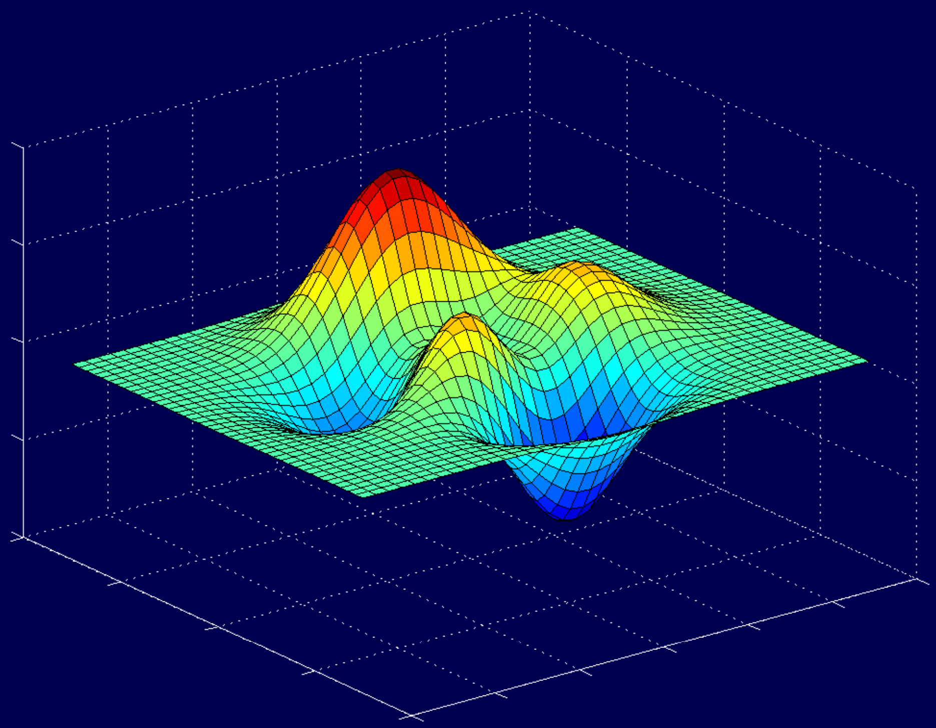


Blurred image

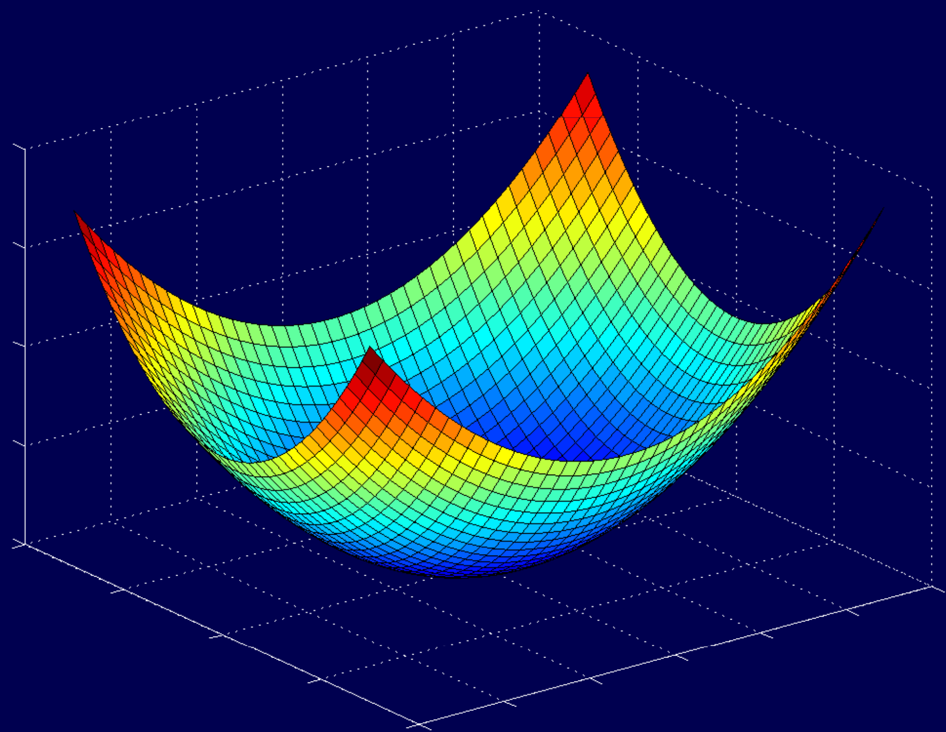
Deblurred image

$$u_{deb} = \arg\min_{u} \int_{\Omega} (b * u - f)^2 \, dx + \lambda \int_{\Omega} |\nabla u| \, dx$$

Lions, Osher, Rudin, '92



Non-convex energy

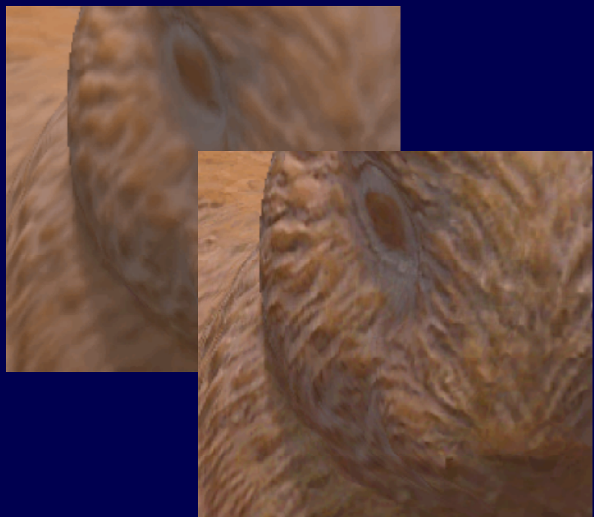


Convex energy

Overview

Multiview reconstruction

Single view reconstruction



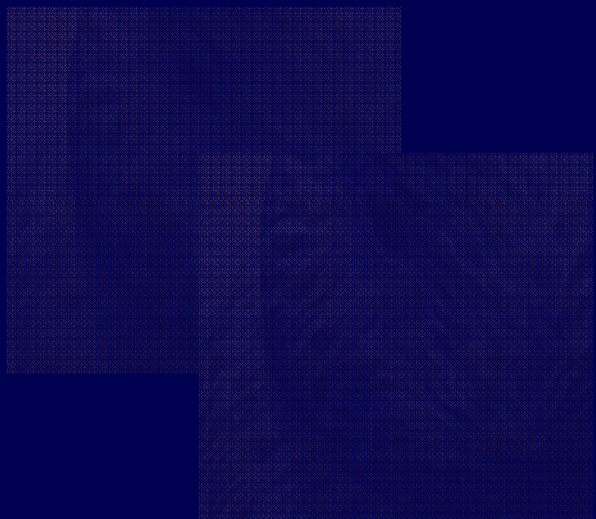
Super-resolution Texture

Silhouette Consistency

Overview

Multiview reconstruction

Single view reconstruction

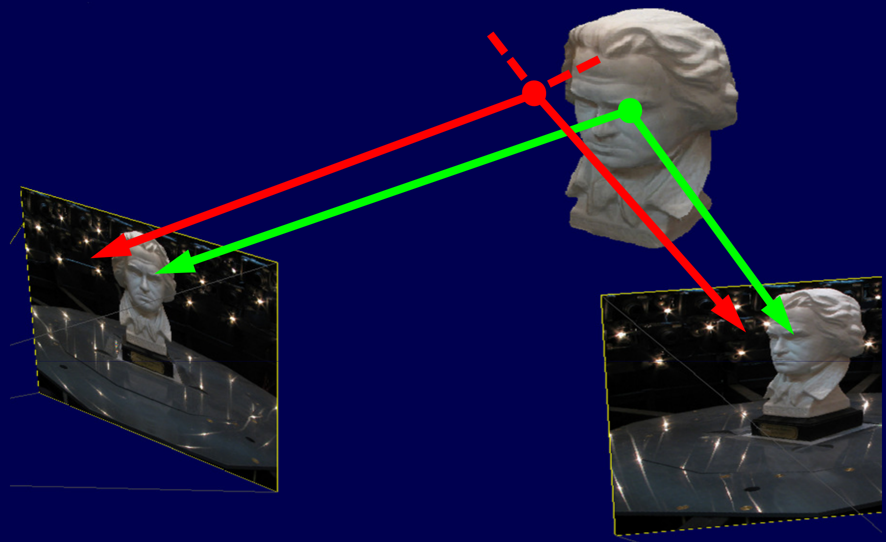


Super-resolution Texture

Silhouette Consistency

3D Reconstruction from Multiple Views

$$\rho : \left(V \subset \mathbb{R}^3\right) \to [0,1]$$



$$E(S) = \int_S \rho(s)\, ds$$

3D Reconstruction: *Faugeras, Keriven '98,   Duan et al. '04*

Segmentation: *Kichenassamy et al. '95,   Caselles et al. '95*

Optimal solution is the empty set: $\arg\min_S E(S) = \emptyset$

Resort:

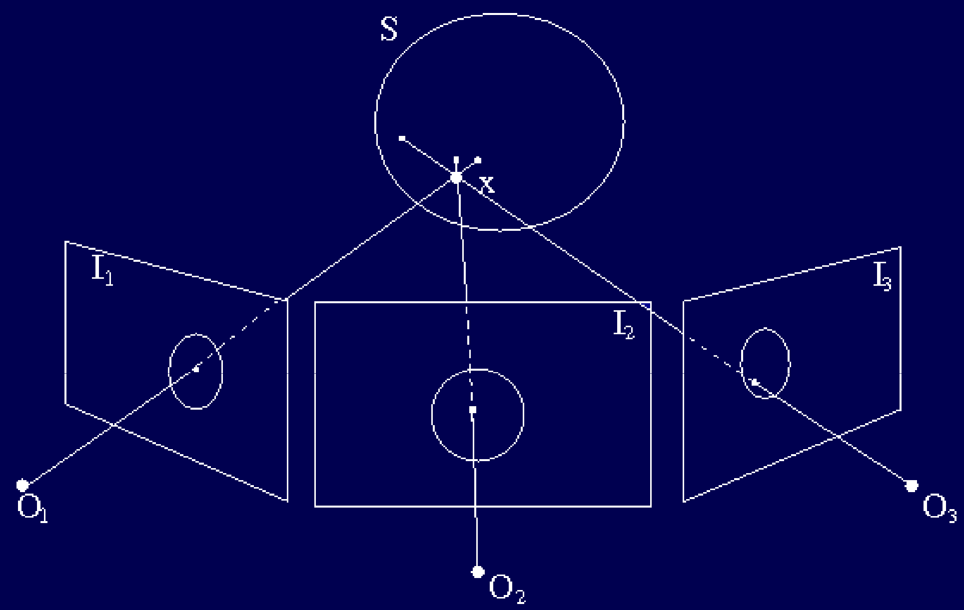
- Local optimization: *Faugeras, Keriven TIP '98*
- Generative object/background modeling: *Yezzi, Soatto '03,…*
- Constrain search space: *Vogiatsis, Torr, Cipolla CVPR '05*
- Intelligent ballooning: *Boykov, Lempitsky BMVC '06*

Solution 1: Volumetric Photoconsistency

$$E(S) = \int_{S_{in}} \rho_{obj}(x)dx + \int_{S_{out}} \rho_{bck}(x)dx + \int_S \rho(s)ds$$

$$\rho_{obj},\ \rho_{bck} : V \longrightarrow [0,1]$$

$$S_{in},\ S_{out} \subset V$$



Kolev, Klodt, Brox, Cremers, IJCV '09

A Convex Formulation

$$E(S) = \int_{S_{in}} \rho_{obj}\, dx \;+\; \int_{S_{out}} \rho_{bck}\, dx \;+\; \int_S \rho\, ds$$

implicit representation

$$E(u) = \int_V \rho_{obj}(x)(1-u(x)) \;+\; \rho_{bck}(x)u(x)\, dx \;+\; \int_V \rho(x)|\nabla u|\, dx, \quad \text{s. t. } u : V \rightarrow \{0,1\}$$

relaxation

$$E(u) = \int_V \rho_{obj}(x)(1-u(x)) \;+\; \rho_{bck}(x)u(x)\, dx \;+\; \int_V \rho(x)|\nabla u|\, dx, \quad \text{s. t. } u : V \rightarrow [0,1]$$

Global Optima of the Binary Problem

$$E(u) = \int_V \rho_{obj}\,(1 - u(x)) \;+\; \rho_{bck}\,u(x)\,dx \;+\; \int_V \rho\,|\nabla u|\,dx, \qquad (*)$$

$$\text{s. t. } u : V \to [0,1]$$

Theorem: Thresholding a minimizer u^* of the relaxed problem $(*)$ leads to an optimal solution of the original binary problem:

$$u_{opt}(x) = 1_{u^* \geq \mu}(x) = \begin{cases} 1, & \text{if } u^*(x) \geq \mu \\ 0, & \text{if } u^*(x) < \mu \end{cases}$$

for any threshold $\mu \in (0,1)$.

Nikolova, Esedoglu, Chan '05,

Kolev, Klodt, Brox, Esedoglu, Cremers, EMMCVPR '07, IJCV '09

Threshold Theorem

Let

$$u^* : \Omega \to [0, 1]$$

be a (real-valued) minimizer of

$$E(u) = \int_\Omega f\, u + |\nabla u|\, dx.$$

Then for any threshold $\mu \in (0, 1)$, the binary function

$$1_{u^* \geq \mu}(x)$$

is a global minimizer of the original binary problem.

Proof: 1) $u(x) = \int_0^1 1_{u \geq \mu}(x)\, d\mu$ (layer cake formula)

2) $$\int_\Omega |\nabla u|\, dx = \int_0^1 \int_\Omega |\nabla 1_{u \geq \mu}(x)|\, dx\, d\mu$$ (coarea formula)

$$E(u) = \int_\Omega f u + |\nabla u|\, dx = \int_0^1 \int_\Omega f 1_{u \geq \mu} + |\nabla 1_{u \geq \mu}|\, dx\, d\mu = \int_0^1 E\left(1_{u \geq \mu}\right) d\mu$$

If $1_{u^* \geq \mu}$ is not minimizer, i.e. there exists a set $\Sigma \subset \Omega$

$$E(1_\Sigma) < E(1_{u^* \geq \mu})$$

$$\Rightarrow E(1_\Sigma) = \int_0^1 E(1_\Sigma)\, d\mu < \int_0^1 E(1_{u^* \geq \mu})\, d\mu = E(u^*)$$ (i.e. u^* not minimizer)

Comparison

Level sets	Convex formulation	Graph cuts
Continuous Local optima	Continuous Global optima*	Discrete Global optima*

* for certain functionals only

Klodt, Schoenmann, Kolev, Schikora, Cremers, ECCV '08

Comparison

Level sets	Convex formulation	Graph cuts
Continuous	Continuous	Discrete
Local optima	Global optima*	Global optima*
	Parallel implementations	

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		Memory limitations

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Klodt, Schoenmann, Kolev, Schikora, Cremers, ECCV '08

Comparison

Level sets	Convex formulation	Graph cuts
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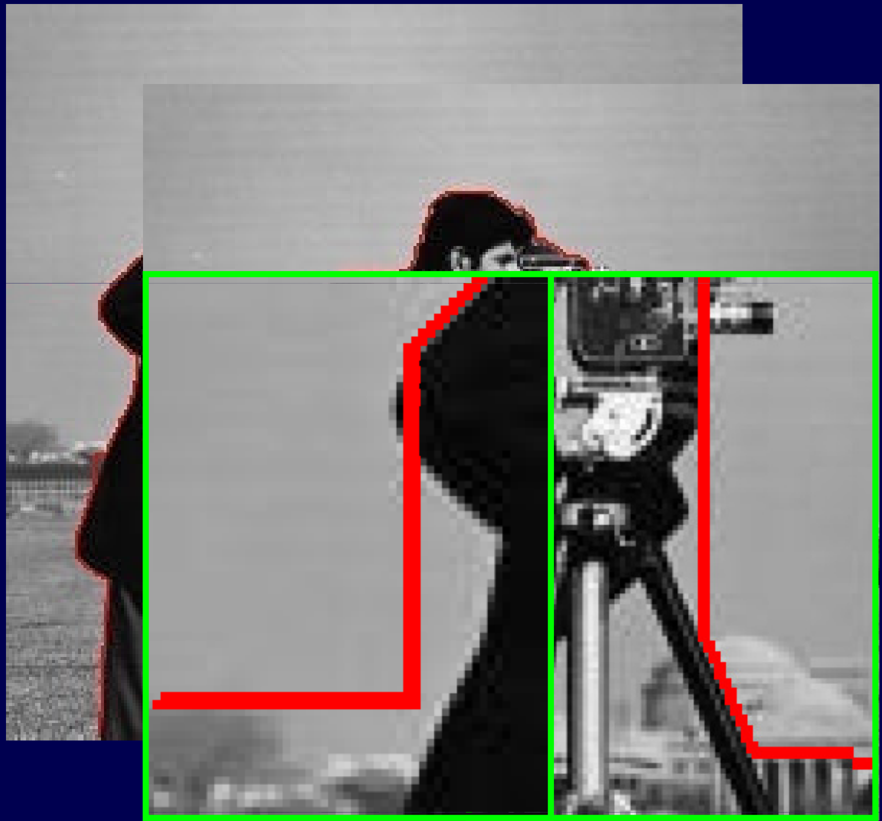
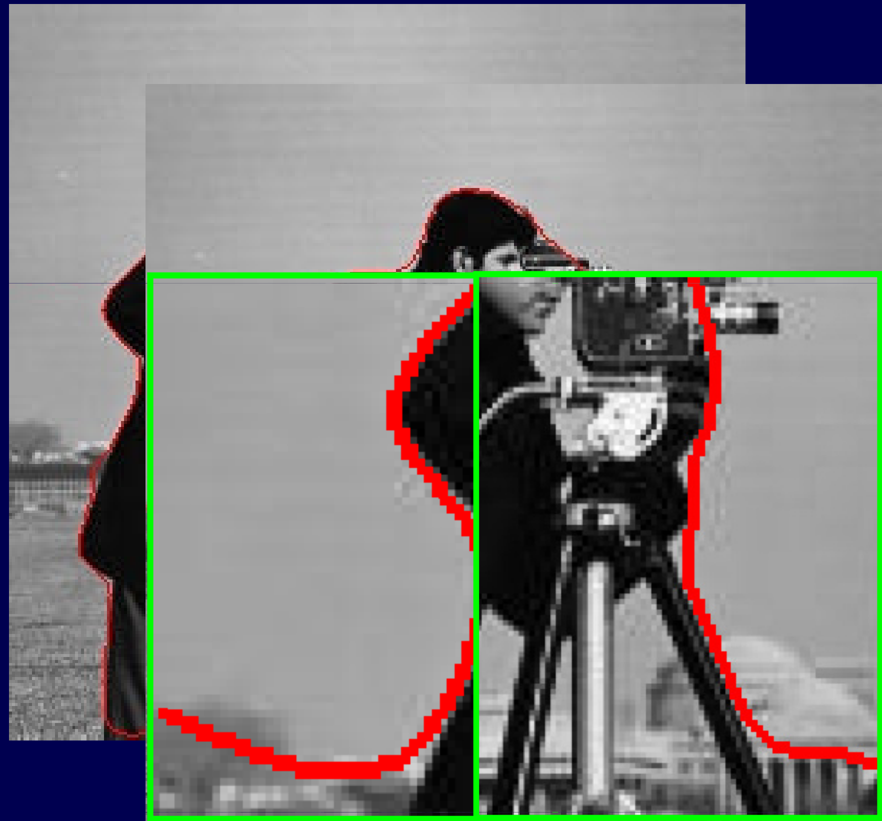
Local optima	Global optima*	Global optima*
	Parallel implementations	
		Memory limitations
		Metrication errors

* for certain functionals only

Klodt, Schoenmann, Kolev, Schikora, Cremers, ECCV '08



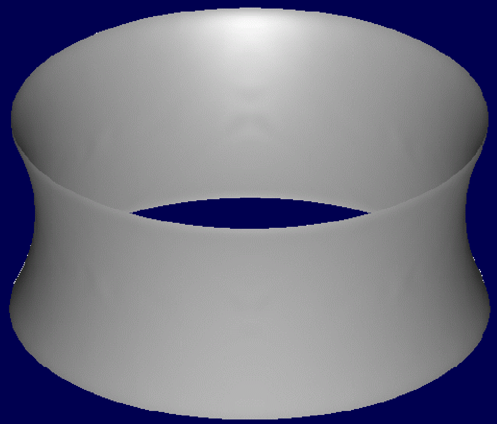
Discrete graph cut optimization
(4-connected grid)



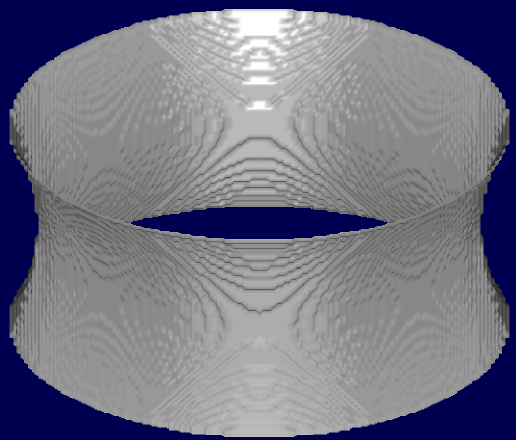
Continuous convex formulation
(4-connected grid)

Improvement: Larger neighborhoods
(*Boykov, Kolmogorov '03*, *Kirsanov, Gortler '04*)

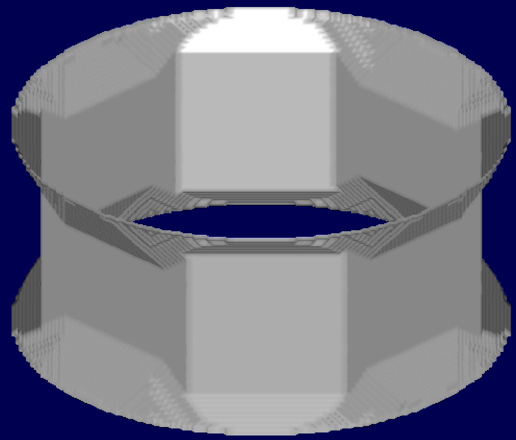
A Minimal Surface Problem: The Catenoid



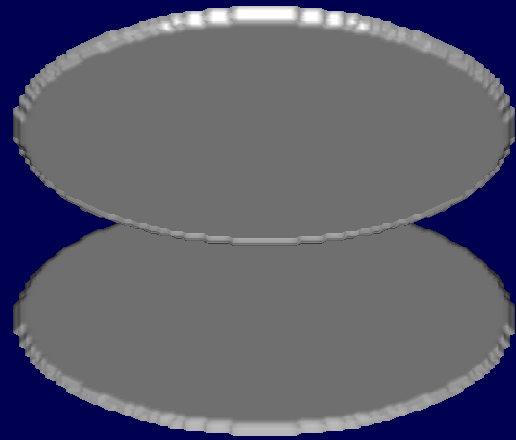
True solution



Convex formulation
(6-connected grid)



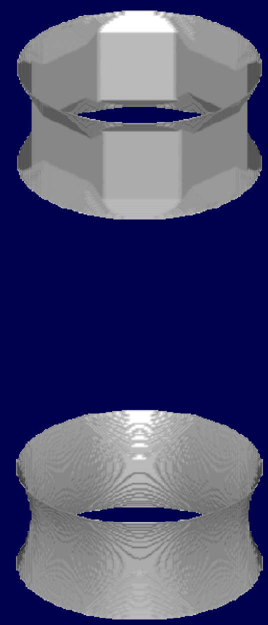
Graph cut
(26-connected grid)



Graph cut
(6-connected grid)

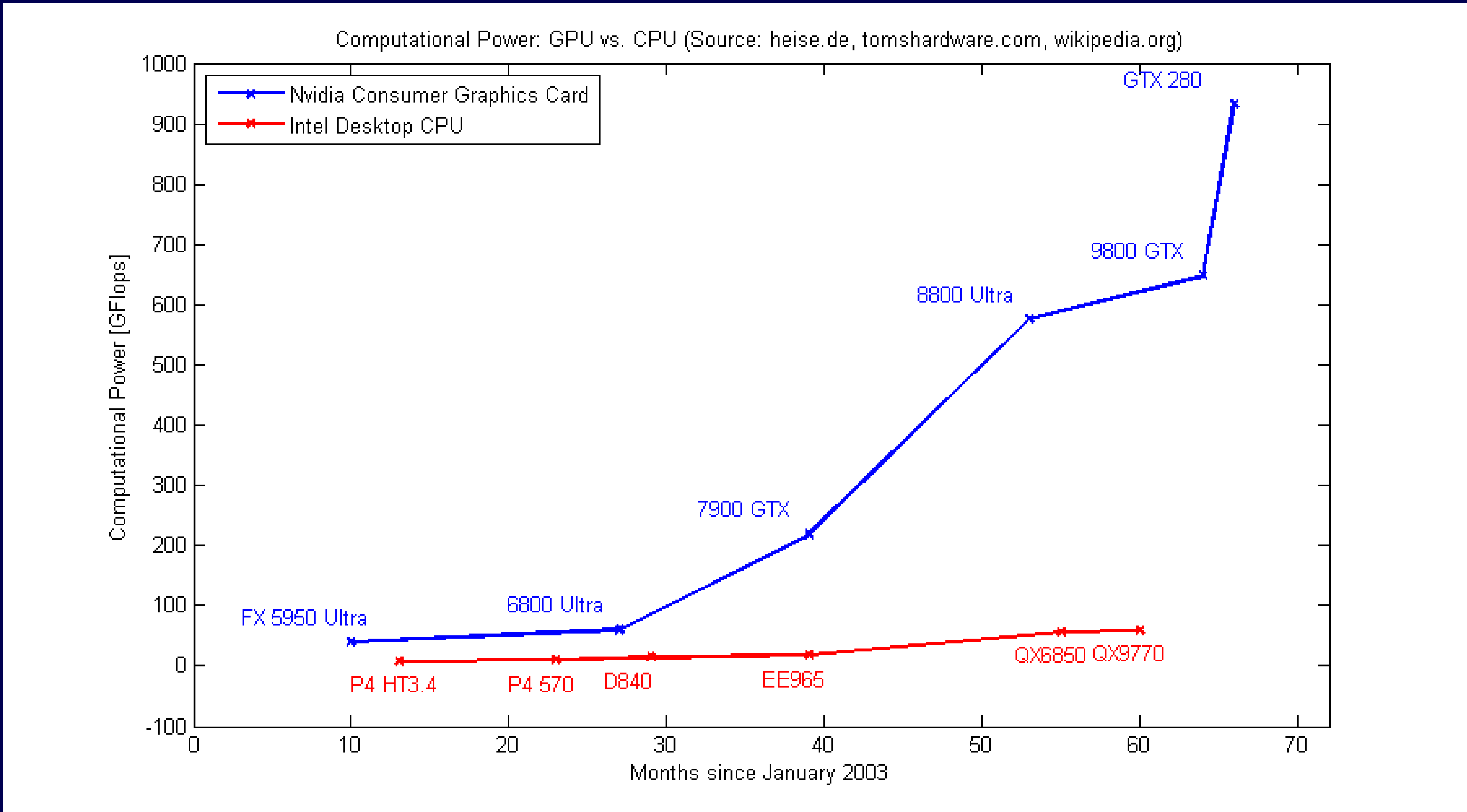
Klodt, Schoenmann, Kolev, Schikora, Cremers, ECCV '08

12 min. 32 sec. (CPU)
9 min. 34 sec. (CPU)
8 sec. (GPU)



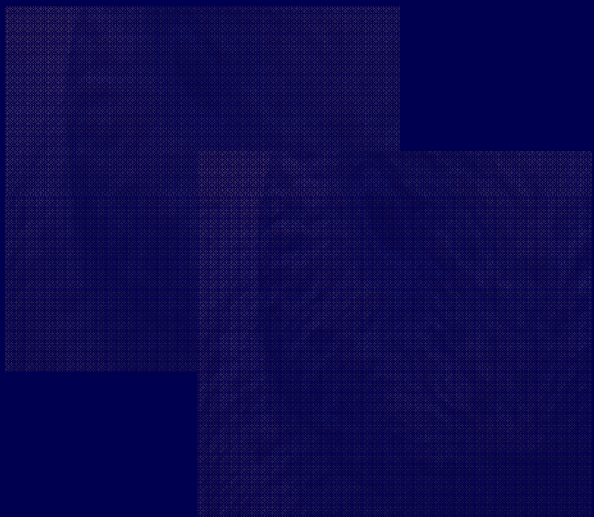
Klodt, Schoenmann, Kolev, Schikora, Cremers, ECCV '08

Speedups in GPU Computation



Multiview reconstruction

Single view reconstruction



Super-resolution Texture

Silhouette Consistency

Can we recover geometry from a single image?

Yes: Shape-from-shading, shape-from-focus, shape from symmetry,…

Silhouette-based approaches:

Horry et al. Siggraph '97, Criminisi et al. IJCV '00,

Hoiem et al. Siggraph '05, Prasad et al. CVPR '06,…

Goal: Simple variational approach with minimal user interaction.

Solution: Fixed-volume silhouette-consistent minimal surface.

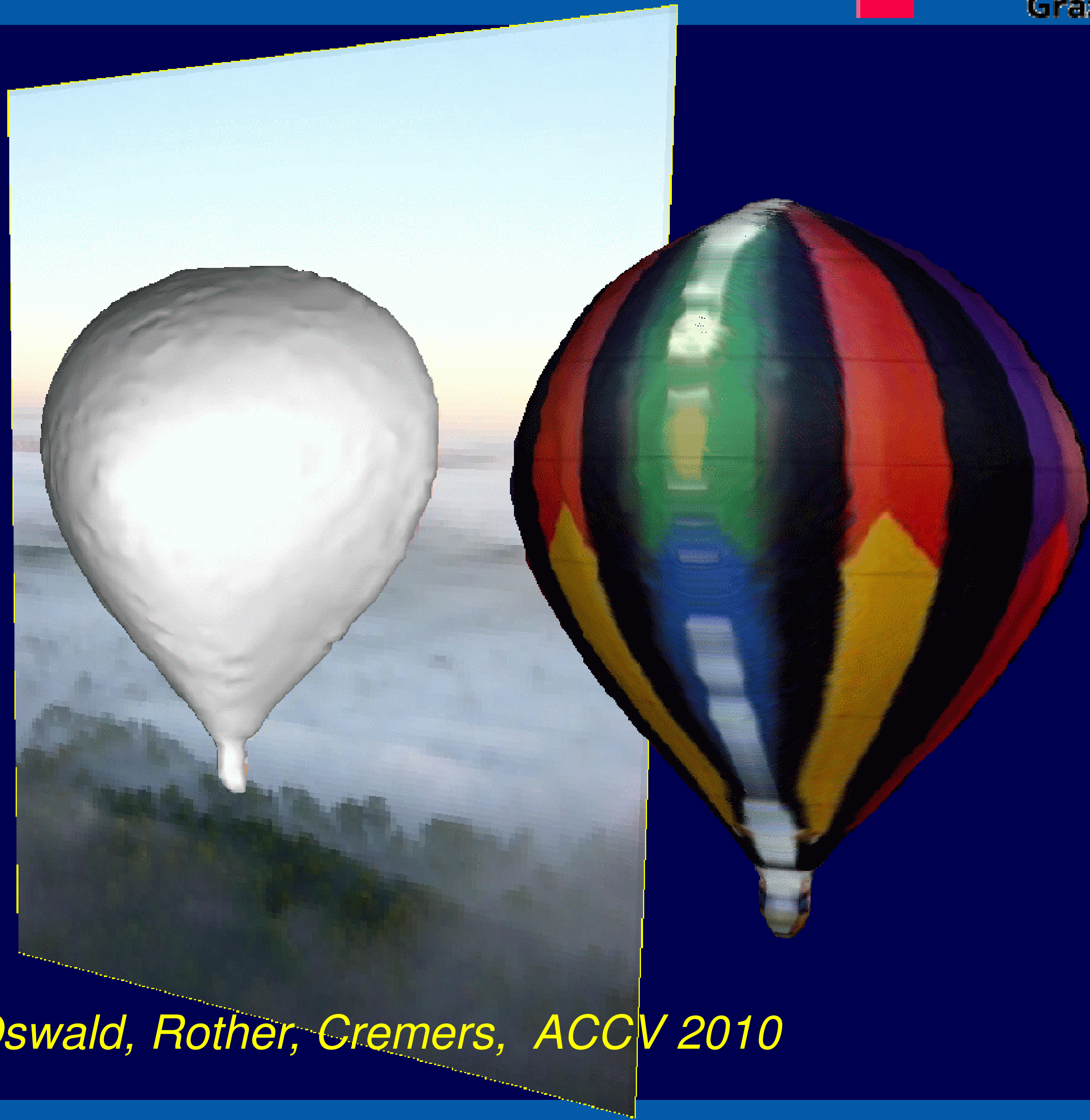
$$\min_S |S| \quad \text{s.t.} \;\; \text{Vol}(S) = V_0, \; \pi(S) = S_0$$

*Toeppe, Oswald, Rother, Cremers, ACCV 2010**

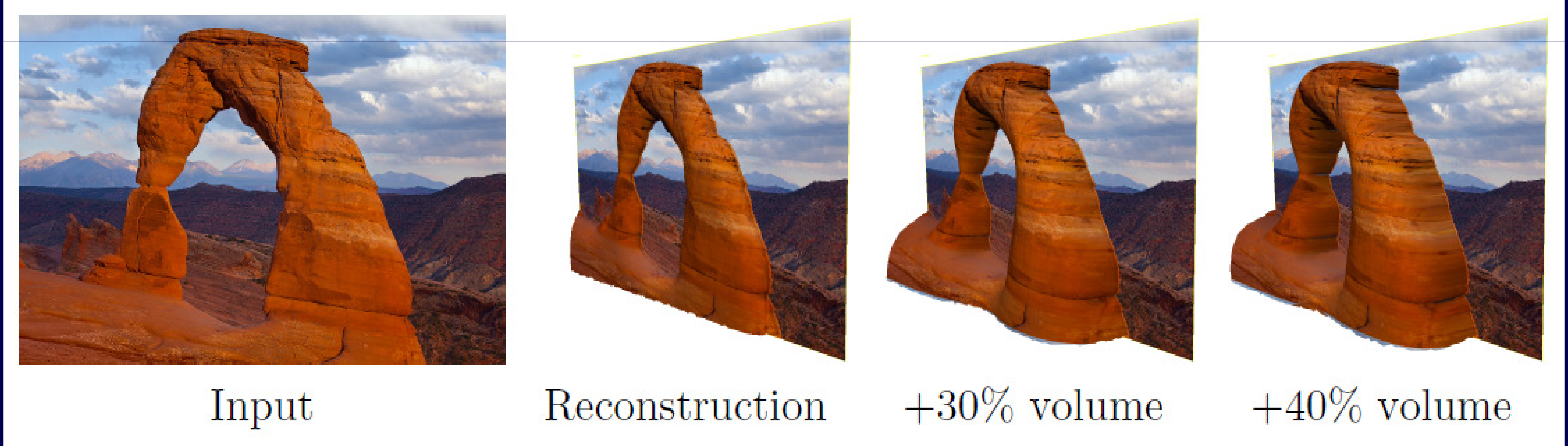
** Best Paper Honorable Mention*

In collaboration with Microsoft Research

Single View Reconstruction



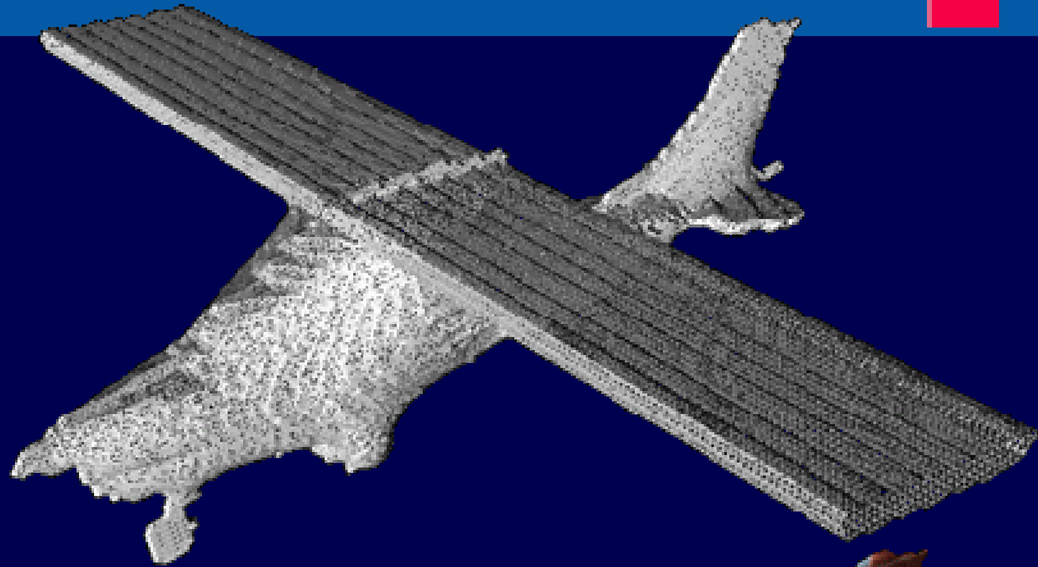
Toeppe, Oswald, Rother, Cremers, ACCV 2010



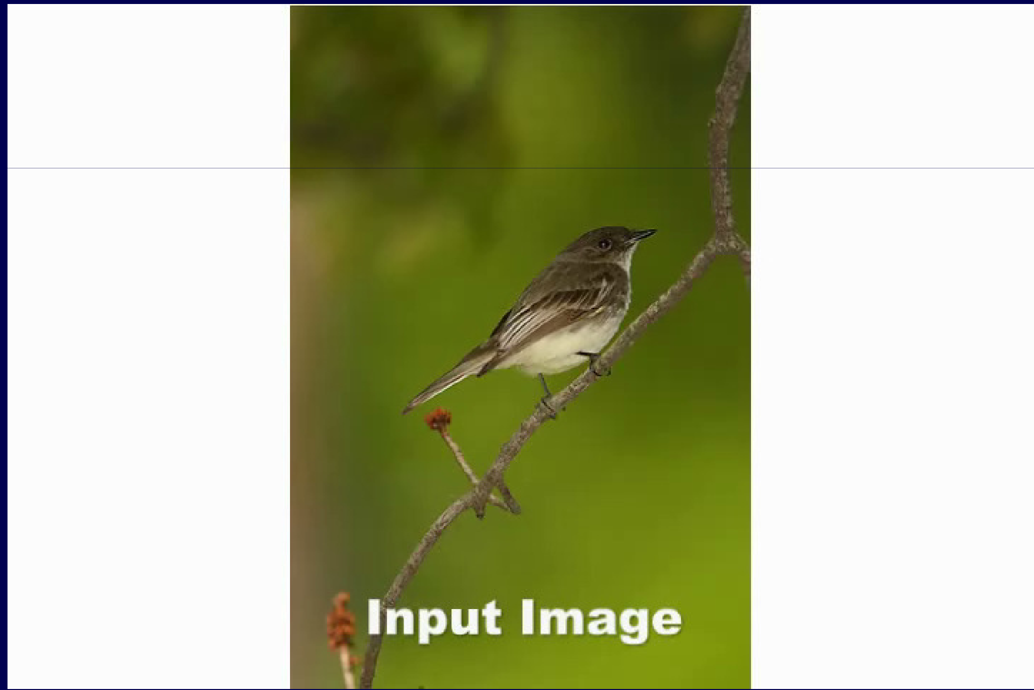
Toeppe, Oswald, Rother, Cremers, ACCV 2010

Single View Reconstruction

Toeppe, Oswald, Rother, Cremers, ACCV 2010



Toeppe, Oswald, Rother, Cremers, ACCV 2010



*Toeppe, Oswald, Rother, Cremers, ACCV 2010**

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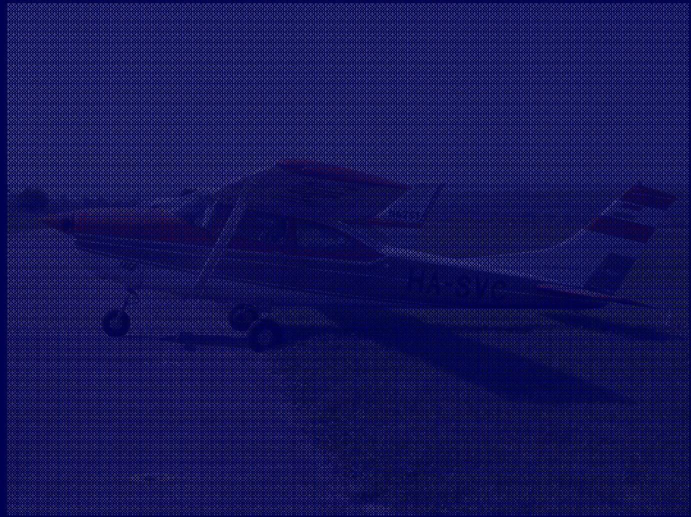
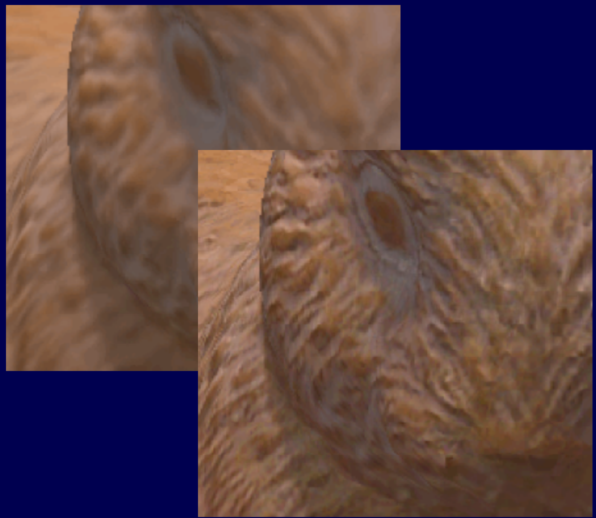
In collaboration with Microsoft Research

Multiview reconstruction



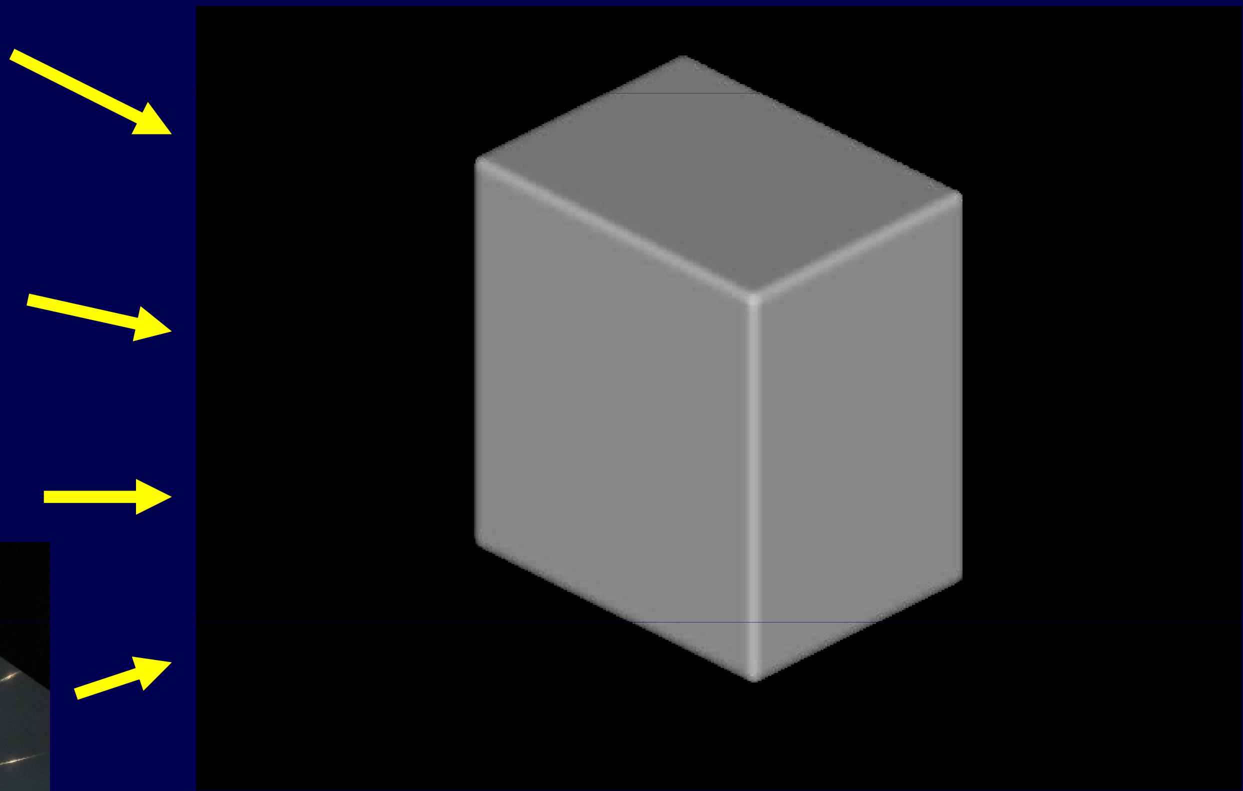
Single view reconstruction



Super-resolution Texture

Silhouette Consistency

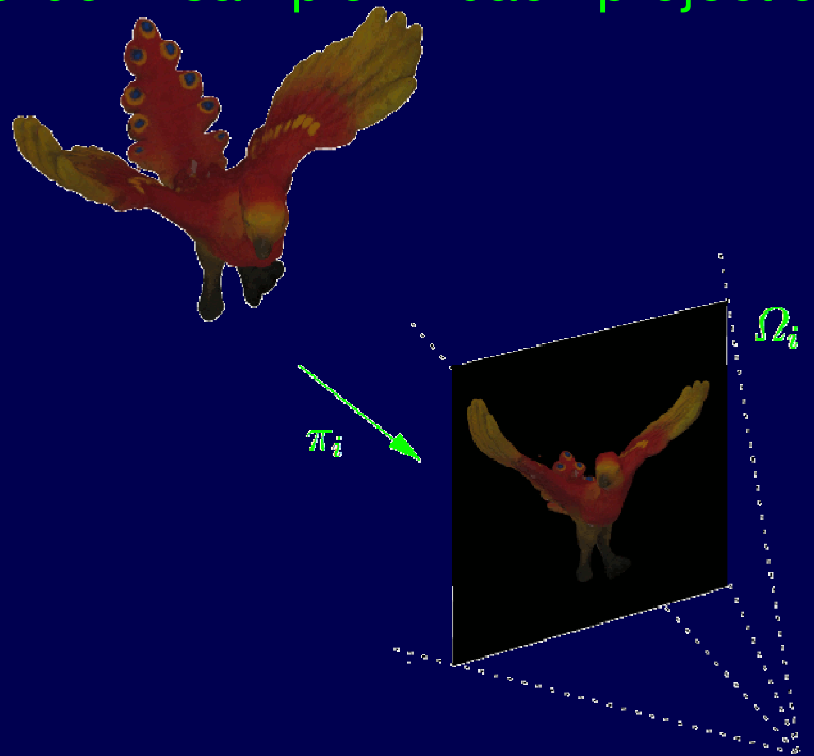
Surface Evolution during Minimization



Given all images $\mathcal{I}_i : \Omega_i \rightarrow \mathbb{R}^3$, determine the surface color $T : S \rightarrow \mathbb{R}^3$

$$\min_T \sum_{i=1}^{n} \int_{\Omega_i} \Big(b * (T \circ \beta_i) - \mathcal{I}_i \Big)^2 dx + \lambda \int_S \|\nabla_S T\| ds$$

blur & downsample   back-projection



Goldlücke, Cremers, ICCV '09, DAGM '09

Given all images $\mathcal{I}_i : \Omega_i \to \mathbb{R}^3$, determine the surface color $T : S \to \mathbb{R}^3$

$$\min_T \sum_{i=1}^n \int_{\Omega_i} \left(b * (T \circ \beta_i) - \mathcal{I}_i \right)^2 dx + \lambda \int_S \|\nabla_S T\| ds$$

blur & downsample   back-projection

Euler-Lagrange equation gives a PDE on the surface:

$$\mathrm{div}_S \left(\frac{\nabla_S T}{\|\nabla_S T\|_S} \right) + \sum_{i=1}^n \frac{v_i}{\lambda} \left((\mathcal{J}_i \mathcal{D}_i) \circ \pi_i \right) = 0$$

where $\mathcal{D}_i = \bar{b} * (b * (T \circ \beta_i) - \mathcal{I})$ and $\mathcal{J}_i = \left\| \frac{\partial \beta_i}{\partial x} \times \frac{\partial \beta_i}{\partial y} \right\|^{-1}$.

Conformal parameterization of the surface ⟶ PDE on charts.

Goldlücke, Cremers, ICCV '09, DAGM '09

Super-Resolution Texture Map

*Goldlücke, Cremers, ICCV '09, DAGM '09**

** Best Paper Award*

Weighted average

Super-resolution texture

*Goldlücke, Cremers, ICCV '09, DAGM '09**

** Best Paper Award*

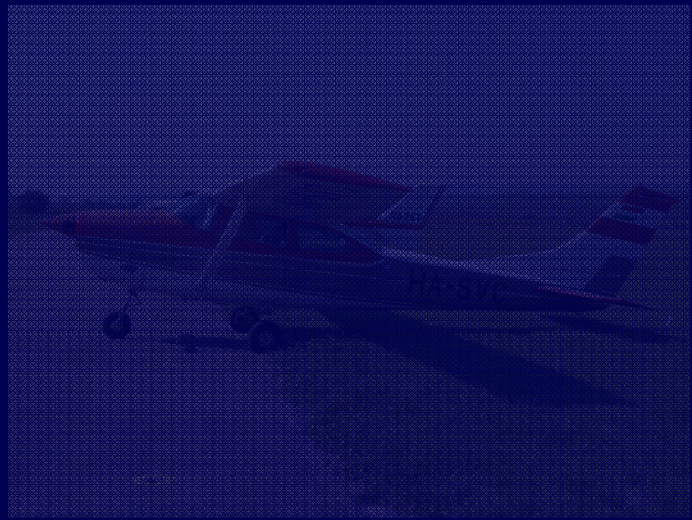
Closeup of input image

Super-resolution texture

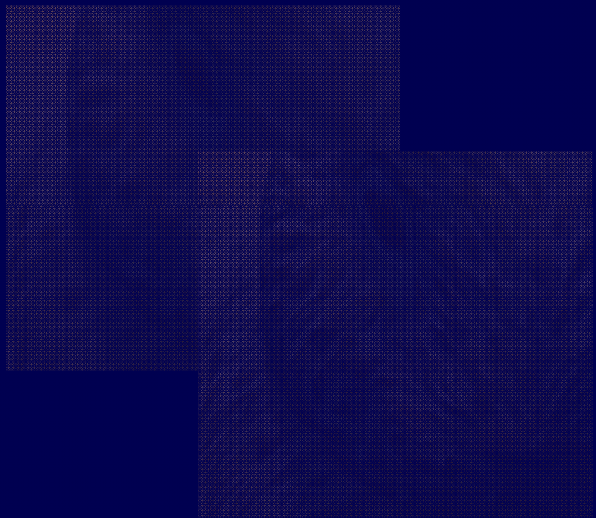
* Best Paper Award

*Goldlücke, Cremers, ICCV '09, DAGM '09**

Multiview reconstruction



Single view reconstruction



Super-resolution Texture

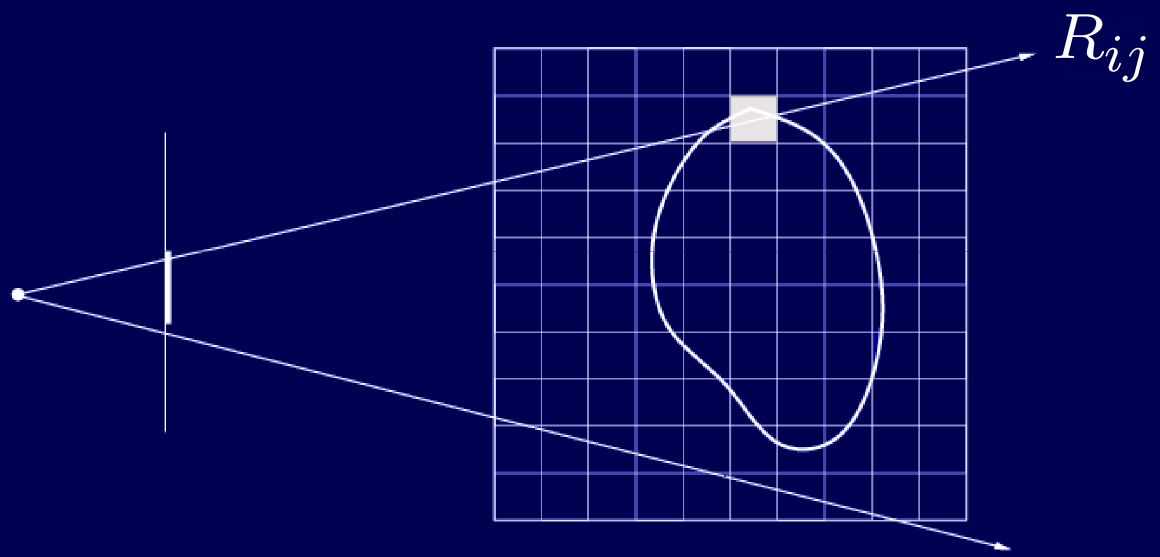
Silhouette Consistency

$$\min_S \int_S \rho \, dS$$

$$\text{s. t.} \quad \pi_i(S) = S_i \;\; \forall\, i = 1, \ldots, n$$

$$\pi_i : V \rightarrow \Omega_i$$

$$S_i \subset \Omega_i$$



Kolev, Cremers, ECCV '08, PAMI 2010

$$E(S) = \int_S \rho(x)\, dS,$$

$$\text{s. t.} \qquad \pi_i(S) = S_i \quad \forall\, i = 1, \dots, n$$

implicit representation & relaxation

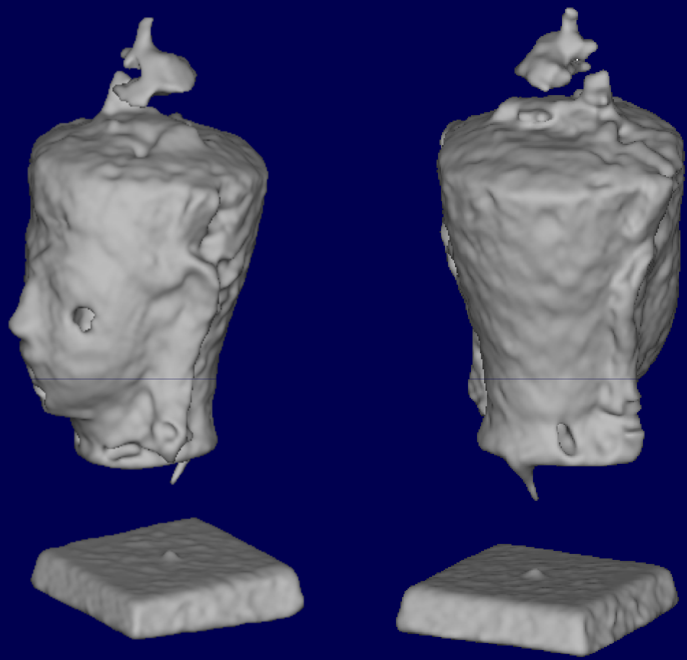
$$E(u) = \int_V \rho(x) |\nabla u(x)|\, dx$$

$$\Sigma = \begin{cases} \text{s. t.} \qquad \sout{u : V \to \{0,1\}} \quad u : V \to [0,1] \\ \int_{R_{ij}} u(x)\, dx \geq \delta \quad \text{if } j \in S_i \\ \int_{R_{ij}} u(x)\, dx = 0 \quad \text{if } j \notin S_i \end{cases}$$

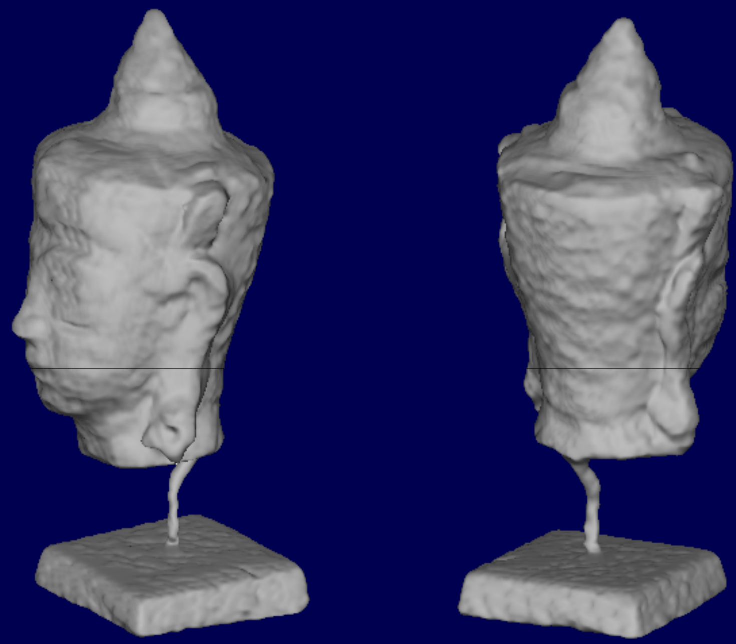
Proposition: The set Σ of silhouette-consistent solutions is convex.

Kolev, Cremers, ECCV '08, PAMI 2010

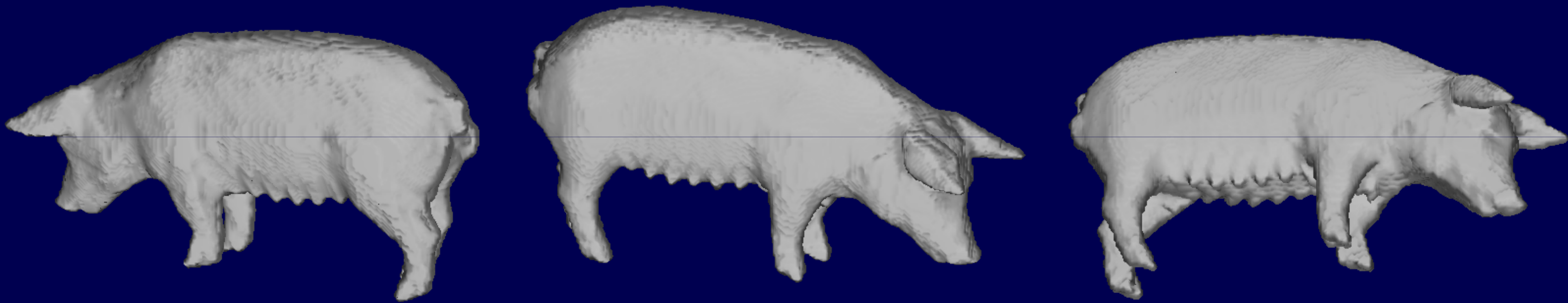
Reconstruction of Shiny Metal



propagation scheme



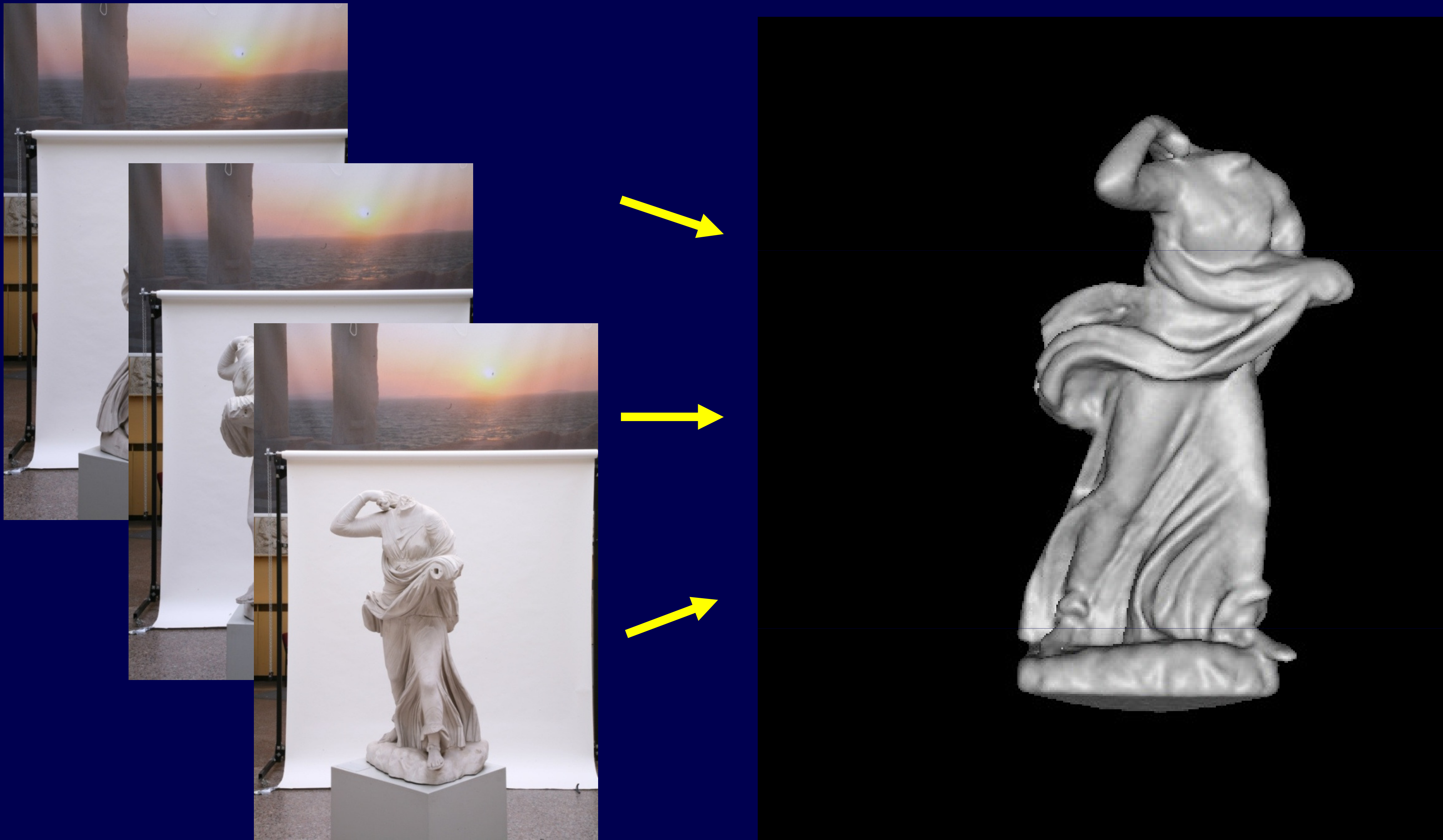
silhouette constraints



Reconstruction of Fine-scale Structures

Image data courtesy of Yasutaka Furukawa.

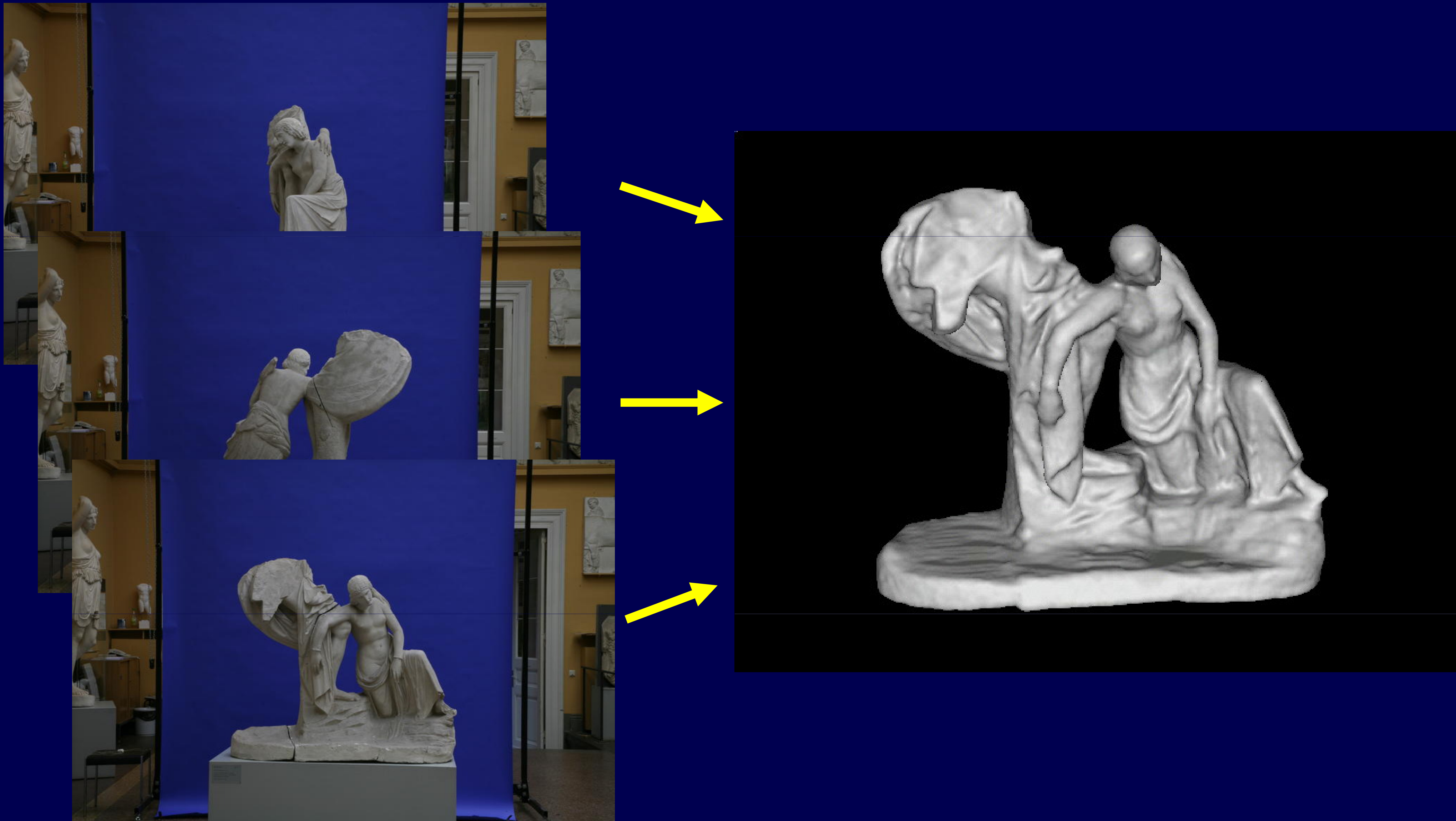
Reconstruction of Statues



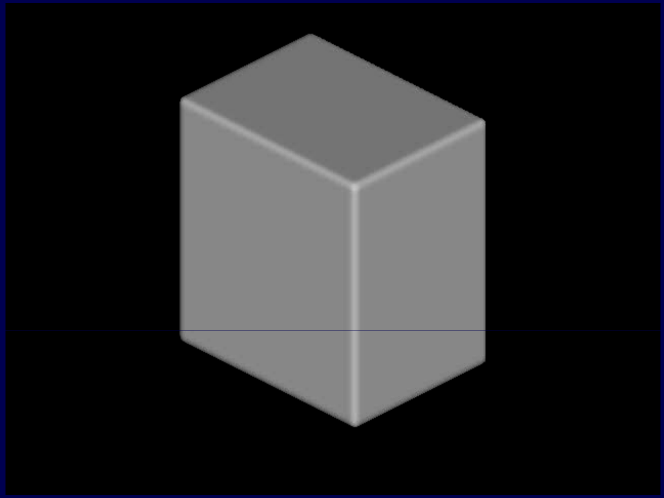
Kolev, Cremers, ECCV '08, PAMI 2010

Reconstruction of Statues

Kolev, Cremers, ECCV ’08, PAMI 2010



Kolev, Cremers, ECCV '08, PAMI 2010



Multiview reconstruction
via convex relaxation

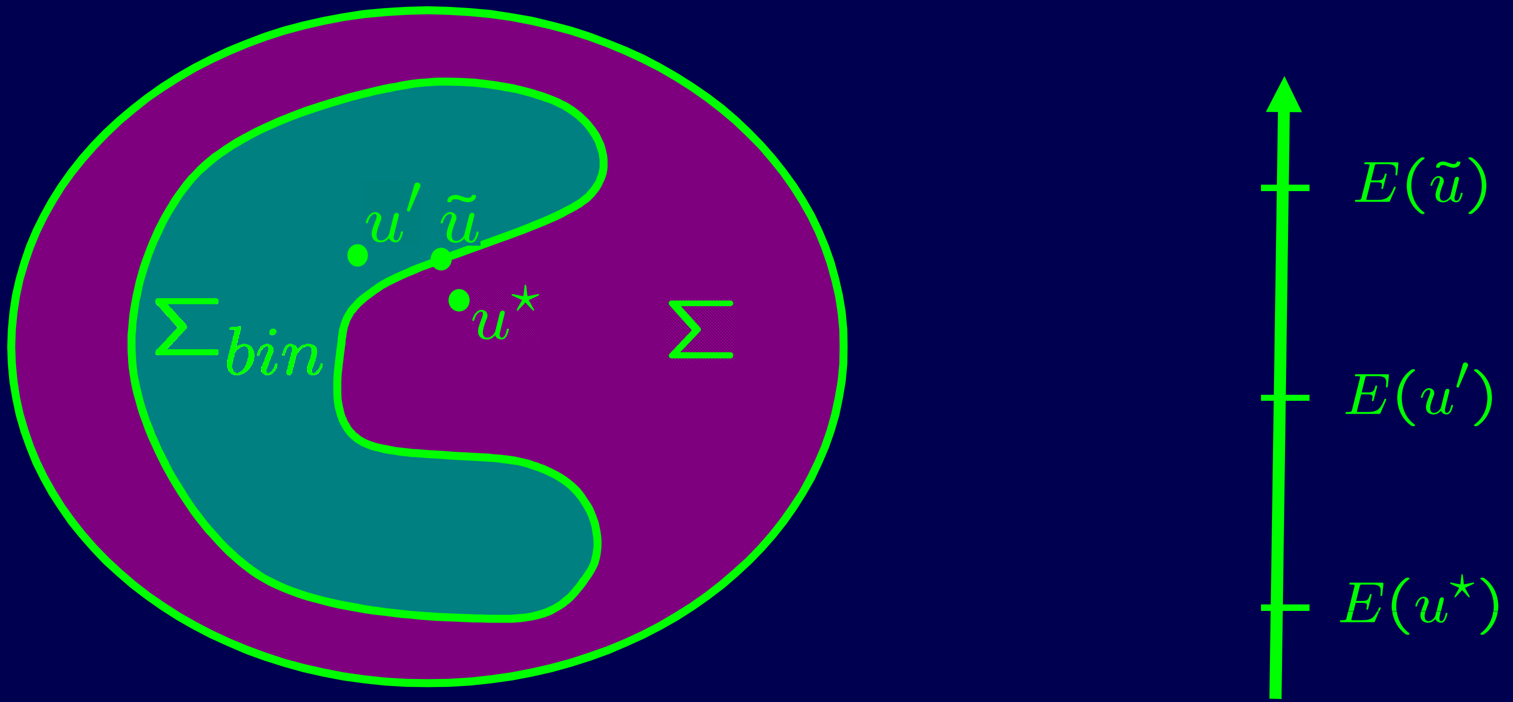
Single View Reconstruction

Superresolution textures

Stereo & silhouettes via convex
functionals over convex sets

Optimality Statement

$$u^\star = \arg\min_{u \in \Sigma} E(u) \qquad u' = \arg\min_{u \in \Sigma_{bin}} E(u)$$



$$E(\tilde{u}) - E(u') \leq E(\tilde{u}) - E(u^\star)$$

Kolev, Cremers, ECCV '08, PAMI 2010

Euler-Lagrange equation

$$\operatorname{div}\left(\rho\frac{\nabla u}{|\nabla u|}\right) = 0$$

linearization

$$g := \frac{\rho}{|\nabla u|}$$

$$\operatorname{div}(g\nabla u) = 0$$

discretization

$$\begin{pmatrix} \star & \star & & & & \star & & \\ \star & \star & \star & & & & \star & \\ & \star & \star & \star & & & & \star \\ & & \star & \star & \star & & & \\ & & & \star & \star & \star & & \\ \star & & & & \star & \star & \star & \\ & \star & & & & \star & \star & \star \\ & & \star & & & & \star & \star \end{pmatrix}$$

sparse system of
linear equations,
solved by
Successive Overrelaxation (SOR)

$$E(u) = \int_V \rho(x) |\nabla u(x)| \, dx$$

$$\text{s. t.} \qquad u : V \to [0,1]$$

$$\int_{R_{ij}} u(x)\, dx \geq 1 \;\; \text{if } j \in S_i$$

$$\int_{R_{ij}} u(x)\, dx = 0 \;\; \text{if } j \notin S_i$$

↓ thresholding

$$R^S_{obj} = \{x \in V \mid u(x) > \mu\}$$

$$R^S_{bck} = \{x \in V \mid u(x) < \mu\}\,, \text{ where}$$

$$\mu = \min\left\{\left(\min_{i\in\{1,\dots,n\}, j \in S_i} \; \max_{x \in R_{ij}} u^*(x)\right) \;,\; 0.5\right\}$$

Kolev, Cremers, ECCV '08, PAMI 2010

One of several input images

Super-resolution estimate

Schoenemann, Cremers, CVPR '08

Toeppe, Oswald, Rother, Cremers, ACCV 2010